Problem #3. (20 points)

For the circuit shown in Figure 3 find:

a. The Thevenin equivalent at terminals **a** and **b**. Draw the Thevenin equivalent and clearly indicate terminals **a** and **b**.

b. If a resistor with a value of **R = 2.228** [kΩ] is placed across terminals **a** and **b**, what is the new $\mathbf{R_{th}}$ at terminals **a** and **b**?

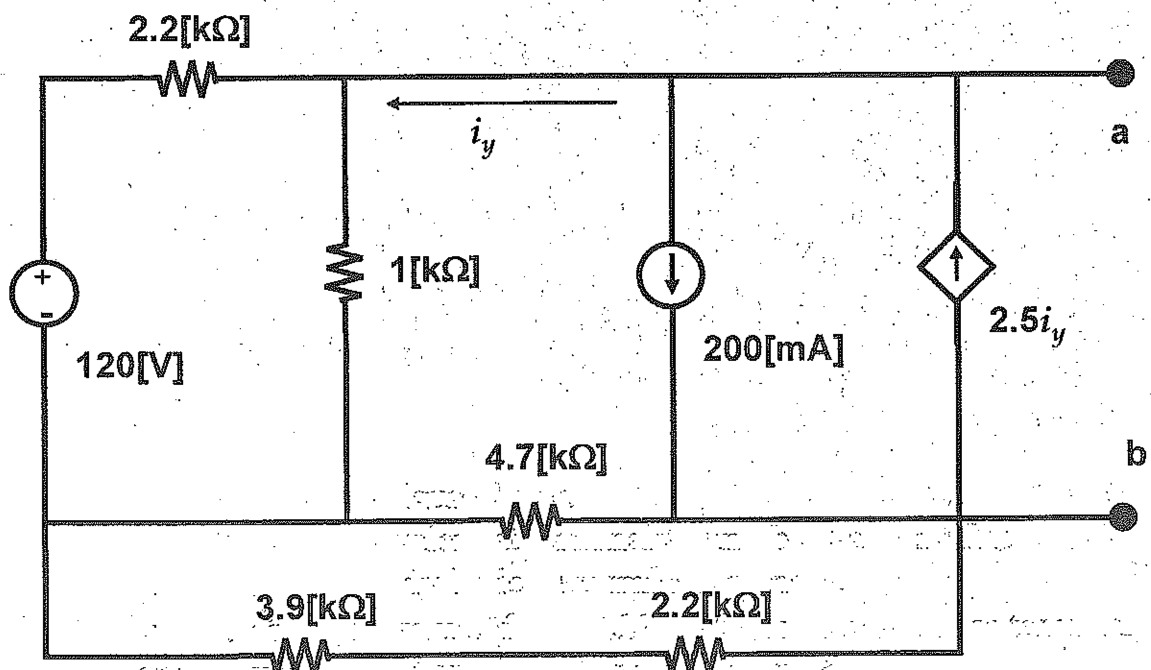


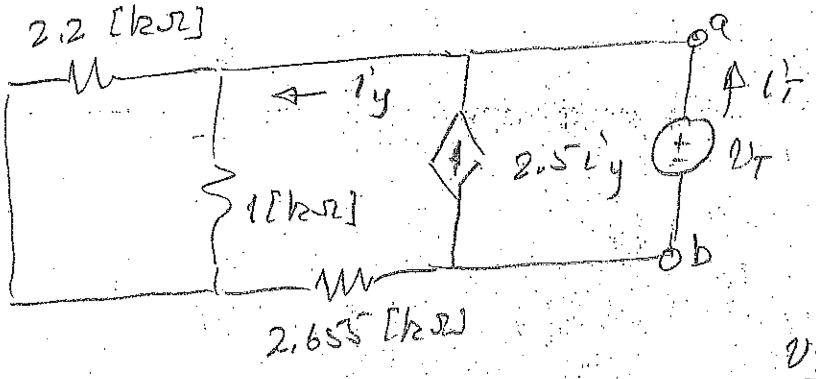
Figure 3

a) We begin with R_{TH}: place a test source at a, b, combine $(3.9[k\Omega] + 2.2[k\Omega]) // 4.7[k\Omega]$, and deactivate the 120 [V] and 200 [mA] sources. We get

→

Room for extra work

$(R_{eq} = 3343\ [\Omega])$



Now $\quad i_T' = i_y' - 2.5 i_y' \qquad i_y' = \dfrac{v_T \times 10^{-3}}{2.655 + (2.2 \parallel 1.0)}$

$$i_T' = -1.5 i_y' = -1.5\, v_T\, (2.992 \times 10^{-4}) = -0.4488\ [mA] \text{ if } v_T = 1\,[V]$$

$$\therefore \frac{v_T}{i_T'} = \frac{-1}{1.5(2.992 \times 10^{-4})} = -2228\ [\Omega]$$

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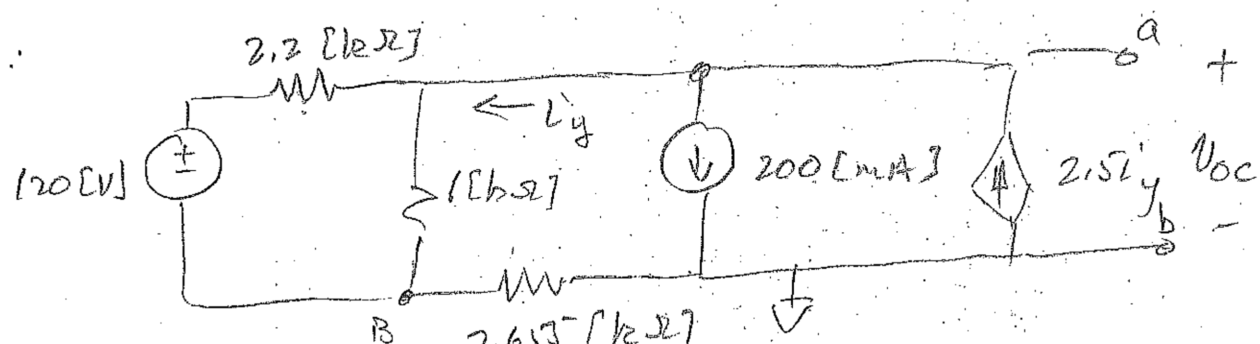
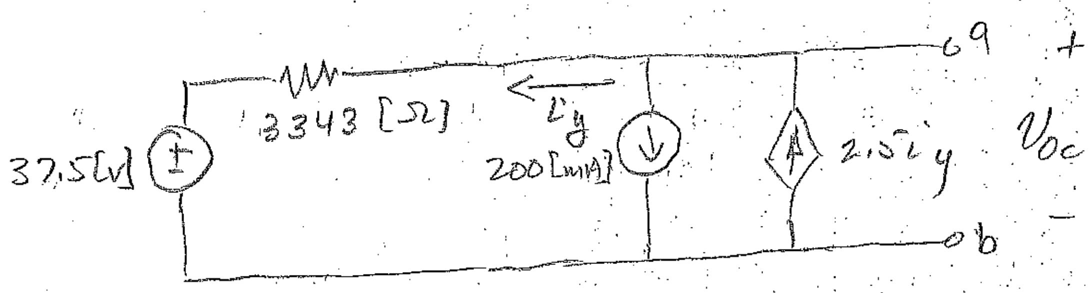
v_{OC}:



Two source transformations simplifies this to:



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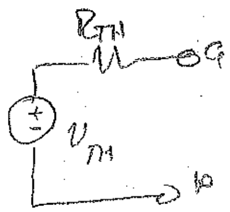
v_{TH} + 8

R_{TH} + 8

R_{TH}' + 2



+2

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KCL $\quad -2.5 i_y' + 0.2 + i_y' = 0 \Rightarrow i_y' = 0.1333$ [A]

KVL $\quad -37.5 - i_y'(3343) + V_{oc} = 0 \Rightarrow V_{oc} = 483.1$ [V]

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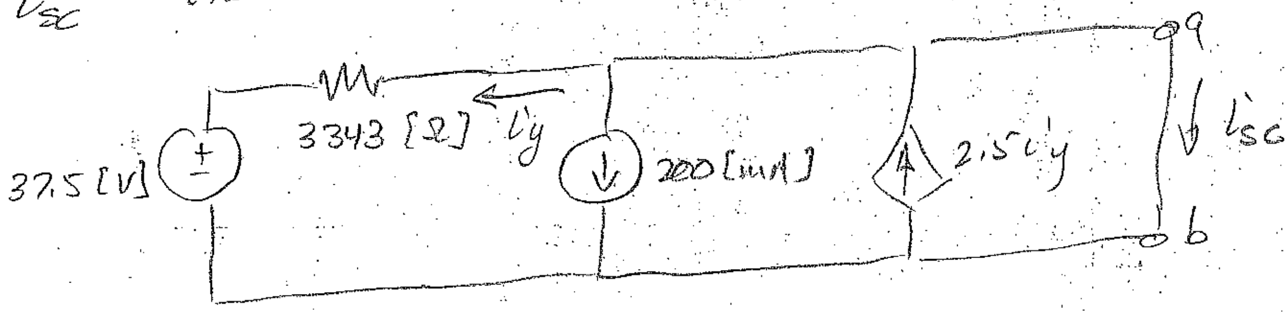
i_{sc} (not needed but we do it to check...)

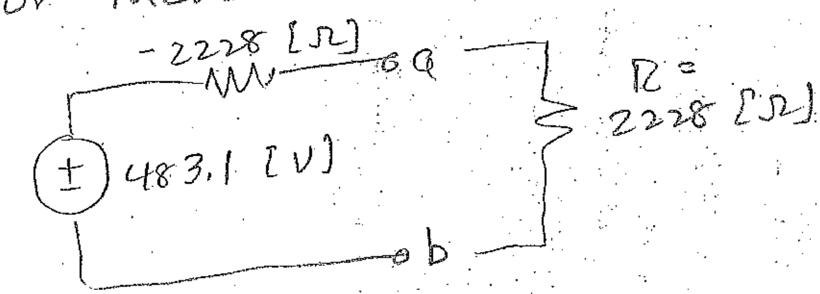


$$i_y' = \frac{-37.5}{3343} = -11.22 \text{ [mA]}$$

$$i_{sc}' - 2.5 i_y' + 0.2 + i_y' = 0 \Rightarrow i_{sc}' = -0.2168 \text{ [A]}$$

$$\therefore R_{TH} = V_{oc}/i_{sc} = \frac{483.1}{-0.2168} = -2228 \text{ [Ω]}$$ ✓

So our Thevenin Equivalent is...



b) with $R = 2228$ [Ω] at a, b, the new R_{TH} is

$$R_{TH}' = 2228 // -2228 = \infty \; !$$

The switch was in position 'a' for a long time, and moved to b at $t=0$.

Problem 4 (25 points)
For the circuit shown in Figure 4, at **t < 0**, it is known that $i_1 = i_2$. Find $i_2(t)$ for **t > 0**.

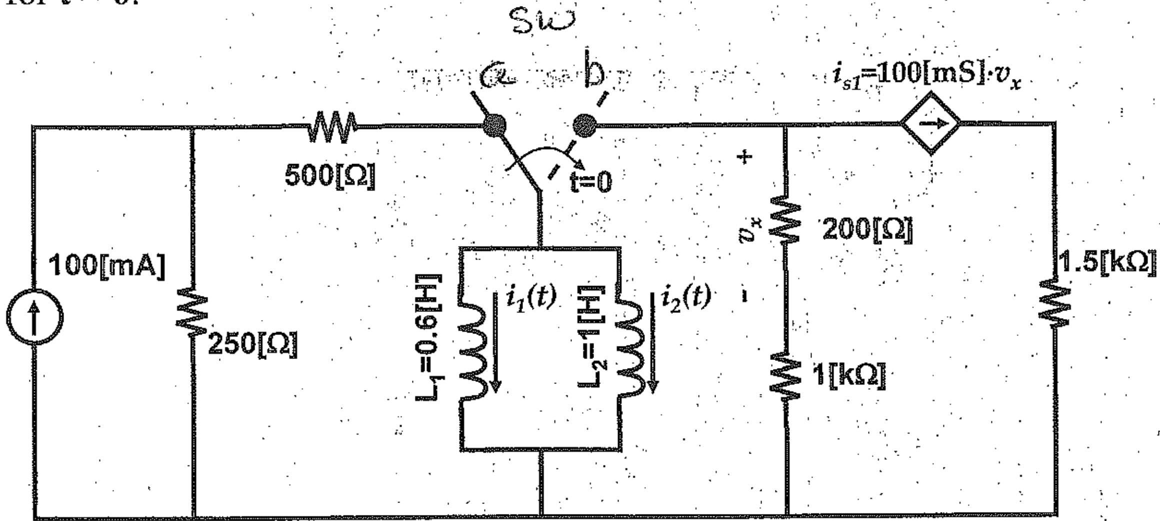
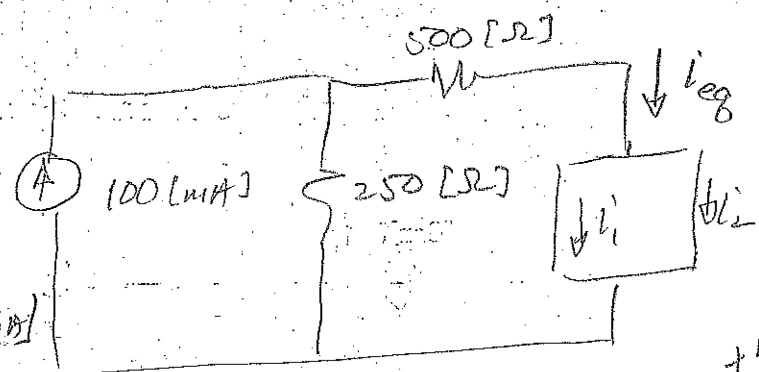


Figure 4

① Initial conditions:

$L_1 \to$ short
$L_2 \to$ short



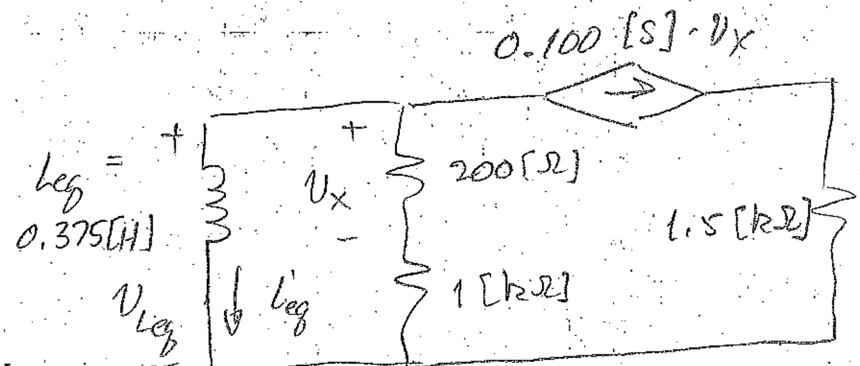
$$i_{eq} = 0.1 \times \frac{250}{250+500} = 33.33\,[mA]$$

+4

$$\therefore\ i_1 = i_2 = 16.67\,[mA] = i_1(0^-) = i_1(0^+) = i_2(0^-) = i_2(0^+)$$

Re-draw for $t>0$. Use $L_{eq} = \dfrac{L_1 L_2}{L_1 + L_2} = 0.375\,[H]$

$i_{eq}(0^+) = 33.33\,[mA]$

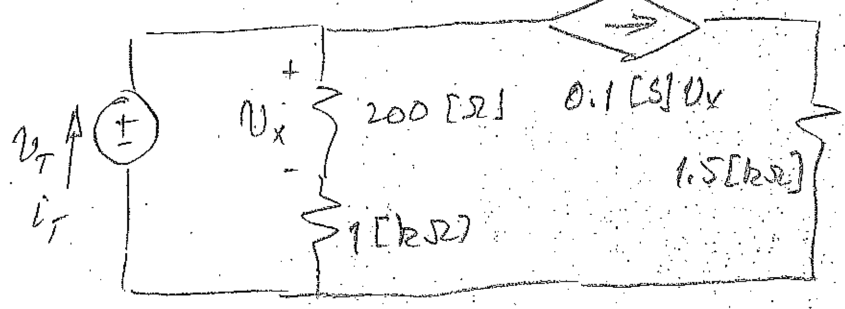


Room for extra work

con't

Room for extra work

⑥ Find τ_L : we need R_{TH} seen by L_{eq}:



$$v_x = v_T \frac{200}{200+1000} = \frac{1}{6} v_T$$

$$i_T = \frac{v_T}{1200} + 0.1 v_x = \frac{v_T}{1200} + 0.1 \frac{v_T}{6} = 17.5 \times 10^{-3} v_T$$

+5, +2 $$\underbrace{R_{TH} = \frac{v_T}{i_T} = 57.14 \text{ [Ω]}}_{} \Rightarrow \underbrace{\tau_L = \frac{L}{R} = 6.563 \text{ [ms]}}_{}$$

③ Final condition: no independent sources

+2 $$\Rightarrow i_{L_{eq}}(\infty) = 0.$$

+4 $$i_{eq}(t) = i_{eq}(\infty) + (i_{eq}(0^+) - i_{eq}(\infty)) e^{-t/\tau_L}$$

$$= 0 + (33.33 - 0) e^{-t/6.563[ms]} \text{ [mA]} \quad t \geq 0 \text{ [ms]}$$

+2 $$v_{eq}(t) = L_{eq} \frac{di_{eq}}{dt} = 0.375 (0.03333)\left(\frac{-1}{6.563 \times 10^{-3}}\right) e^{-t/6.563[ms]}$$

$$= -1.904 e^{-t/6.563[ms]} \text{ [V]} \quad t > 0 \text{ [ms]}$$

+6 We now integrate to get $i_2(t)$:

$$i_2(t) = \frac{1}{L_2} \int_0^t v_{eq}(t)\, dt + i_2(0^+) = \frac{1}{1[H]} \int_0^t (-1.904) e^{-t/6.563[ms]} dt + 0.01667$$

$$i_2(t) = [12.5 (e^{-t/6.563[ms]} - 1) + 16.67] \text{ [mA]} \quad t \geq 0 \text{ [ms]}$$

Problem #6 (20 points)
For the circuit shown in Figure 6, the loads $\mathbf{L_1}$, $\mathbf{L_2}$, and $\mathbf{L_3}$ are described as follows:
$\mathbf{L_1}$ is a resistance **R** in series with a reactance of ~~[illegible]~~ $+j39.2$ [Ω],
$\mathbf{L_2}$ is absorbing **2921.4 [W]** at a **pf** of **0.832 lagging**,
$\mathbf{L_3}$ is absorbing **2482.7 [VA]** at a **pf** of **0.196 leading**.
Find:

a. The value of **R**.
b. The **total reactive power** delivered by the source.

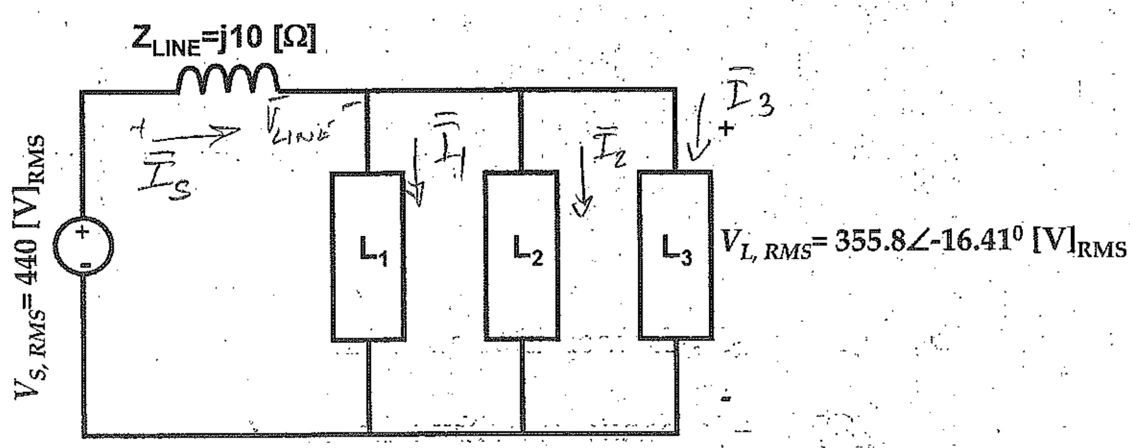


Figure 6.

$$\bar{I}_S = \frac{440 - 355.8\angle{-16.41^\circ}}{j10} = \begin{cases} 14.12\angle{-44.76^\circ} \\ 10.03 - j9.946 \end{cases} [A]_{rms}$$

$$S_{del\ by\ V_S} = \bar{V}_S \cdot \bar{I}_S^* = 6194.5\angle{44.46^\circ}\ [VA] = 4421 + j4339$$

The real power delivered by V_S is 4421 [W]. This must be equal to the real power absorbed by the three loads together.

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$$\bar{V}_{LINE} = j10\,\bar{I}_S = \left.\begin{matrix} 99.42 + j100.3 \\ 141.2\angle{45.2^\circ} \end{matrix}\right\} [V]_{rms}$$

Room for extra work

LOAD 2 : $|S_{L2}| = \dfrac{2921.4}{0.832} = 3511.3$ [VA] $\theta = 33.7^\circ$

$pf = 0.832 \Rightarrow rf = \sqrt{1-(0.832)^2} = 0.5548$

lag

$\Rightarrow S_{L2} = 2921.4 + j\,3511.3(0.5548) = \left.\begin{matrix} 2921.4 + j\,1948.1 \\ 3511.3\angle 33.70^\circ \end{matrix}\right\}$ [VA]

LOAD 3 $\theta = 78.7^\circ$

$pf = 0.196 \Rightarrow rf = \sqrt{1-(0.196)^2} = 0.9806$

lead

$\Rightarrow S_{L3} = 2482.7(0.196) - j\,2482.7(0.9806)$

$= \left.\begin{matrix} 487.5 - j\,2434.5 \\ 2482.8\angle -78.68^\circ \end{matrix}\right\}$ [VA]

Now $\bar{I}_2^* = \dfrac{S_{L2}}{\bar{V}_L} = \left.\begin{matrix} 9.869\angle 50.11^\circ \\ 6.329 + j\,7.572 \end{matrix}\right\}$ $[A]_{rms}$

$\bar{I}_3^* = \dfrac{S_{L3}}{\bar{V}_L} = \left.\begin{matrix} 6.978\angle -62.27^\circ \\ 3.247 - j\,6.176 \end{matrix}\right\}$ $[A]_{rms}$

$\bar{I}_1 = \bar{I}_S - \bar{I}_2 - \bar{I}_3 = \left.\begin{matrix} 8.559\angle -86.98^\circ \\ 0.4500 - j\,8.547 \end{matrix}\right\}$ $[A]_{rms}$

a) $Z_1 = \dfrac{\bar{V}_L}{\bar{I}_1} = \left.\begin{matrix} 41.57\angle 70.57^\circ \\ 13.83 + j\,39.20 \end{matrix}\right\}$ $[\Omega]$ $\Rightarrow \underline{R = 13.8\ [\Omega]}$

b) $Q_{del,s} = 4339$ [VAR] from previous page