Experimental Verification of Kirchhoff's Voltage Law and Kirchhoff's Current Law

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Abstract

We have experimentally tested Kirchhoff's Voltage Law and Kirchhoff's Current Law by measuring the sum of the voltage drops around several closed paths, and the sum of the currents at several nodes, in two resistive circuits. A low resistance circuit was constructed using resistors in the range of 1[kΩ], and a high resistance circuit was constructed using resistors in the range of 10[MΩ]. Kirchhoff's Current Law, which states that the algebraic sum of the currents at a node is zero, was found to be accurate to within 1% error. Kirchhoff's Voltage Law, which states that the algebraic sum of the voltage drops around a closed loop is zero, was found to be accurate to within 1% error when applied to the low resistance circuit, but when applied to the high resistance circuit gave errors of 10 to 20%. The reason for this discrepancy is not understood, but is believed to be related to the operation of the voltmeter when used to measure voltages across large resistances. With the exception of the voltage law applied to high resistance circuits, we conclude that Kirchhoff's Laws accurately predict the behavior of resistive circuits.

Introduction

The range of complexity exhibited by modern electronic systems is quite broad. One encounters electrical systems as simple as a switch that closes to ring a doorbell, or as complex as the central processing unit of a high speed computer. In many areas of electrical engineering, fundamental ideas that guide the understanding and design of electrical systems are surprisingly few. A notable example is that of circuit theory which, it may be argued, is founded on three simples laws: Ohm's Law, Kirchhoff's Voltage Law (KVL), and Kirchhoff's Current Law (KCL).

Ohm's Law and Kirchhoff's Laws place constraints on voltages and current within a circuit, thus providing important information about these variables. In particular, Ohm's Law provides a relationship between the voltage and current associated with a resistor, while KVL and KCL provide constraints on the sum of the voltages around a closed loop and the sum of currents at a circuit node. Ohm's Law and Kirchhoff's Laws are applied frequently in the analysis and design of electrical circuits. In complex circuits they may be cast into more sophisticated forms that disguise their simplicity, but they nevertheless provide a basis for the understanding of virtually all electrical systems. Since a great many working electronic circuits in present use were designed and analyzed using these laws, their validity is hard to question. Nevertheless, it is reasonable to explore their accuracy through experimentation.

The present work documents an experiment designed to test the validity of Kirchhoff's Laws. Ohm's Law was not examined here. To test KVL and KCL, two resistive circuits were constructed. The circuits were similar except that one was built with resistors in the range of 1[kΩ] and one with resistors in the range of 10[MΩ]. These two configurations were chosen because if was deemed important to test KVL and KCL under conditions in which typical circuit currents were on the order of mA as well as μA. In each circuit, measurements were made of the sum of the voltages around several closed paths and of the sum of the currents at several nodes. It was found that within experimental error, KCL successfully predicted the sum of the currents at a node in all configurations tested. However, KVL was found to be accurate only on the circuit containing resistors in the 1[kΩ] range and predicted poorly the voltages around two closed loops in the circuit containing resistors in the 10[MΩ] range. The results obtained in the high resistance circuit are not presently understood, but several possible explanations are explored in the *Discussion* section.

Research Question

Although we assume that Kirchhoff's Voltage and Current Laws are accurate, and that they apply to circuit theory that one encounters in an electrical engineering curriculum, we nevertheless wish to explore them in the laboratory. To that end, we pose the following research questions.

> To what extent can Kirchhoff's Voltage and Current Law be validated using standard, bench-top laboratory equipment? What errors arise in investigating KVL and KCL?

Background

The predictions of Kirchhoff's Laws are summarized in this section. A discussion of KVL and KCL may be found in most books on circuit theory [1].

Kirchhoff's Voltage Law states that "the algebraic sum of all the voltages *v* around any closed path in a circuit equals zero" [1], that is,

$$\sum_i v_i = 0, \tag{1}$$

where the sum is taken around a closed path. Kirchhoff's Current Law states that "the algebraic sum of all the currents *i* at any node in a circuit equals zero" [1], that is,

$$\sum_i i_i = 0, \tag{2}$$

where the sum is carried out at a particular node. The sums are illustrated in Figure 1, where a simple resistive circuit is shown along with one closed path and one node at which KVL and KCL may be evaluated. Included in the figure are equations for the KVL and KCL corresponding to the path and node in the figure.

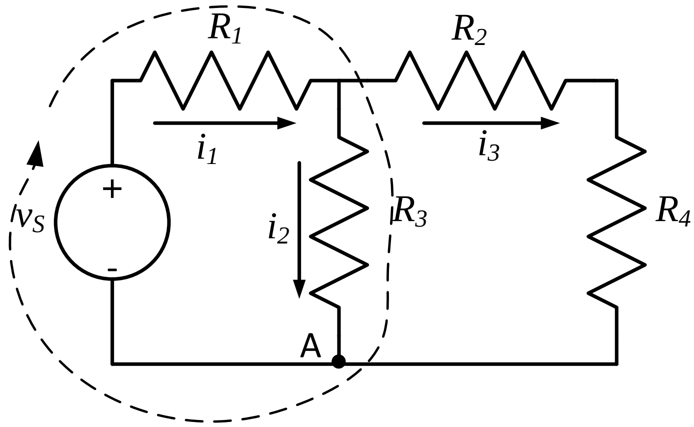

Figure 1. A simple circuit to illustrate KVL and KCL. A dotted line indicates a path along which we may apply KVL: $-v_S + i_1 R_1 + i_2 R_3 = 0$. Also, an essential node "A" is indicated, at which we may apply KCL: $-i_2 - i_3 + i_1 = 0$.

It is interesting to introduce simple physical interpretations of KVL and KCL. Kirchhoff's Voltage law arises from the nature of the electric field and the electrostatic force, which are conservative. Thus, the work done in moving a charged particle in the presence of a force depends only on where the charged particle starts and ends and not on the path in between. That the electric field is conservative allows us to write

$$\oint \bar{E} \cdot d\bar{l} = 0, \tag{3}$$

where E is the electric field and the integral is taken around a closed path. It can be shown that KVL follows from the application of Equation (3) to an electric circuit. [2].

Kirchhoff's Current Law is based on conservation of charge, and the notion that charge does not "pile up" at a node in a circuit. Current is a flow of charge; if the number of electrons entering the node were not equal to the number leaving it, charge would build up at the node. That would result in generation of electric fields that would resist further buildup of charge and tend to redistribute it away from the node. Further, conservation of charge implies that charge cannot leave the node by simply disappearing! KCL is a consequence of these ideas.

Methods

As part of the verification of Kirchhoff's laws, two resistive circuits were constructed on a breadboard using ¼ [W], 5% tolerance resistors. Where a voltage source was required, a Heath model 2918 tri-power supply was connected to the circuit. For voltage and current measurements, a Fluke model 8050A digital multimeter was used with standard multimeter probes.

Figure 2 illustrates the first of two circuits constructed for this experiment. [3] Shown in the figure are current nodes labeled A through D, X, and Y, as well as one of several possible closed paths. Current nodes A and B were used to evaluate Kirchhoff's Current Law, while the closed paths shown was one of several used to calculate Kirchhoff's Voltage Law. The values of the resistors and of the voltage source indicated in Figure 2 are nominal, that is, no measurements were taken of the actual values. As presented in the Discussion section, accurate knowledge of voltage and resistance values was not necessary to obtain accurate results.

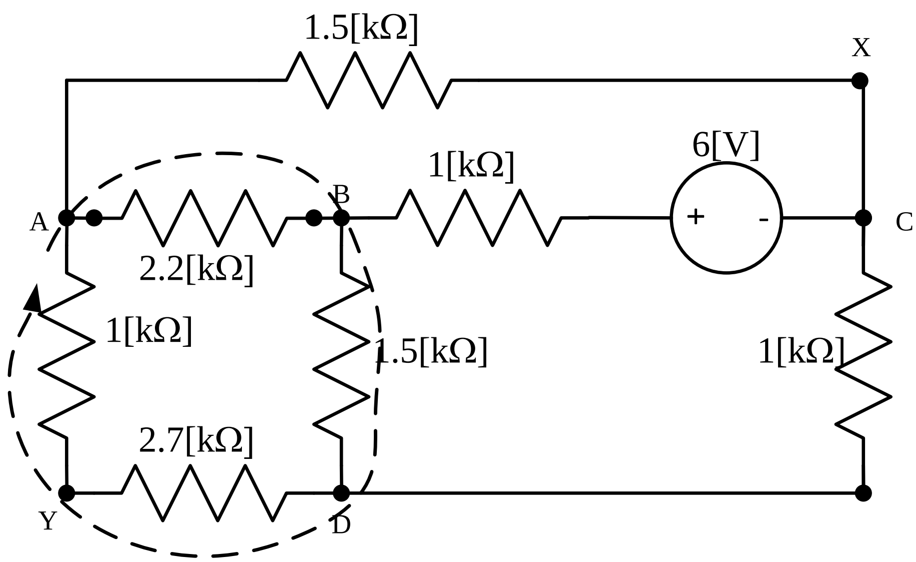

Figure 2. The first of two resistive circuits constructed for this work (taken from Reference 3).

The second of the two circuits is shown in Figure 3. [3] Four current nodes labeled A through D are shown, as is one of several possible closed paths. Again the indicated resistor and voltage values are nominal. The principle difference between the circuits of Figures2 and 3 is in the value of the resistors; the circuit of Figure 2 contains resistors in the [kΩ] range, and that of Figure 3 contains resistors in the [MΩ] range.

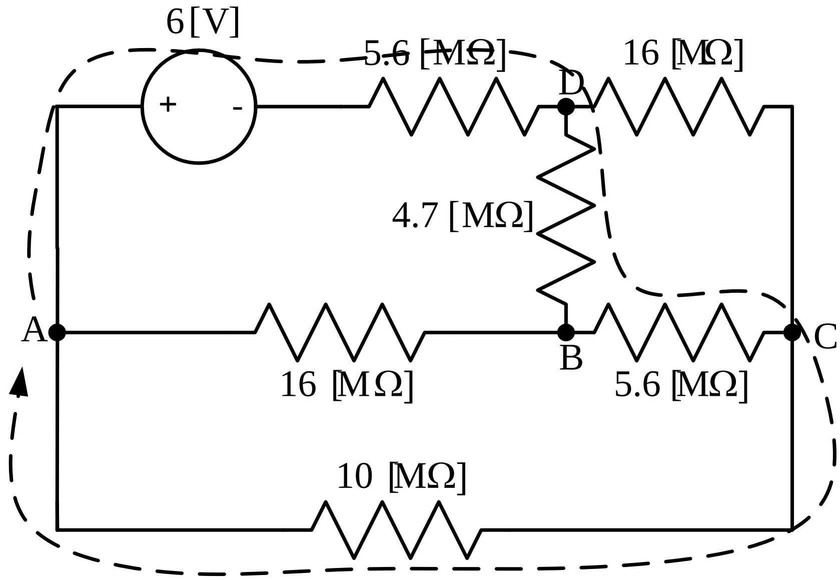

Figure 3. The second of two resistive circuits constructed for this work. The circuit contains larger resistors than those of Figure 2 (taken from Reference 3).

Kirchhoff's Voltage Law and Kirchhoff's Current Law were tested on the circuits shown in Figures 2 and 3. The test of KVL was performed by measuring voltage drops along closed paths indicated by pairs of nodes. For example, the voltage drop around the closed path shown in Figure 2 was taken as the sum of the voltage drops between nodes A and B, nodes B and D, and nodes D and A.

A test of KCL was made by disconnecting, one at a time, the end of each circuit element arriving at a particular node and measuring the current leaving that node through the element. This was done, however, only at "essential nodes", i.e., those at which at least three circuit elements were connected. The current leaving the node was measured by connecting the positive terminal of the ammeter to the node and the common terminal to the disconnected end of the appropriate circuit element. As an example, the current leaving node A of Figure 2 though the 2.2[kΩ] resistor was labeled i_{AB}.

Voltage measurements were made with the Fluke multimeter in the dc voltage mode, and typically at 20[V] full scale. Voltage readings were found to be stable to 0.1[mV], which was the resolution of the multimeter. Current measurements were made with the Fluke multimeter in the dc current mode, and typically at 200[mA] full scale, except that currents less than 0.2[mA] were measured at 2[mA] full scale. Current readings were found to be stable to the maximum precision of the multimeter, which was 0.001[mA] at the 2[mA] full scale range, and 0.01[mA] at the 200[mA] full scale range.

For this experiment, the percent error could not be reported as a fractional difference between a measured value and a reference value, because in this case the reference is 0 (it is the sum of the voltages around a closed path!). Therefore, the percent error E indicated in Table 1 was calculated by comparing the measured sum of voltage drops to the average of the absolute values of the voltage drops along the closed path, that is,

$$E = \frac{\sum v - 0}{\sum |v| / N} x 100\% . \quad (4)$$

Results

Table 1 reports the results of individual voltage drops, the sum of the voltage drops, and the percent error for each of several closed paths in the circuit of Figure 2. As described in the Methods Section, the closed paths were constructed of smaller paths denoted by pairs of nodes; these smaller paths are included in the table entries. The first row in Table 1, for example, corresponds to the closed path drawn in Figure 2. Measurements on other closed paths are similarly indicated.

Table 1. Tabulation of voltage drops around several closed paths for the circuit of Figure 2.

Path: voltage drop [V]				Σv [V]	% error
AB: 2.211	BD: -2.175	DA: -0.037	---	-0.001	-0.07
AC: -1.418	CD: 1.456	DA: -0.037	---	0.001	0.10
CB: 3.642	BD: -2.185	DA: -0.037	AC: -1.418	0.002	0.11
AX: -1.418	XY: 1.429	YA: -0.010	---	0.001	0.01

As an example, consider the first row of data in Table 1. The sum of the voltage drops is -0.001[V] and the average of the absolute values of the voltage drops AB, BD, and DA is 1.47[V]; thus, the error is (-0.001 – 0)/1.47 x 100% = -0.07%.

Note that the last row in Table 1 corresponds to a closed path that does not go directly through circuit elements. This path was chosen specifically to demonstrate that the closed path around which Kirchhoff's Voltage Law is evaluated need not follow circuit elements directly.

Kirchhoff's Current Law was evaluated at nodes A and B of the circuit shown in Figure 2. The results of the individual current measurements, as well as the sum of the currents and the percent error, are shown in Table 2. The notation preceding each current measurement follows the convention described in the Methods Section. Thus, the first row in Table 2 gives the measurement of currents leaving node A through each of the three branches; the second row gives the currents leaving node B.

As in Table 1, percent errors in Table 2 were evaluated by comparing the sum of the currents with the average of the absolute values of the currents at a node.

Table 2. Tabulation of currents leaving several nodes for the circuit of Figure 2.

branch: current [mA]			Σi [mA]	% error
AB: 0.944	AD: 0.0085	AC: -0.953	0.00	0.00
BA: -0.943	BD: -1.406	BC: 2.388	0.039	2.5

Table 3. Tabulation of voltage drops around several closed paths for the circuit of Figure 3.

Path: voltage drop [V]				Σv [V]	% error
AC: -1.400	CD: -1.161	DA: 2.755	---	0.194	10.9
AD: -2.748	DB: 0.874	BC: 0.408	CA: 1.402	-0.066	-4.87

Table 4. Tabulation of currents leaving several nodes for the circuit of Figure 3.

branch: current [mA]			Σi [mA]	% error
AB: -0.21	AD: 0.35	AC: -0.15	-0.01	-4.22

Measurement of the voltage drops around two closed paths in the circuit of Figure 3 are given in Table 3. Table 4 shows the results of current measurements at node A of Figure 3. Note that the currents reported in Table 4 are in [μA] and not [mA] as in Table 2. Tabulation of percent errors was performed as in Tables 1 and 2.

Discussion

In general, the results obtained in this experiment were as expected from consideration of circuit theory. The results of measurements on the "low resistance" circuit of Figure 1, which were given in Tables 1 and 2, agree well with the predictions of KVL and KCL. The sums of voltage drops around three closed paths were no more than 2[mV] in magnitude, giving errors on the order of 0.1%; the sum of currents at a node was 0 in one case and only 39[mA] (2.47% error) in another.

The results of measurements on the "high resistance" circuit of Figure 3, given in Tables 3 and 4, were not as easy to interpret. The percent error in the current measurement was higher than that for the low resistance circuit, perhaps because typical currrent in the high resistance circuit was small. However, the absolute error was quite small (-0.01[mA]) and we feel justified in suggesting that KCL was verified in this case. Interestingly, a discrepancy between theory and experiment for the high resistance circuit arose in the KVL measurement. There we observed absolute and percent errors much larger than in any other measurement, so that it is not completely clear that KVL was verified. The reason for this discrepancy is not understood, but we consider possible sources of error in the following discussion.

To check whether we had made a simple error in connecting the voltmeter to the circuit or in reading the meter, we performed both KVL measurements on the high resistance circuit a second time and performed the first measurement in Table 3 a third time. This simple test of reproducibility gave identical results in almost all cases; the voltage drop across two particular resistors changed by 0.002[V] in one case and by 0.001[V] in another. Although these new results changed the error slightly, the change was not nearly sufficient to account for the large errors indicated in Table 3.

We observed during the course of these measurements that the value of the voltage source was “drifting”, i.e., changing slowly with time. The rate of drift was quite small (perhaps 0.05[V] over several minutes), so that the voltage source changed negligibly during the course of a single KVL or KCL measurement. Therefore, drift in the voltage source is probably not responsible for the large errors in Table 3.

Another source of error in electronic measurements is uncertainty in the values of resistors and voltage sources. However, the present results could not have been affected by this problem. Because the sums of voltage drops and currents are 0, precise values of individual resistances and voltage sources are not relevant. Had we substituted, for example, a resistor or voltage source for one twice is value, individual voltage drops and currents would have changed, but the sum of voltages around closed paths and of currents at a node should have remained 0. We therefore conclude that the 5% tolerance of the resistors used in this experiment had no bearing on the results. In addition, we note that the systematic error introduced by the voltmeter and current meter is considerably smaller (less than 0.1%) than typical errors encountered here. Further, such errors would have been present in all measurements to a similar degree. Thus we do not suspect that systematic error in the voltage and current measurements is responsible for the discrepancies in the voltage measurements in the high resistance circuit.

The only likely remaining source of error in Table 3 is the voltmeter itself. We checked for drift in the reading by keeping the probes connected to the circuit for about one minute, but did not notice a significant change. We therefore suspect that the errors in Table 3 are due to details in the operation of the voltmeter when large resistances are present. We are presently planning new experiments to test this idea.

Conclusion

We have tested the predictions of Kirchhoff's Voltage and Current Laws by measuring the sum of voltages around several closed paths, and the sum of currents at several nodes in two resistive circuits. The "low resistance" circuit was built using resistors in the range of 1[kΩ], and the "high resistance" circuit was built using resistors in the range of 10[MΩ]. Measurements on the low resistance circuit gave voltage and current sums very close to zero, and thus conformed to the predictions of Kirchhoff's Laws.

Measurements on the high resistance circuit conformed to KCL; however, the sum of voltages around two closed paths in the circuit gave significant errors. We investigated several possible sources of error, but could not account for the discrepancies observed. We suspect that these errors arise from operation of the voltmeter where large resistances are present, but we do not know in detail what is causing the error.

Aside from the question of voltage measurements across large resistors, we conclude that Kirchhoff's Voltage and Current Laws accurately predict the sum of the voltage drops around a closed path and the sum of the currents at a node in the resistive circuits examined here. Further, because of the arbitrary nature of the circuits investigated here, we feel confident in concluding that in fact KVL and KCL accurately predict the behavior of resistive circuits.

References

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