Name: ____ **(Print)**

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Date: _____________

ECE 2300 -- Quiz #5

S.R. Brankovic Section – TuTh 8:30 AM

April 6, 2006

KEEP THIS QUIZ CLOSED AND FACE UP UNTIL YOU ARE TOLD TO BEGIN.

1. This quiz is closed book, closed notes.
2. Show all work on these pages. Show all work necessary to complete the problem. A solution without the appropriate work shown will receive no credit. A solution which is not given in a reasonable order will lose credit. **<u>If your work continues on to another page, indicate clearly where your work can be found. Failure to indicate this clearly will result in a loss of credit.</u>**
4. Show all units in solutions, intermediate results, and figures. Units in the quiz will be included between square brackets.
5. Do not use red ink. Do not use red pencil.
6. You will have 30 minutes to work on this quiz.

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Problem #1.

In Figure 1, shown below, the circuit with three identical capacitive elements is presented. For the time domain t < 0, the switch A is closed and switch B is opened and switch C is closed for a long time. At t = 0, the switch A opens and stays open and at t=0.1 [s] switch B closes and stays closed and at t = 1[s] switch C opens and stays open. Find: a) the value of the capacitance $\boldsymbol{C}$, for which the current $\boldsymbol{i_x}$ at t=0.2 [s] equals 1% of the current $\boldsymbol{i_o}$ supplied by the independent current source, b) discuss the relation between the $\boldsymbol{C}$ and current $\boldsymbol{i_x}$ *for* $0.1[s] \leq t \leq 1\ [s]$ *a*nd c) find the total energy dissipated on 1[Ω] resistor in the time interval $0 \leq t \leq \infty [s]$ assuming the $i_0 = 1$[A].

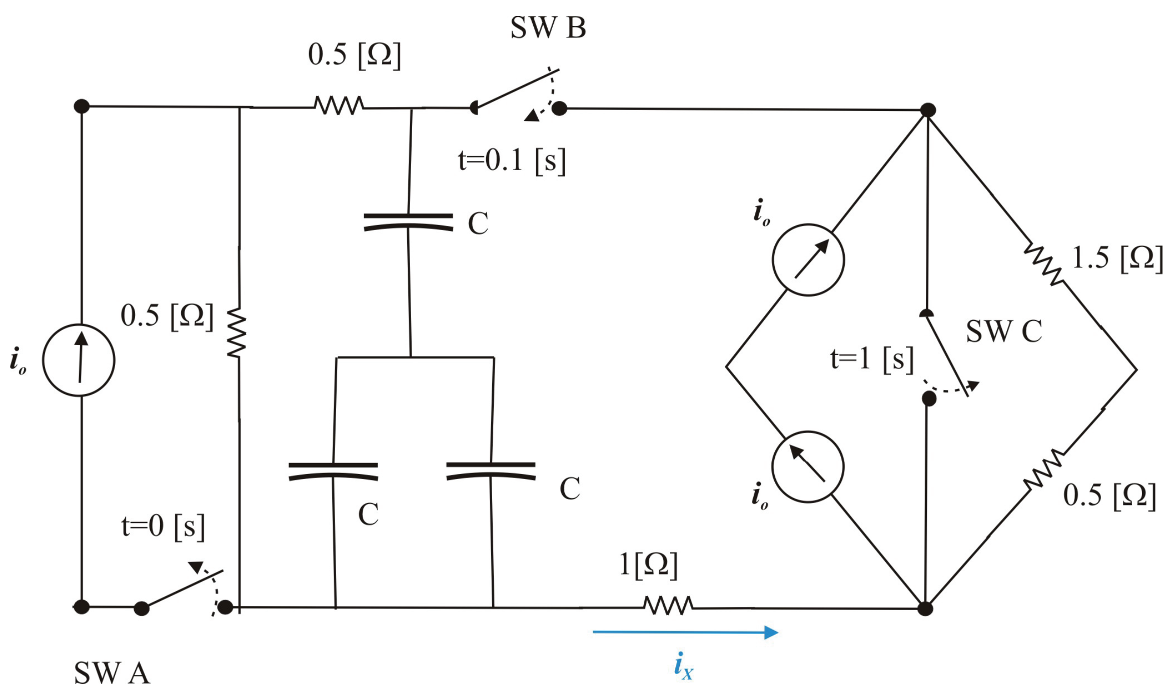

Figure 1.

Solution:

To start solving this problem we need to represent the capacitor combination with its equivalent C_E=2/3C; Figure 2.

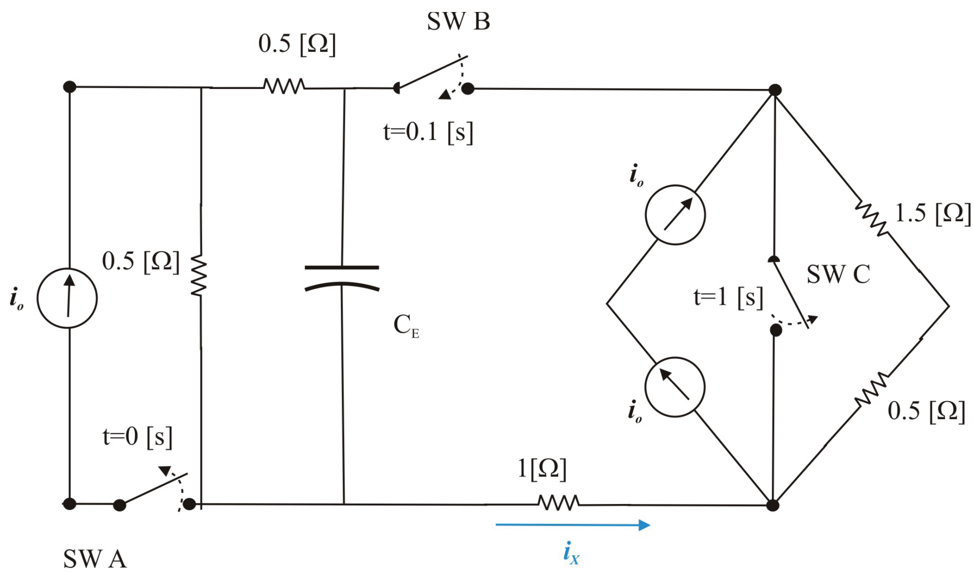

Figure 2. Equivalent circuit with C_E

For time domain t<0 the circuit to consider is shown in Figure 3:

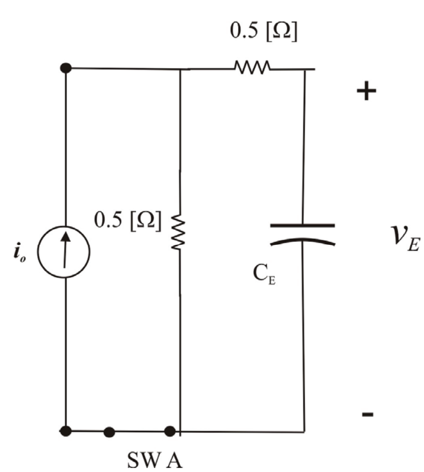

Figure 3. Circuit for time domain t<0

The voltage v_E on the terminals of the equivalent capacitor is then $v_E(t<0) = i_0 \cdot 0.5[\Omega]$ because at steady state conditions the capacitors is considered as open circuit and the current is not flowing through the upper 0.5 [Ω] resistor. Since the voltage across the capacitor can not have instantaneous change, then at t = 0, when switch A opens,

$v_E(t=0[s]) = i_0 \cdot 0.5[\Omega]$. This is our initial condition for the $v_E(0 \le t \le 0.1[s])$ expression (Figure 4).

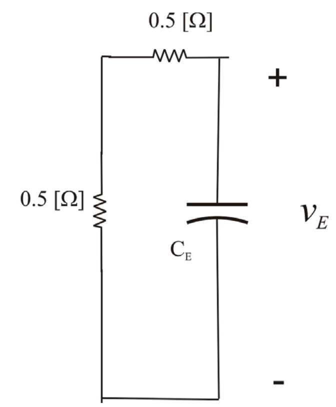

Figure 4. Circuit for 0≤t≤0.1[s]

The circuit in Figure 4 represent the natural response circuit and the expression for voltage v_E dependence for time domain 0[s]≤t≤0.1[s] is given as:

$$v_E(0[s] \le t \le 0.1[s]) = v_E(0)e^{-\frac{t}{\tau}} \quad (1)$$

The time constant for this circuit is given as $R_E \cdot C_E = (0.5[\Omega]+0.5[\Omega]) \cdot 2/3C = 2/3C \cdot [\Omega]$

Knowing the $v_E(0)$, the full expression for v_E is then:

$$v_E(0[s] \le t \le 0.1[s]) = i_0 \cdot 0.5[\Omega] \cdot e^{-\frac{t}{\frac{2}{3}C \cdot [\Omega]}} \quad (2)$$

At t = 0.1 [s], the switch B closes and the new situation for 0.1[s]≤t≤1[s] is described in Figure 5. Note that two sources and resistors in parallel are shorted and they do not matter until the switch C opens. We have again natural response circuit situation and we look for the solution for v_E as:

$$v_E(0.1[s] \le t \le 1[s]) = v_E(0.1[s])e^{-\frac{t-0.1[s]}{\tau_1}} \quad (3)$$

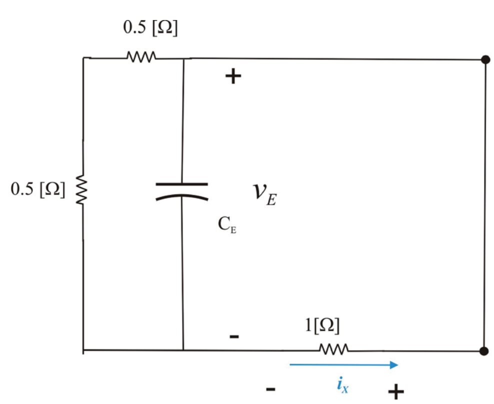

Figure 5. Circuit for 0.1[s]≤t≤1[s]

The initial condition for v_E in this case we find putting t = 0.1[s] in to Eq.(2), so we have;

$$v_E(0.1[s]]) = i_0 \cdot 0.5[\Omega] \cdot e^{-\frac{0.1[s]}{\frac{2}{3}C \cdot [\Omega]}} \qquad (4)$$

The new time constant for this circuit τ_1 is now given as $R_{E1} \cdot C_E = 0.5[\Omega] \cdot 2/3C$, so the expression for v_E is then:

$$\boxed{v_E(0.1[s] \le t \le 1[s]) = i_0 \cdot 0.5[\Omega] \cdot e^{-\frac{0.1[s]}{\frac{2}{3}C \cdot [\Omega]}} \cdot e^{-\frac{t-0.1[s]}{\frac{1}{3}C \cdot [\Omega]_1}} = i_0 \cdot 0.5[\Omega] \cdot e^{-\frac{(3t-0.15[s])}{C \cdot [\Omega]}}} \qquad (5)$$

The current going trough the 1 [Ω] resistor (Figure 5) then can be found as:

$$i_x = \frac{v_E(0.1[s] \le t \le 1[s])}{1[\Omega]} = 0.5 \cdot i_0 \cdot e^{-\frac{(3t-0.15[s])}{C \cdot [\Omega]}} \qquad (6)$$

At t = 0.2[s], the i_x/i_0 = 0.01(see the text of the problem), which yields expression from Eq(6) to be:

$$0.01 = 0.5 \cdot e^{-\frac{0.45[s]}{C \cdot [\Omega]}}, \qquad (7)$$

After applying logarithm to both sides, we get:

$$\boxed{-3.91 = -\frac{0.45[s]}{C \cdot [\Omega]} \Rightarrow C = 0.11 \frac{[s] \cdot [A]}{[V]} = 0.11[F]} \qquad (8)$$

(Note that chousing the active sing convention on 1 [Ω] resistor facilities the mathematical handling of the logarithm operation. If chosen a passive sign convention on 1 [Ω] resistor, then the ABS value of the currents ratio has to be taken as the argument of logarithm). *From the equation (8), we see that the general dependence of i_x on C is the function of from;*

$$\boxed{i_x = B \cdot e^{-A/C},} \qquad (9)$$

where A is constant. B represents the ratio between the resistors 0.5[Ω]/1[Ω] multiplied by the current i_0 and some other constant. It is straight forward to conclude that i_x will be increasing for increasing values of C. So, in another words, the current i_x at $0.1[s] \le t \le 1[s]$ will be exponentially dependent on the ability of the circuit to store the charge during the interval t<0 on the capacitor combination.

At t=1[s], SW C opens and the shorted part of the circuit becomes relevant, the new situation is presented in Figure 6.

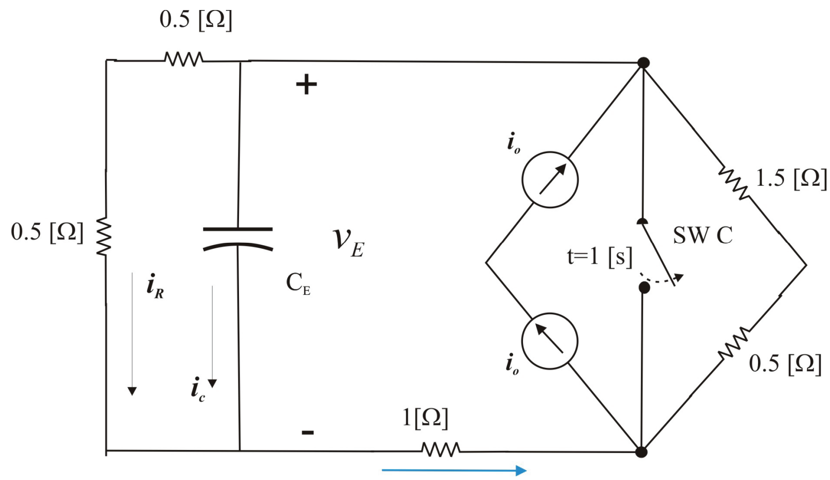

Figure 6. Circuit for t≥1[s]

The total energy absorbed (dissipated) by 1[Ω] resistor is:

$$W_T = W_{0.1[s]\le t\le 1[s]} + W_{t\ge 1[s]} \tag{10}$$

According to Figure 5;

$$W_{0.1[s]\le t\le 1[s]} = \int_{0.1[s]}^{1[s]} \frac{v_E^2(t)}{1[\Omega]}dt = \int_{0.1[s]}^{1[s]} \frac{\left(0.5[V]\cdot e^{-\frac{(3t-0.15[s])}{0.11[s]}}\right)^2}{1[\Omega]} dt = 0.0003[J] \tag{11}$$

Looking the circuit in Figure 6, one would realize that after infinitely long time the current flowing through the 1[Ω] resistor will still have the finite value of 1 [A]. Indirectly this means that the dissipated energy on this resistor for t≥1[s] will be infinitely large for infinitely long time ($W_{t\ge 0} = \infty$). **<u>This gives the final answer for W_T to be $W_T = \infty$ [J].</u>**

If someone wants to find this answer on more explicit way, the story could continue as follows:

The circuit in this case (t≥1[s]) is considered as step response circuit, and the initial condition (voltage) can be found from Eq(5) having t=1[s], and C=0.11[F],

$$v_E(1[s]) = i_0 \cdot 0.5[\Omega]\cdot e^{-25.9} = 2.82\cdot 10^{-12}[\Omega]\cdot i_0 \tag{12}$$

The $v_E(\infty)$ can be found considering the circuit in Figure 6 with capacitor C_E acting as open circuit. From Ohms law and CDR we get that;

$$v_E(\infty) = (0.5[\Omega]+0.5[\Omega])\cdot 2i_0 \frac{0.5[\Omega]+1.5[\Omega]}{0.5[\Omega]+0.5[\Omega]+1.5[\Omega]+0.5[\Omega]+1[\Omega]} = 1[\Omega]\cdot i_0 \tag{13}$$

The time constant for this circuit is:

$$\tau_2 = R_E C_E = \frac{3}{4}[\Omega] \cdot \frac{2}{3} \cdot 0.11[F] = 0.55[s]. \tag{14}$$

For $i_0 = 1[A]$ the final expression for v_E for $t \geq 1[s]$ is then:

$$v_E(t \geq 1[s]) = 1[V] + (2.82 \cdot 10^{-12}[V] - 1[V]) \cdot e^{-\frac{t-1[s]}{0.55[s]}}; \tag{15}$$

The $2.82 \cdot 10^{-12}$[V] *i.e.* the initial value of v_E for this time domain is very small number, and in practical sense, it can be considered as zero, so the expression in (15) can be simplified as:

$$v_E(t \geq 1[s]) \approx 1[V] - 1[V] \cdot e^{-\frac{t-1[s]}{0.55[s]}} \tag{16}$$

From Figure 6, for t≥1[s], the voltage on 1[Ω] resistor can be found from KVL and VDR after the source transformation is applied on the parallel connection of the current sources with resistors (Figure 7) and using (14) we get;

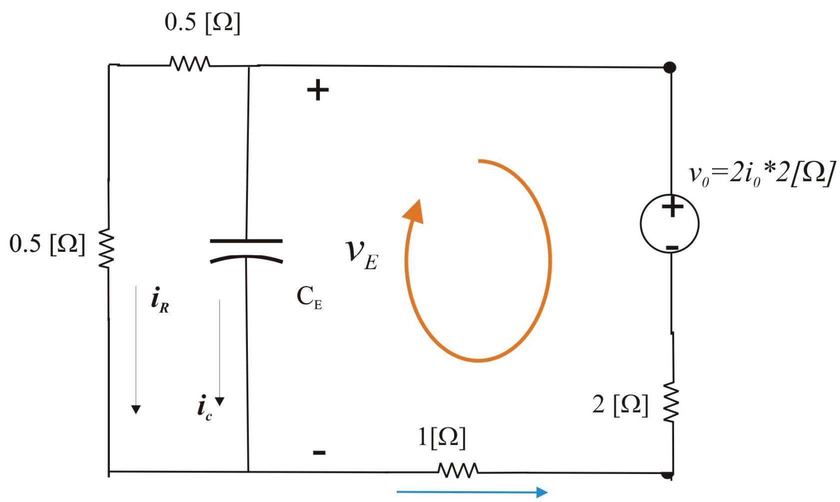

Figure 7. Circuit for t≥1[s] after applying source transformation operation.

$$v_{1\Omega}(t) = \frac{1}{3}(v_E(t \geq 1[s]) - 4[V]) = -1[V] - 0.33[V] \cdot e^{-\frac{t-1[s]}{0.55[s]}} \tag{17}$$

And

$$W_{t \geq 1[s]} = \int_1^\infty \frac{v_{1\Omega}^2(t)}{1[\Omega]} dt = \int_{1[s]}^\infty \frac{\left(-1[V] - 0.33[V] \cdot e^{-\frac{t-1[s]}{0.55[s]}}\right)^2}{1[\Omega]} dt = \infty[J] \tag{18}$$

Finally, from (16) and (18) we get that

$$W_T = \infty\ [J]$$