Name: ____ **(Print)**

Signature ____

Date: _____________

ECE 2300 -- Quiz #6

S.R. Brankovic Section – TuTh 8:30 AM

Apr. 27th, 2006

KEEP THIS QUIZ CLOSED AND FACE UP UNTIL YOU ARE TOLD TO BEGIN.

1. This quiz is closed book, closed notes. You can have one crib sheet.
2. Show all work on these pages. Show all work necessary to complete the problem. A solution without the appropriate work shown will receive no credit. A solution which is not given in a reasonable order will lose credit. **<u>If your work continues on to another page, indicate clearly where your work can be found. Failure to indicate this clearly will result in a loss of credit.</u>**
4. Show all units in solutions, intermediate results, and figures. Units in the quiz will be included between square brackets.
5. Do not use red ink. Do not use red pencil.
6. You will have 30 minutes to work on this quiz.

________________/100 %

Problem #1.

For the circuit shown in Figure 1, having $\omega > 0$,

a) Find which statement is true: 1) $i_S(t)$ lags $v_L(t)$, 2) $i_S(t)$ leads $v_L(t)$, or 3) $i_S(t)$ and $v_L(t)$ have the same phase?
b) If the frequency ω=1 rad/s, find the average power absorbed by the load ?

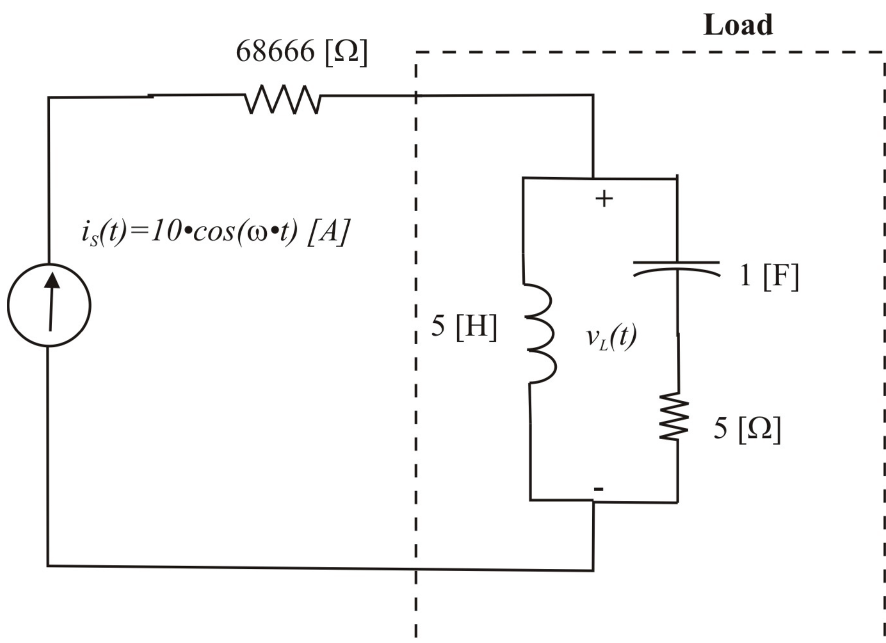

Figure 1.

Solution:

In order to start solving this problem, the first thing we need to do is to transform our circuit into the phasor domain, Figure 2, Note that on the figure, the units of the impedance have been already derived assuming that we search for numerical solution of ω given in [rad/s].

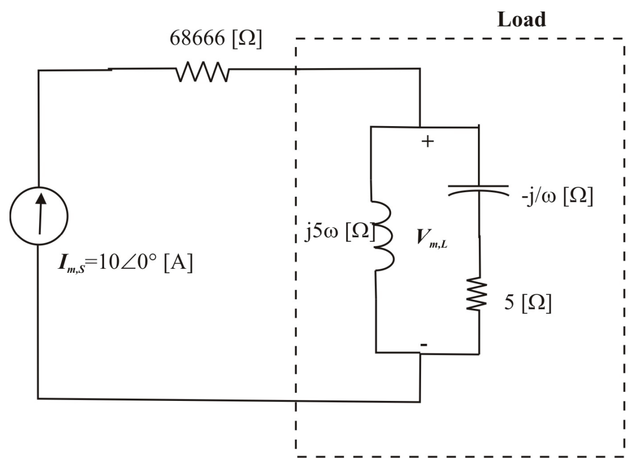

Figure 2.

The extra step to simplify the above circuit is to replace the load-impedances combination with their equivalent, Figure 3,

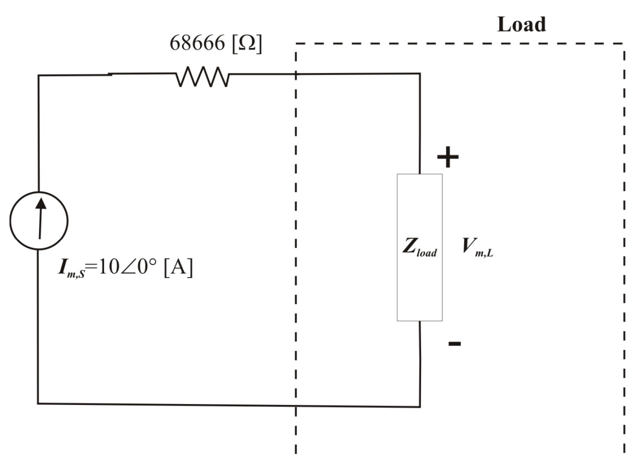

Figure 3.

Now, it is straight forward that the $\boldsymbol{V_{m,L}}$ is given as:

$$\boldsymbol{V_{m,L}} = \boldsymbol{I_{m,S}} \cdot \boldsymbol{Z_{load}} \tag{1}$$

To proceed further we need to found $\boldsymbol{Z_{load}}$ as a function of ω, From Figure 2, one can write:

$$\boldsymbol{Z_{load}} = (5[\Omega] - \frac{\boldsymbol{j}}{\omega}[\Omega])\,\|\,5\omega \cdot \boldsymbol{j}[\Omega] = \frac{\left(5 - \frac{\boldsymbol{j}}{\varpi}\right)\cdot 5\omega \cdot \boldsymbol{j}}{5 + \boldsymbol{j}\cdot\left(5\omega - \frac{1}{\varpi}\right)}[\Omega] = \frac{25\varpi \cdot \boldsymbol{j} + 5}{5 + \boldsymbol{j}\cdot\left(5\omega - \frac{1}{\varpi}\right)}[\Omega] \quad (2)$$

Furthermore, to get more friendly expression for $\boldsymbol{Z_{load}}$, one can do following;

$$\boldsymbol{Z_{load}} = \frac{25\varpi \cdot \boldsymbol{j} + 5}{5 + \boldsymbol{j}\cdot\left(5\omega - \frac{1}{\varpi}\right)} \cdot \frac{5 - \boldsymbol{j}\cdot\left(5\omega - \frac{\boldsymbol{j}}{\varpi}\right)}{5 - \boldsymbol{j}\cdot\left(5\omega - \frac{1}{\varpi}\right)}[\Omega] = \frac{125\omega^2}{25 + \left(5\omega - \frac{1}{\varpi}\right)^2} + \boldsymbol{j}\cdot\frac{100\omega + \frac{5}{\omega}}{25 + \left(5\omega - \frac{1}{\varpi}\right)^2}[\Omega] \ (3)$$

Now, form eq(1) one get:

$$\boldsymbol{V_{m,L}} = 10\angle 0^\circ[\boldsymbol{A}] \cdot \left\{\frac{125\omega^2}{25 + \left(5\omega - \frac{1}{\varpi}\right)^2} + \boldsymbol{j}\cdot\frac{100\omega + \frac{5}{\omega}}{25 + \left(5\omega - \frac{1}{\varpi}\right)^2}\right\}[\Omega], \quad (4)$$

After converting phasor of the current in to the Cartesian form and performing multiplication,

$$\boldsymbol{V_{m,L}} = \frac{1250\omega^2}{25 + \left(5\omega - \frac{1}{\varpi}\right)^2} + \boldsymbol{j}\cdot\frac{10\cdot\left(100\omega + \frac{5}{\omega}\right)}{25 + \left(5\omega - \frac{1}{\varpi}\right)^2}[\boldsymbol{V}] \quad (5)$$

Now the phase angle of the voltage $\boldsymbol{V_{m,L}}$ is given as:

$$\theta = \mathbf{arctan}\frac{\dfrac{10\cdot\left(100\omega + \frac{5}{\omega}\right)}{25 + \left(5\omega - \frac{1}{\varpi}\right)^2}}{\dfrac{1250\omega^2}{25 + \left(5\omega - \frac{1}{\varpi}\right)^2}} = \mathbf{arctan}\frac{\left(100\omega + \frac{5}{\omega}\right)}{125\omega^2} = \mathbf{arctan}\frac{\boldsymbol{f_1(\omega)}}{\boldsymbol{f_2(\omega)}} \quad (6)$$

The evaluation of the sign of the fraction follows as ;

$$\begin{aligned} &\omega > 0, \Rightarrow f_1(\omega) > 0, \text{and } f_2(\omega) > 0 \\ &\Rightarrow \tan\frac{f_1(\omega)}{f_2(\omega)} > 0, \text{and } \boxed{\theta = \arctan\frac{f_1(\omega)}{f_2(\omega)} > 0} \end{aligned} \tag{7}$$

<u>Meaning that for any physical value of ω, the current will be lagging the voltage.</u>

If someone noticed that;

$$\boxed{\lim_{\omega\to+\infty}\frac{\left(100\omega+\frac{5}{\omega}\right)}{1250\omega^2}=0, \Rightarrow \lim_{\omega\to+\infty}\arctan\frac{\left(100\omega+\frac{5}{\omega}\right)}{1250\omega^2}=0} \tag{8}$$

would get an extra 10% in score. <u>The above expression means that the phase angle θ between voltage and current for the circuit in Figure 1, will get smaller and smaller as the frequency of the current increases, and for some ultimately large frequencies (for example, above 10^9 rad/s) the current $i_S(t)$ and voltage $v_L(t)$ can be considered to be in phase.</u> If some of you choose to specialize in high frequency measurements and signal processing, you will get the chance to learn more about these peculiarities.

Recalling that;

$$p_{average} = P = \left|\boldsymbol{I}_{RMS,S}\right|^2 \cdot \mathbf{Re}\{\boldsymbol{Z}_{load}\} = \frac{\left|\boldsymbol{I}_{m,S}\right|^2}{2} \cdot \mathbf{Re}\{\boldsymbol{Z}_{load}\} \tag{9}$$

From the eq(3) the real part of $\boldsymbol{Z}_{\text{load}}$ is;

$$\boxed{\mathbf{Re}\{\boldsymbol{Z}_{load}\} = \frac{125\omega^2}{25+\left(5\omega-\frac{1}{\varpi}\right)^2}[\Omega]} \tag{10}$$

For ω = 1 rad/s

One gets;

$$\boxed{p_{average} = 50[A]^2 \cdot 3.05[\Omega] = 152.5[W]} \tag{11}$$