Name: Solutions (please print)

Signature: ______________________________

ECE 2201 – Final Exam
December 6, 2017

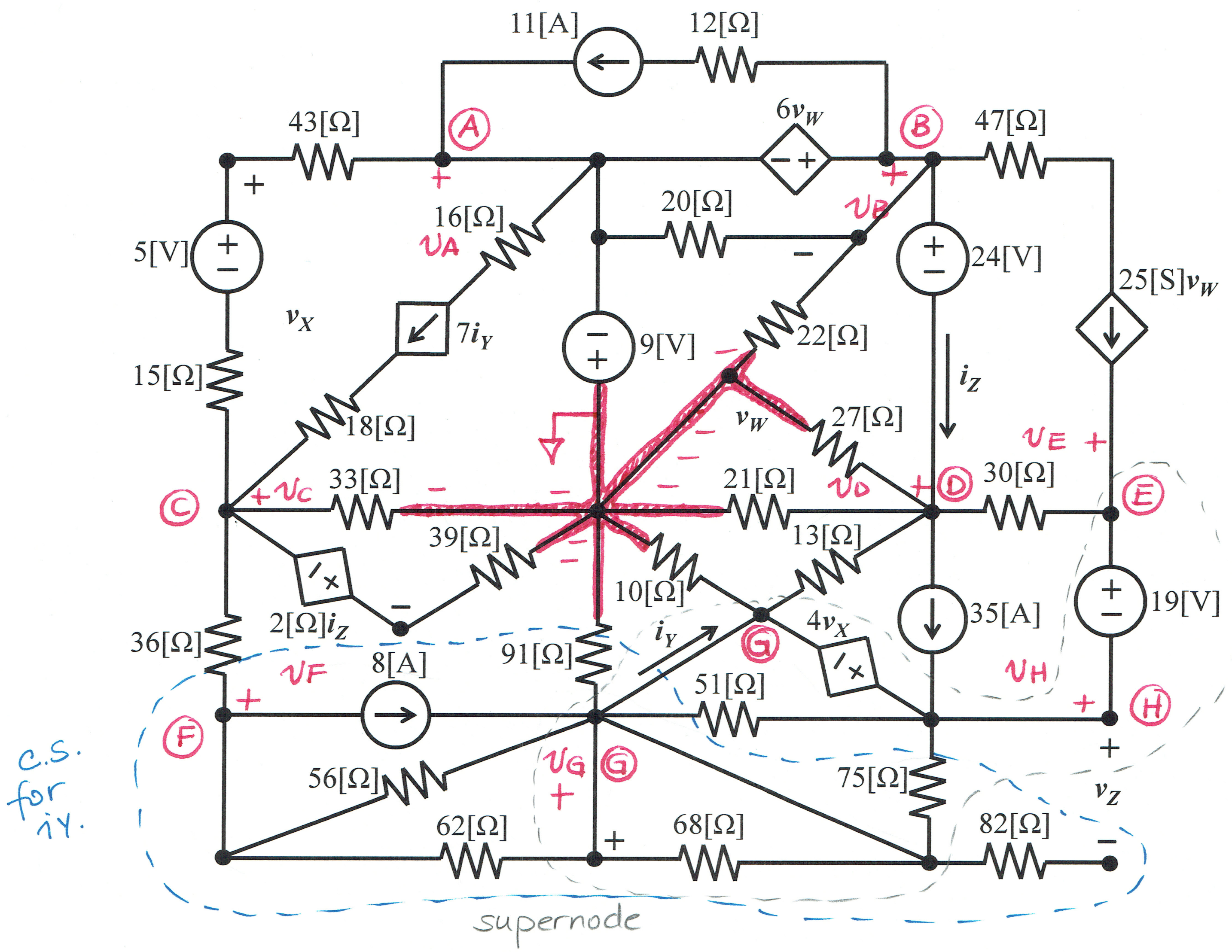
Keep this exam closed and face up until you are told to begin.

1. This exam is closed book, closed notes. You may use one 8.5" x 11" crib sheet, or its equivalent.
2. Show all work on these pages. Show all work necessary to complete the problem. A solution without the appropriate work shown will receive no credit. A solution which is not given in a reasonable order will lose credit.
3. Show your work clearly. If the grader cannot follow or understand your work, you will lose credit.
4. Show all units in solutions, intermediate results, and figures. Units in the quiz will be included between square brackets.
5. Do not use red ink. Do not use red pencil.
6. You will have 170 minutes to work on this exam.

1.	________	/30
2.	________	/30
3.	________	/35
4.	________	/35
5.	________	/35
6.	________	/35

Total = 200

1. {30 Points} Use the node-voltage method or the mesh-current method to write a complete set of equations that could be used to solve this circuit. Do not simplify the circuit. Do not attempt to simplify or solve your equations. **Define all your variables clearly.**



(A) $v_A = -9[V]$

(A+B) $v_B - v_A = 6v_W$

(B+D) $v_B - v_D = 24[V]$

(C) $$\frac{v_C - v_A + 5[V]}{(43+15)[\Omega]} - 7i_Y + \frac{v_C}{33[\Omega]} + \frac{v_C + 2[\Omega]i_Z}{39[\Omega]} + \frac{v_C - v_F}{36[\Omega]} = 0$$

(F) $$\frac{v_F - v_C}{36[\Omega]} + 8[A] + \frac{v_F - v_G}{56[\Omega]} + \frac{v_F - v_G}{62[\Omega]} = 0$$

Room for extra work

(E+H+G) $-25[S]v_W + \frac{v_E - v_D}{30[\Omega]} - 35[A] + \frac{v_G - v_D}{13[\Omega]} + \ldots$

$\frac{v_G}{10[\Omega]} + \frac{v_G}{91[\Omega]} - 8[A] + \frac{v_G - v_F}{56[\Omega]} + \frac{v_G - v_F}{62[\Omega]} = 0$

(E+H) $v_E - v_H = 19[V]$

(G+H) $v_H - v_G = 4v_X$

(v_W) $v_G - v_B = v_W$

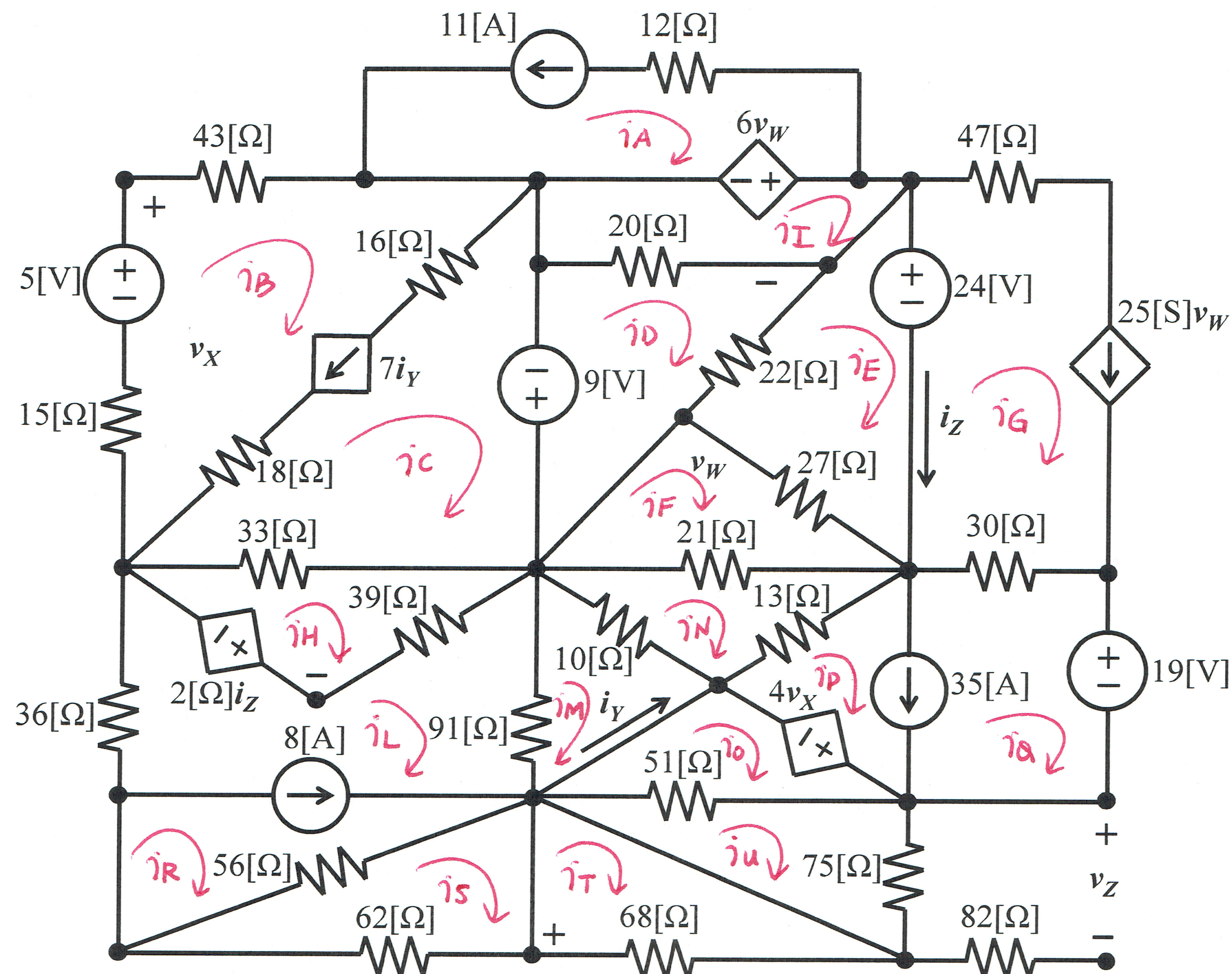
(v_X) $-v_X + 43[\Omega] \cdot \frac{v_C - v_A + 5[V]}{(43+15)[\Omega]} + v_A - v_C - 2[\Omega]i_Z = 0$

(i_Y) $\frac{v_F - v_C}{36[\Omega]} + \frac{v_G}{91[\Omega]} + i_Y + \frac{v_G - v_H}{51[\Omega]} + \frac{v_G - v_H}{75[\Omega]} = 0$

(i_Z) $-i_Z + \frac{v_D - v_E}{30[\Omega]} + 35[A] + \frac{v_D - v_G}{13[\Omega]} + \frac{v_D}{21[\Omega]} + \frac{v_D}{27[\Omega]} = 0$

$9 - 1 + 4 = 12$ equations

1. {30 Points} Use the node-voltage method or the mesh-current method to write a complete set of equations that could be used to solve this circuit. Do not simplify the circuit. Do not attempt to simplify or solve your equations. **Define all your variables clearly.**



(iA): iA = −11[A] (iB+iC): iB − iC = 7iY

(iB+iC): 43[Ω]iB − 9[V] + 33[Ω](iC − iH) + 15[Ω]iB − 5[V] = 0

(iD): 9[V] + 20[Ω](iD − iI) + 22[Ω](iD − iE) = 0

(iI): −6vW + 20[Ω](iI − iD) = 0 (iG): iG = 25[S]vW

(iE): 24[V] + 27[Ω](iE − iF) + 22[Ω](iE − iD) = 0

Room for extra work

i_F: $27[\Omega](i_F - i_E) + 21[\Omega](i_F - i_N) = 0$

i_H: $33[\Omega](i_H - i_C) + 39[\Omega](i_H - i_L) + 2[\Omega]i_Z = 0$

$i_L + i_R$: $-2[\Omega]i_Z + 39[\Omega](i_L - i_H) + 91[\Omega](i_L - i_M) + 56[\Omega](i_R - i_S) + \ldots$
$36[\Omega]i_L = 0$

$i_L + i_R$ $i_R - i_L = 8[A]$ i_T: $68 i_T = 0$

i_S: $56[\Omega](i_S - i_R) + 62[\Omega]i_S = 0$ i_U: $75[\Omega]i_U + 51[\Omega](i_U - i_O) = 0$

i_M: $91[\Omega](i_M - i_L) + 10[\Omega](i_M - i_N) = 0$ i_O: $-4v_X + 51[\Omega](i_O - i_U) = 0$

i_N: $21[\Omega](i_N - i_F) + 13[\Omega](i_N - i_P) + 10[\Omega](i_N - i_M) = 0$

$i_P + i_Q$: $4v_X + 13[\Omega](i_P - i_N) + 30[\Omega](i_Q - i_G) + 19[V] = 0$

$i_P + i_Q$ $i_P - i_Q = 35[A]$ i_Z $i_Z = i_E - i_G$ i_Y: $i_Y = i_O - i_M$

v_X: $-v_X + 5[V] - 15[\Omega]i_B - 2[\Omega]i_Z = 0$

v_W: $-v_W + 91[\Omega](i_M - i_L) + 9[V] - 6v_W = 0$

$19 + 4 = 23$ equations (Almost twice as many equations as NVM)

2. {30 points} In the circuit below, the charge carriers are positive. The number of charges leaving the voltage source v_{S2} through terminal 1 is given by

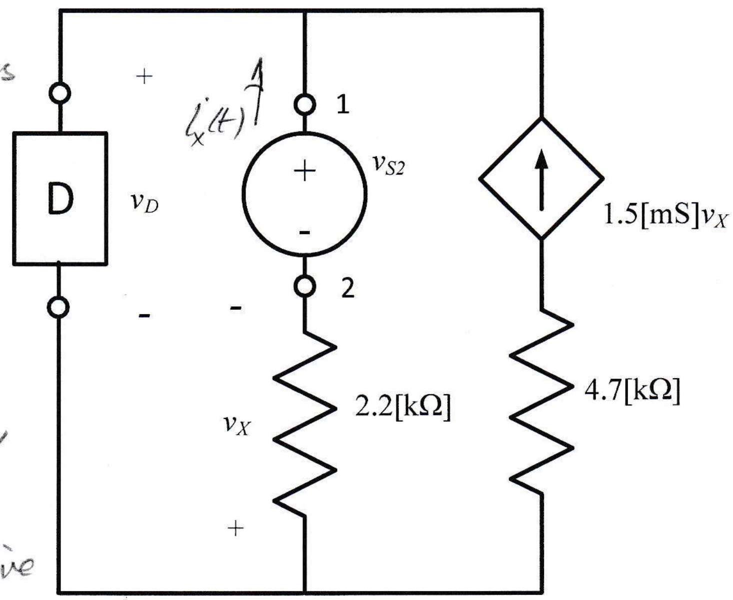
$$n(t) = 3.5\, e^{-40\left[\frac{1}{ms}\right]t}\ [\mu C].$$

The voltage across device D is known to be $v_D = 200$ [V].

i) Find an expression for the power delivered by v_{S2}.
ii) Find the energy delivered to the circuit by device D between 0 and 20 [s].

Positive charge carriers leaving terminal 1 ⇒ $i_x(t)$ is toward terminal 1.

$n(t)$ is decreasing, so $\frac{dn(t)}{dt}$ is negative, but current is in direction of positive charge carriers.



$$i_x(t) = \left|\frac{dn(t)}{dt}\right| = 40(3.5)\left[\frac{\mu C}{ms}\right] e^{-40\left[\frac{1}{ms}\right]t}$$

$$= 140\, e^{-40\left[\frac{1}{ms}\frac{1000\,ms}{s}\right]t} \left[\frac{10^{-6}C}{\mu C}\cdot\frac{10^3 ms}{s}\,\frac{\mu C}{ms}\right]$$

$$= 0.140\, e^{-40000\left[\frac{1}{s}\right]t}\ [A]$$

+5
+4 units

$$v_D = v_{S2} - 2200\, i_x(t) = 200\ [V]$$

+4

$$\therefore v_{S2} = 200 + 308\, e^{-40000\left[\frac{1}{s}\right]t}\ [V]$$

↗

Room for extra work

+ 3

i) $P_{del\ by\ v_{S2}} = v_{S2} \cdot i_x(t)$

$= 28\, e^{-40000[\frac{1}{s}]t} + 43.12\, e^{-80000[\frac{1}{s}]t}$ [W]

+ 6

ii) $P_{del\ by\ D} = -v_D(i_x + 0.0015\, v_x)$ $\quad v_x = 2200\, i_x$

$= -200(0.140\, e^{-40000[\frac{1}{s}]t} + 0.462\, e^{-40000[\frac{1}{s}]t})$

$= -120.4\, e^{-40000[\frac{1}{s}]t}$ [W]

+6

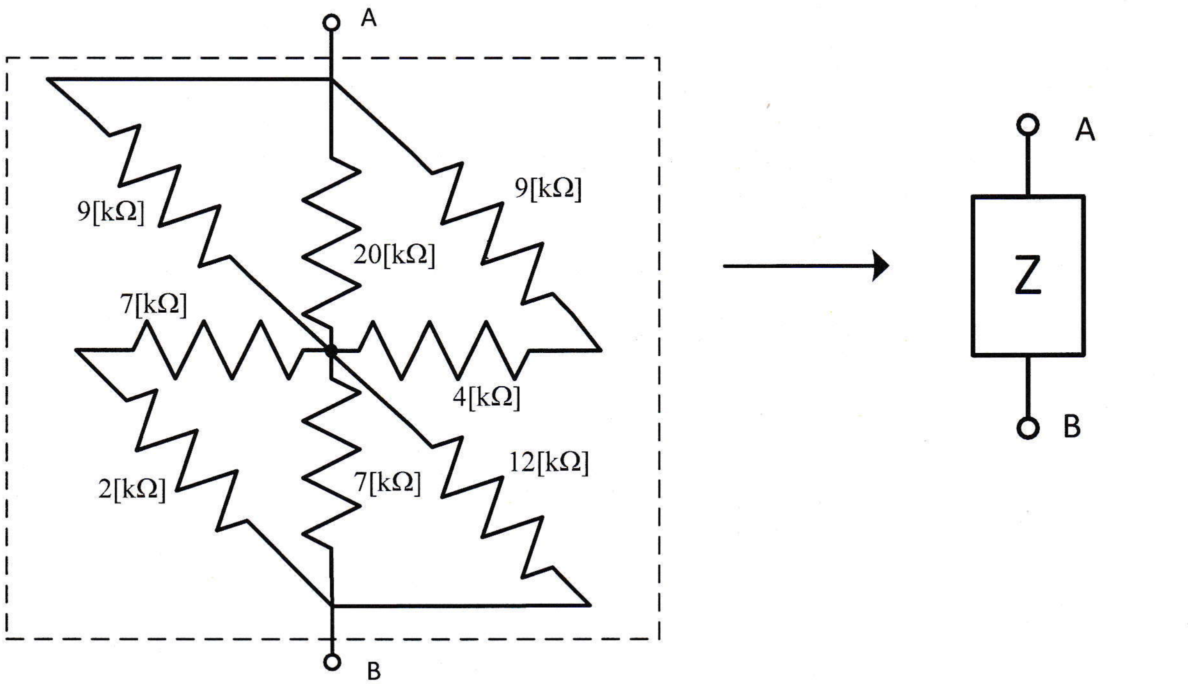
+2 units

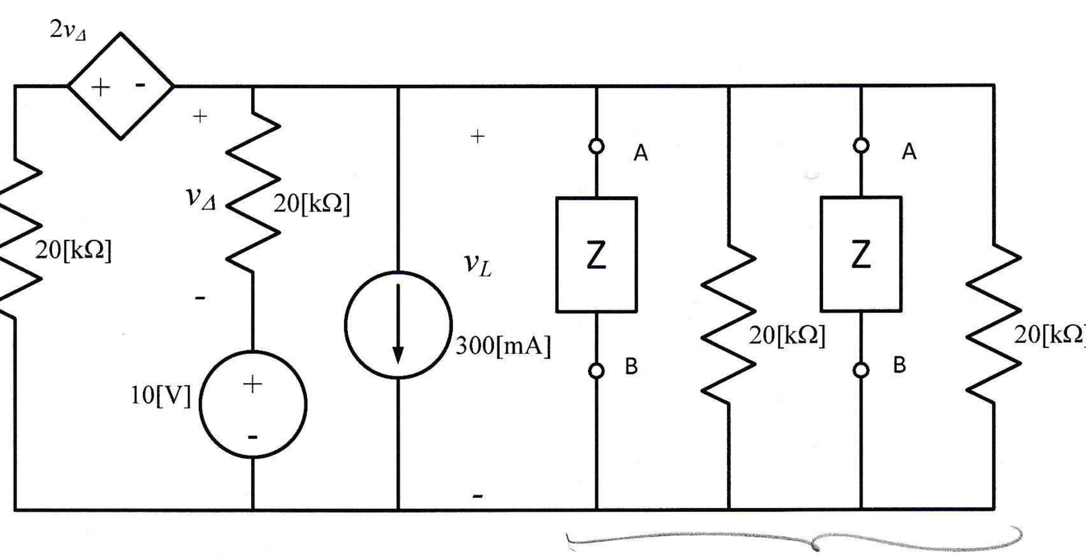
$w_{del\ by\ D} = -\int_0^{20[s]} 120.4\, e^{-40000[\frac{1}{s}]t}\, dt$

$= -\frac{120.4}{-40000[\frac{1}{s}]}[W]\, e^{-40000[\frac{1}{s}]t}\Big|_0^{20[s]}$

$= -3$ [mJ]

3. {35 Points} A resistive network called Z is shown in the upper figures. Network Z is connected into a circuit as shown in the lower figure. Use **superposition** to find the voltage v_L. Be sure to clearly indicate your steps in the process.





Simplify Z:

$R_{AB} \equiv \left(\frac{1}{9}+\frac{1}{20}+\frac{1}{13}\right)^{-1}$ 4.201 [kΩ]

$+\left(\frac{1}{9}+\frac{1}{7}+\frac{1}{12}\right)^{-1}$ 2.965 [kΩ]

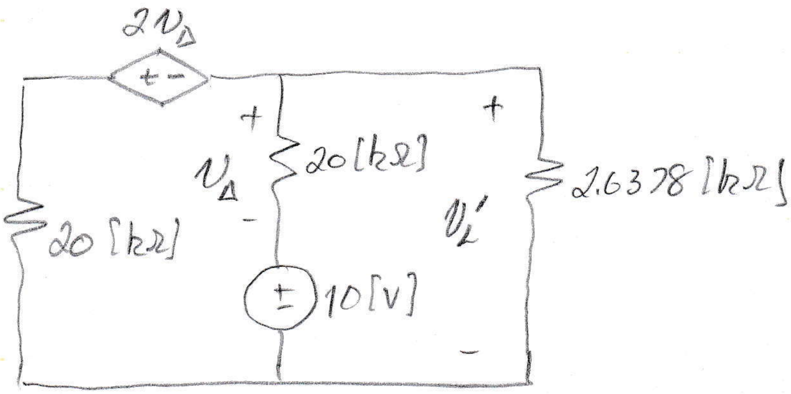
$R_{AB} = 7.166$ [kΩ]

$R_{eq} = \left(\frac{2}{R_{AB}}+\frac{2}{20}\right)^{-1} = 2.638$ [kΩ]

+10

Room for extra work

Deactivate current source:



+10

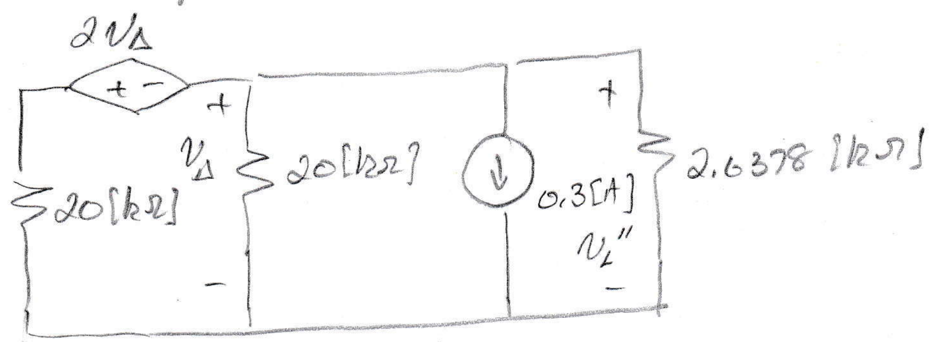
$$\frac{v_L'}{2.6378} + \frac{v_L' - 10}{20} + \frac{v_L' + 2v_\Delta}{20} = 0 \qquad v_\Delta = v_L' - 10$$

$$\Rightarrow v_L' = 2.590 \text{ [V]}$$

(+2)

Deactivate voltage source:

+10



2.6378 ∥ 20 = 2.304

$$\frac{v_L''}{2.6378k} + \frac{v_L''}{20k} + \frac{v_L'' + 2v_\Delta}{20k} + 0.3 = 0 \qquad v_\Delta = v_L''$$

$$\Rightarrow v_L'' = -518.04 \text{ [V]}$$

(+2)

$$v_L = v_L' + v_L'' = 2.59 - 518 = -515.4 \text{ [V]}$$

+5

4. {35 Points} A device can be modeled as a current source in parallel with a resistance as shown in Figure 1. The relationship between the voltage across this device, v_D and current through the device, i_D is shown in Figure 2.
a) Find a model for the device that would be valid for $i_D > 6[mA]$ and draw it showing terminals A and B.
b) When this device is connected to a circuit as shown in Figure 3, the 3[mA] independent current source delivers 10.5[mW] power. Find ***K***.

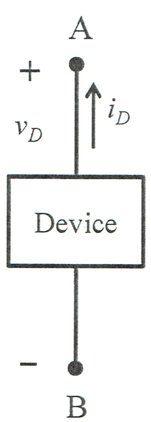


Figure 1

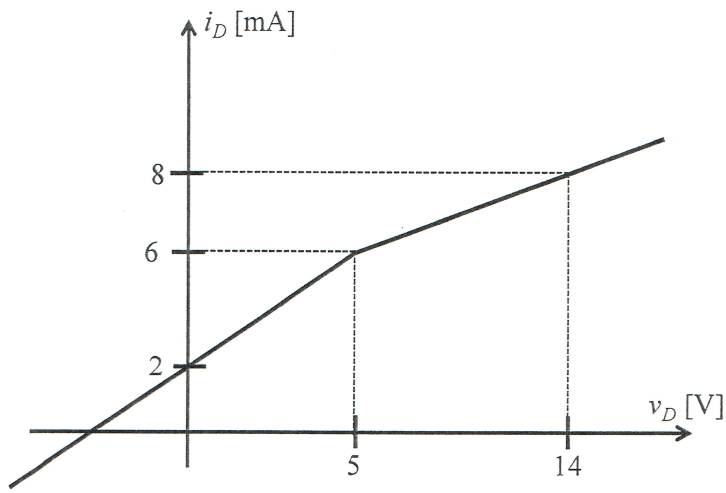


Figure 2

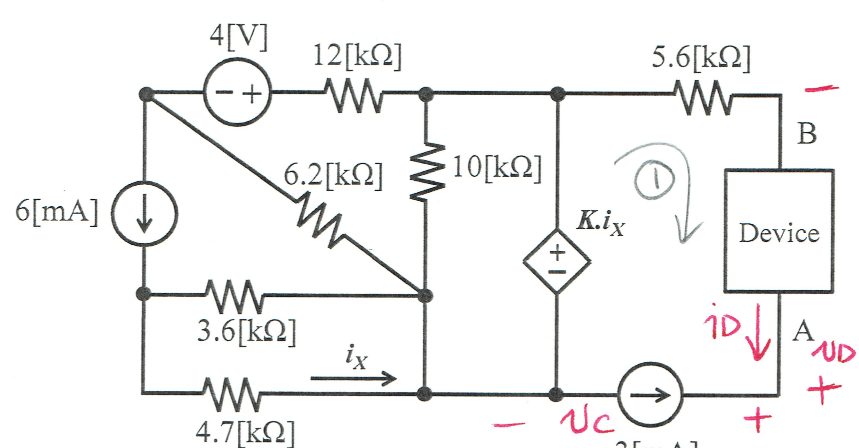
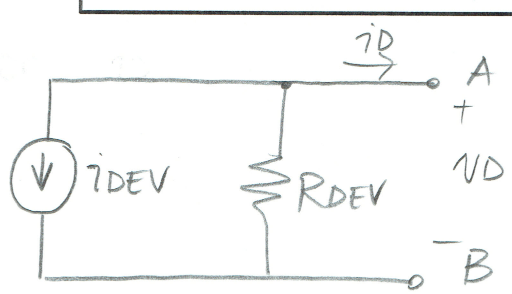


Figure 3

a)



$$i_D + \frac{v_D}{R_{DEV}} + i_{DEV} = 0$$

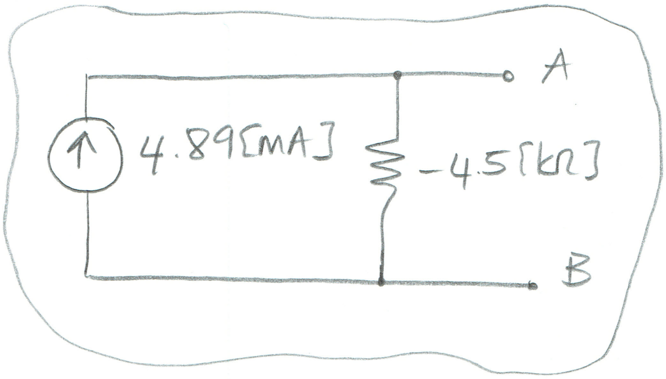
$$6[mA] + \frac{5[V]}{R_{DEV}} + i_{DEV} = 0 \quad (1)$$

$$8[mA] + \frac{14[V]}{R_{DEV}} + i_{DEV} = 0 \quad (2)$$

Room for extra work

Solving (1) and (2), we get: $i_{DEV} = -4.89$ [mA]

$R_{DEV} = -4.5$ [kΩ]



b) From Figure 3, we note that $i_D = -3$ [mA]. Since the model found in part (a) is valid for $i_D \geq 6$ [mA], we can not use it.

$$i_D = \frac{4}{5} [mA/V] \cdot v_D + 2 [mA] \quad \text{for } i_D \leq 6 [mA]$$

If $i_D = -3$ [mA], then $v_D = -6.25$ [V]

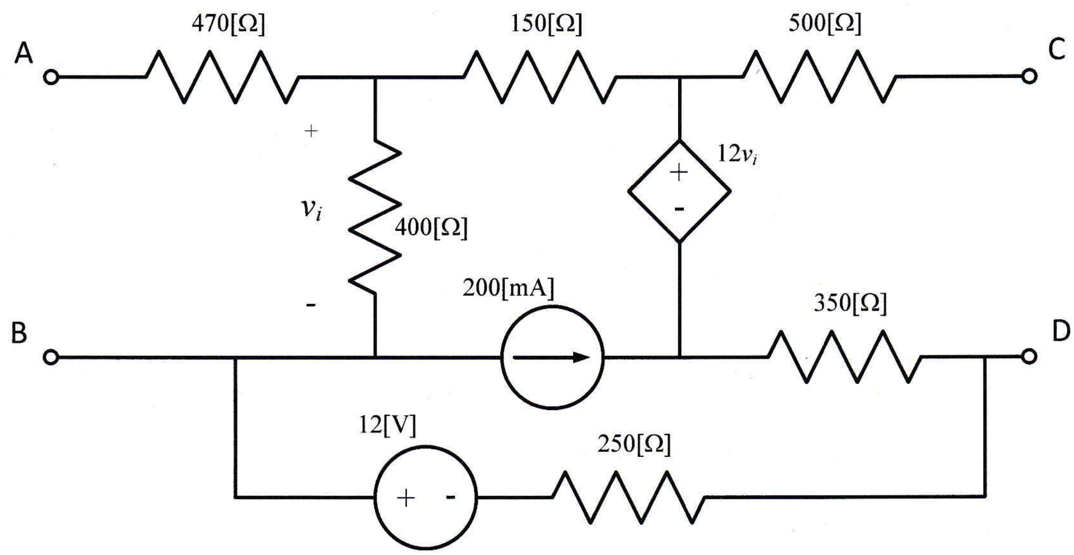
$P_{DEL,3[mA]} = 3 [mA] \cdot v_C = 10.5$ [mW] $\Rightarrow$ $v_C = 3.5$ [V]

$$i_x = 6 [mA] \cdot \frac{3.6}{4.7+3.6} = 2.6 [mA]$$

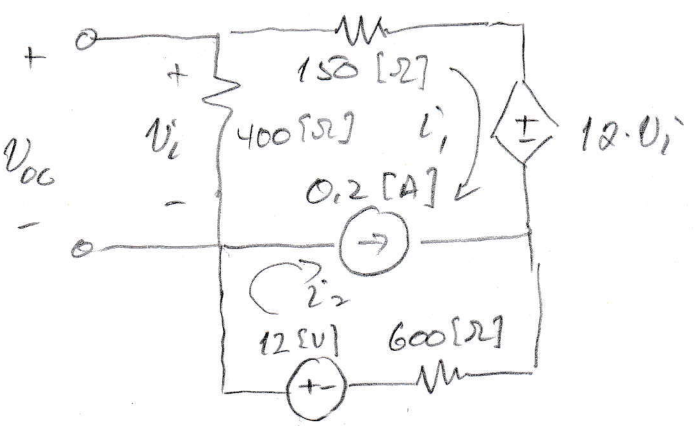
KVL ①: $-K i_x - 3 [mA] \cdot 5.6 [k\Omega] - (-6.25 [V]) + 3.5 [V] = 0$

$$\boxed{K = -2.71 [k\Omega]}$$

5. {35 Points}
i) Find the Thevenin Equivalent of the circuit below at terminals A, B.
ii) A load resistor R_L is to be connected to the circuit at terminals C, D. Find the value of R_L that will maximize power to R_L.



i) V_{oc}:

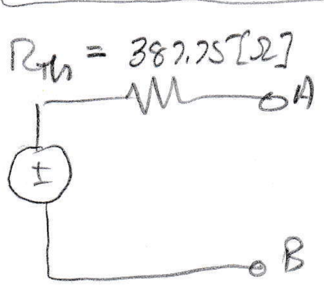


$550 i_1' + 12 v_i' + 600 i_2' - 12 = 0$ $v_i' = -400 i_1'$ +11

$i_2' - i_1' = 0.2$ ⇒ $i_1' = 0.02959$ [A] $i_2' = 0.22959$ [A]

$v_i' = V_{oc} = -11.836$ [V]

$R_{Th} = 387.75$ [Ω]



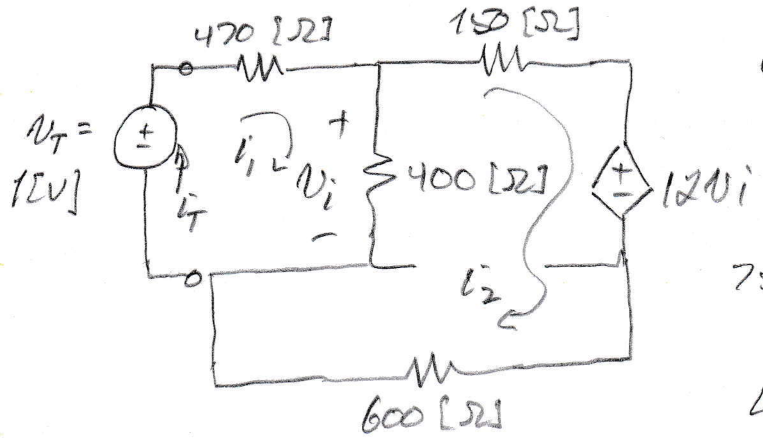
$V_{Th} = -11.84$ [V]

$i_{sc} = -30.52$ [mA] (+11)

Room for extra work

+11

v_T :



$470 i_1' + 400(i_1' - i_2') - 1 = 0$

$v_i' = 400(i_1' - i_2')$

$750 i_2' + 12 v_i' + 400(i_2' - i_1') = 0$

$i_1' = 2.58$ [mA] $v_i' = -0.212$ [V]

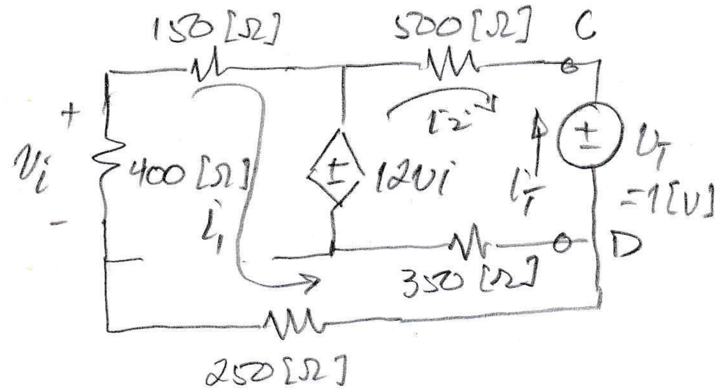
$i_2' = 3.11$ [mA]

$i_T' = i_1' = 2.579$ [mA]

$\Rightarrow R_{Th} = 387.75$ [Ω]

+11

ii)



$150 i_1' + 12 v_i' + 350(i_1' - i_2') + 650 i_1' = 0$

$v_i' = -400 i_1'$

$500 i_2' + 1 + 350(i_2' - i_1') - 12 v_i' = 0$

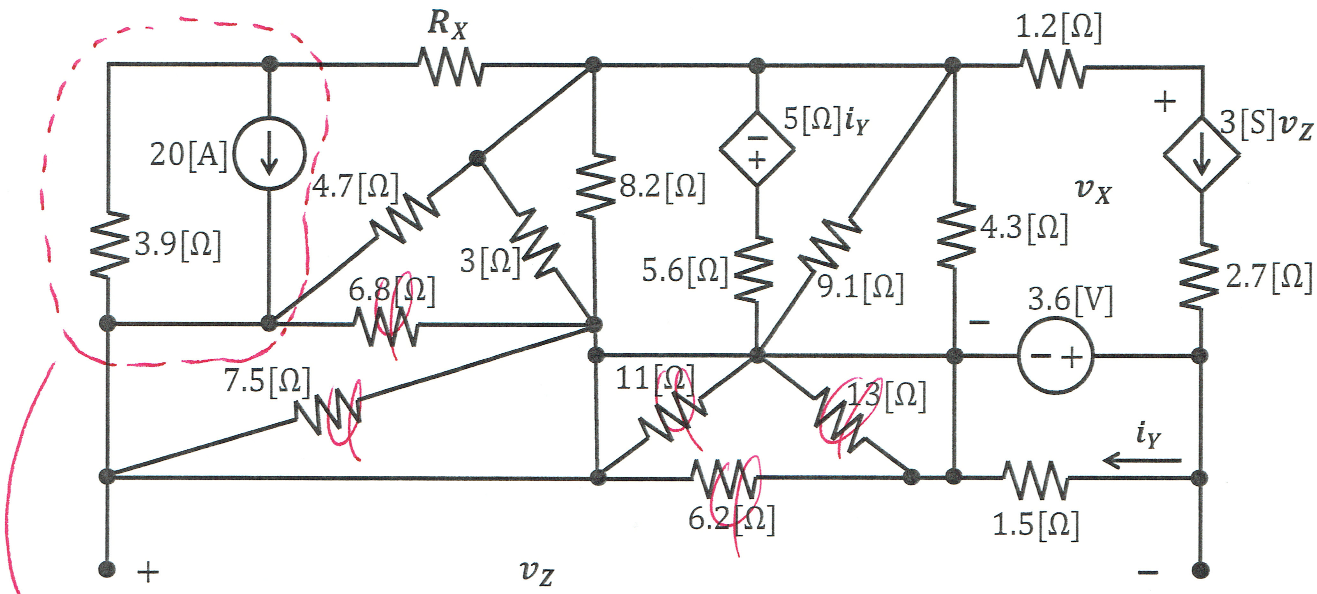
$i_1' = 0.227$ [mA] $i_2' = -2.362$ [mA] $v_i' = -0.0906$ [V]

$i_T' = -i_2' = 2.362$ [mA] $\Rightarrow R_{Th} = 423.37$ [Ω]

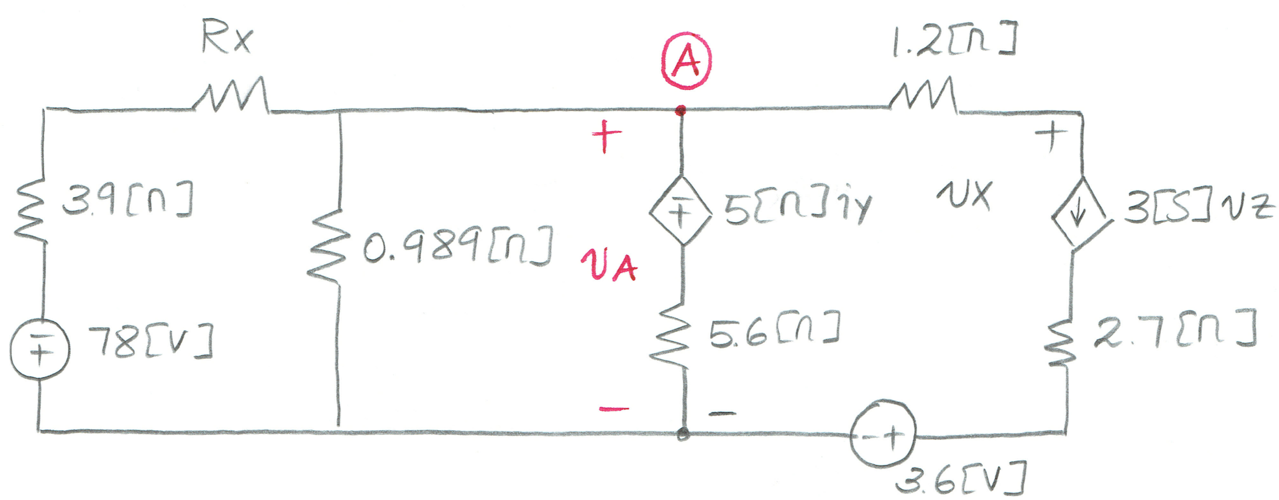
R_L for max power $= R_{Th}$ at C,D $=$ 423.37 [Ω]

+2

6. {35 points} For the following circuit, v_X is measured to be 8.3[V]. Find R_X.



Let's apply source transformation, simplify the circuit and redraw:



$v_Z = -3.6[V]$

$i_Y = \frac{3.6[V]}{1.5[\Omega]} = 2.4[A]$

(Now that we know the value of iy, 1.5[Ω] resistor can be ignored, since it's in parallel w/ a volt. source)

$4.7[\Omega] \parallel 3[\Omega] \parallel 8.2[\Omega] \parallel 9.1[\Omega] \parallel 4.3[\Omega] = 0.989[\Omega]$

Room for extra work

KVL: $-v_A + 1.2[\Omega] \cdot 3[S] \cdot v_Z + v_X = 0$

$v_X = 8.3[V]$ $v_A = -4.66[V]$

$$\frac{v_A + 78[V]}{R_X + 3.9[\Omega]} + \frac{v_A}{0.989[\Omega]} + \frac{v_A + 5[\Omega]i_Y}{5.6[\Omega]} + 3[S]v_Z = 0$$

$$\boxed{R_X = 1.265[\Omega]}$$