Name: ____ **(Print)**

Signature ____

Date: _____________

ECE 2300 -- Quiz #5

S.R. Brankovic Section – MW 11:30 AM

November 21, 2005

KEEP THIS QUIZ CLOSED AND FACE UP UNTIL YOU ARE TOLD TO BEGIN.

1. This quiz is closed book, closed notes.
2. Show all work on these pages. Show all work necessary to complete the problem. A solution without the appropriate work shown will receive no credit. A solution which is not given in a reasonable order will lose credit. **If your work continues on to another page, indicate clearly where your work can be found. Failure to indicate this clearly will result in a loss of credit.**
4. Show all units in solutions, intermediate results, and figures. Units in the quiz will be included between square brackets.
5. Do not use red ink. Do not use red pencil.
6. You will have 25 minutes to work on this quiz.

_________________/100 %

Problem #1.

In Figure 1 shown below the circuit with inductive element is presented. For the time domain t < 0, the switch A is closed and switch B is opened for a long time. At t = 0, the switch A opens and stays open and at t=0.1 [s] switch B closes and stays closed. Find the value of the resistor ***R***, for which current i_x at t=0.2 [s] equals 1% of the current i_o supplied by the independent current source.

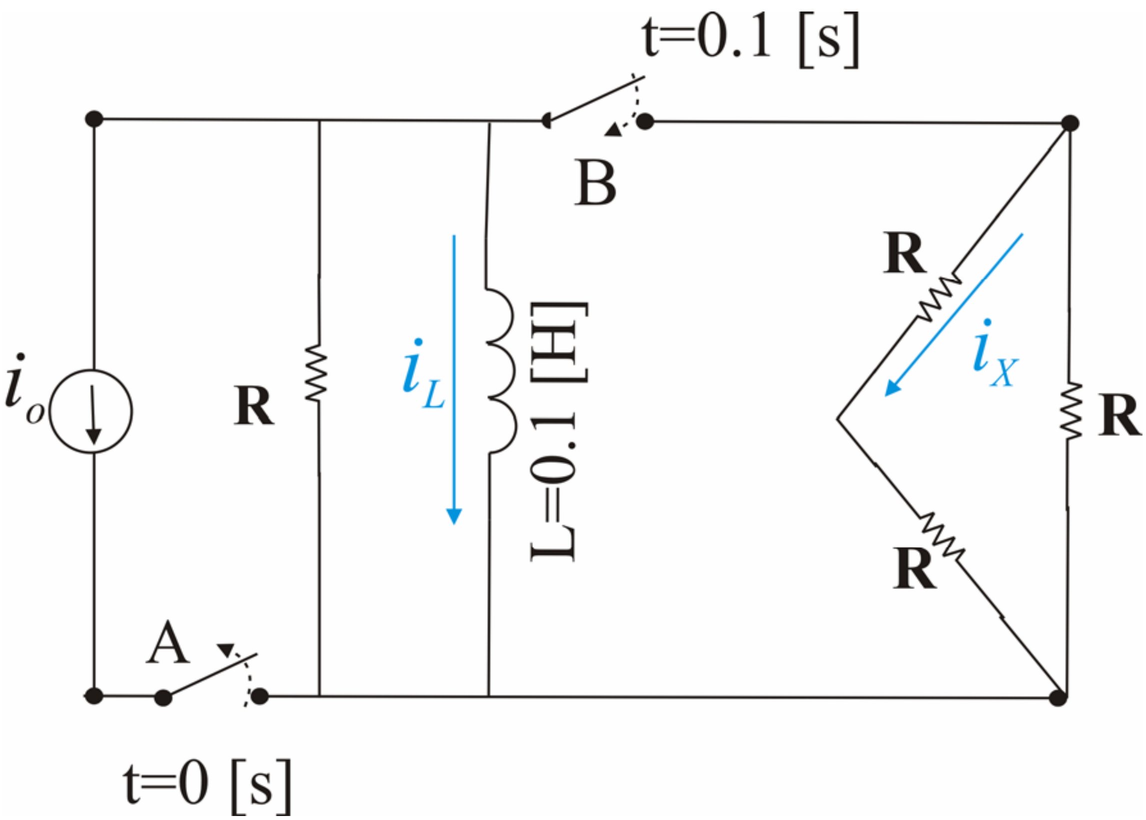

Figure 1.

Solution

After taking a deep breath, and overcoming the initial anxiety, we look the circuit in Figure 1, and just proceed with doing the same steps as we did many times in the class. So the first one is to look the circuit for the time domain $t < 0$, and redraw it if necessary, see Figure 2A.

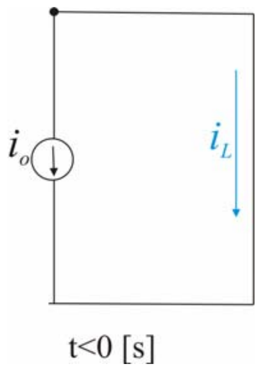

Figure 2A

We are looking the situation at $t < 0$ that was in the circuit **for a long time.** One realizes that the resistor is in parallel with the inductor that has been in steady state. The resistor is considered as shorted so it can be excluded from the analysis. Also, one sees that the inductor for this situation is presented as the short, or just the line. It is straight forward that:

$$i_L(t < 0) = -i_o \tag{1}$$

We know that the current through the inductor can not change its value abruptly and that at $t = 0$ when the switch A is opened, the following expression must be valid;

$$\boxed{i_L(0) = -i_o} \tag{2}$$

Obviously we just defined our initial condition for the time domain $0 \le t \le 0.1[s]$. Let's look he circuit for this new situation, Figure 2B.

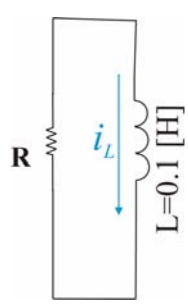

Figure 2B, Circuit for $0 \le t \le 0.1$ $[s]$ domain.

Here we can recognize the circuit with a natural response, and for this situation we know that the inductive current is given by:

$$i_L(t) = i(0) \cdot e^{-\frac{t}{L/R}}, \qquad (3)$$

and substituting the value for *i(0)* from equation (2) we get;

$$\boxed{i_L(t) = -i_o \cdot e^{-\frac{t}{L/R}}} \qquad (4)$$

Now we have to evaluate the equation (4) at *t = 0.1 [s]* moment since this will be our initial current value for the time domain *t ≥ 0.1 [s]*. i.e. when the switch B closes.

$$\boxed{i_L(t_0) = -i_o \cdot e^{-\frac{t_0}{L/R}}; \Rightarrow i_L(0.1[s]) = -i_o \cdot e^{-\frac{0.1[s]}{L/R}} = -i_o \cdot e^{-R\cdot 1[\Omega^{-1}]}} \qquad (5)$$

The circuit for this situation is presented in Figure 2C

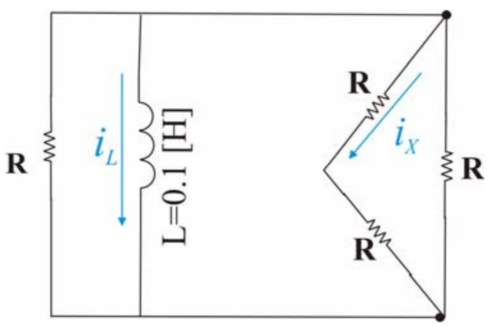

Figure 2C . Circuit from Figure 1 for *t ≥ 0.1 [s]*.

We can consider the situation for *t ≥ 0.1 [s]* as the last stage in our sequential switching problem where the circuit still obeys a natural response type of solution. So, it follows that ;

$$i_L(t \ge t_0) = i_L(t_0) \cdot e^{-\frac{t-t_0}{L/R_{EQ}}}. \qquad (6)$$

Here the L/R_{EQ} is the new time constant for the circuit shown in Figure 2C. Now we have to define the equivalent resistance of the circuit in Figure 2C as it is seen form the terminals of the inductor *L= 0.1 [H]*. If we look closer to the Figure 2C we will realize that:

$$\boxed{R_{EQ} = R \,||\, \{R \,||\, (R+R)\} = 0.4R} \qquad (7)$$

After substituting equation (7) into equation (6) and for the initial value of current i_L at $t = 0.1[s]$ expressed by equation (5) the expression for i_L for time domain $t \geq 0.1\ [s]$ is:

$$i_L(t \geq 0.1[s]) = -i_o \cdot e^{-R \cdot 1[\Omega^{-1}]} \cdot e^{-\frac{t-0.1[s]}{L/0.4R}} \tag{8}$$

After substituting the value for L, (0.1 [H]) and looking the value of i_L at $t = 0.2[s]$ the final expression for current is i_L at $t = 0.2\ [s]$ is;

$$\boxed{i_L(0.2[s]) = -i_o \cdot e^{-1.4R \cdot 1[\Omega^{-1}]}} \tag{9}$$

Now we are ready to find the value of current $i_x(0.2[s])$ in terms of the current $i_L(0.2[s])$. *A*fter applying the current divider rule twice (Figure 2C), the expression for $i_x(0.2[s])$ is, Figure 2C;

$$\boxed{i_x(0.2[s]) = -i_L(0.2[s]) \cdot \frac{R}{R + \frac{2}{3}R} \cdot \frac{R}{R + 2R} = -0.2 \cdot i_L(0.2[s])} \tag{10}$$

If we substitute equation (9) into (10), and using the condition that $i_x(0.2[s])/i_o = 0.01$ we get logarithmic equation that we solve easily for R, and that is **THE END**.

$$\boxed{i_x(0.2[s]) = 0.2 \cdot i_o \cdot e^{-1.4R \cdot 1[\Omega^{-1}]} \Rightarrow R = 2.14[\Omega]} \tag{11}$$