Name: Solutions ____________________ (please print)

Signature: ______________________________

ECE 2201 – Final Exam
May 3, 2018

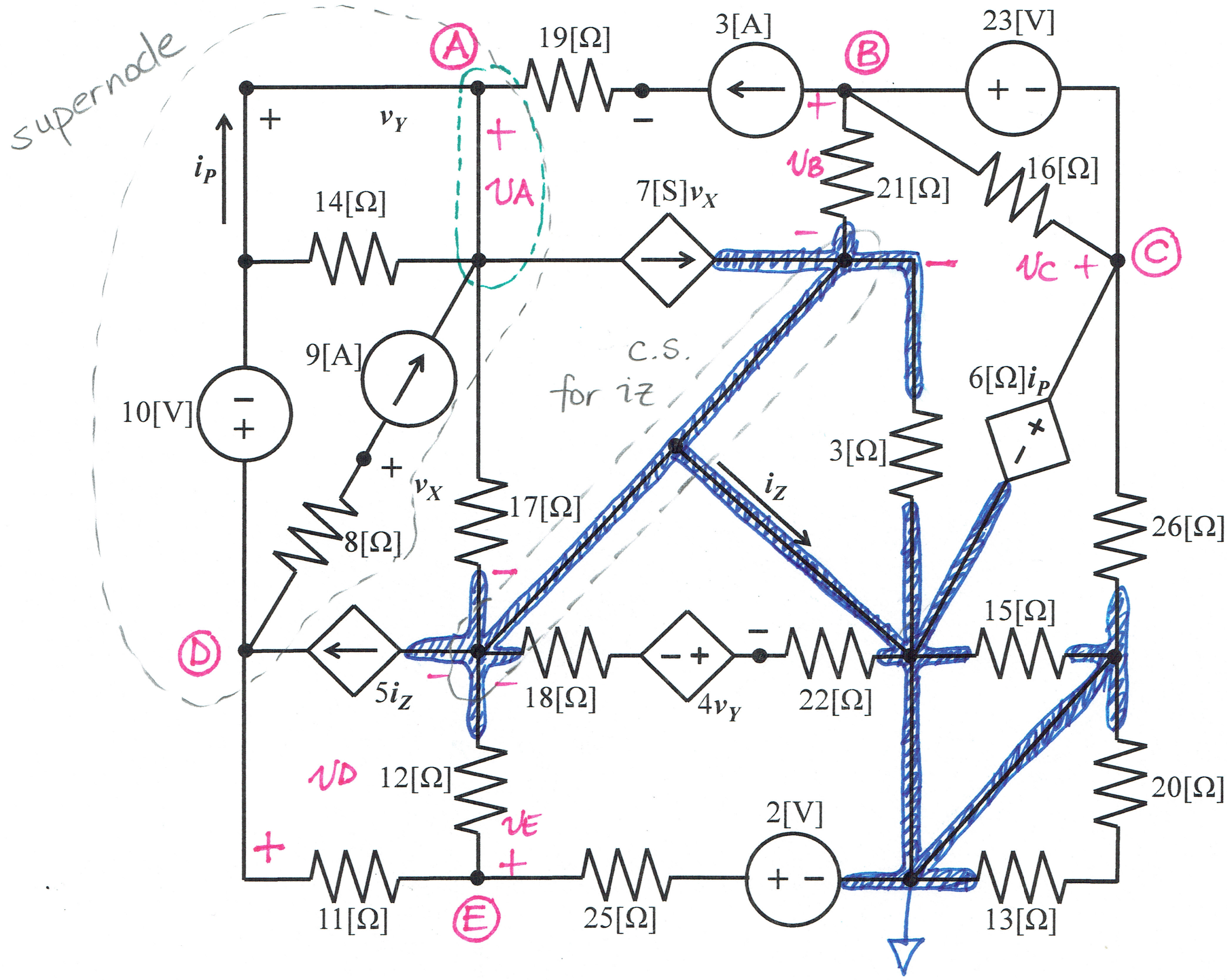
Keep this exam closed until you are told to begin.

1. This exam is closed book, closed notes. You may use one 8.5" x 11" crib sheet, or its equivalent.
2. Show all work on these pages. Show all work necessary to complete the problem. A solution without the appropriate work shown will receive no credit. A solution that is not given in a reasonable order will lose credit. Clearly indicate your answer (for example by enclosing it in a box).
3. It is assumed that your work will begin on the same page as the problem statement. If you choose to begin your work on another page, you must indicate this on the page with the problem statement, with a clear indication of where the work can be found. **If your work continues on to another page, indicate clearly where your work can be found. Failure to indicate this clearly will result in a loss of credit.**
4. Show all units in solutions, intermediate results, and figures. Units in the exam will be included between square brackets.
5. Do not use red ink. Do not use red pencil.
6. You will have 170 minutes to work on this exam.

1.	____________/30
2.	____________/30
3.	____________/35
4.	____________/35
5.	____________/35
6.	____________/35
	Total = 200

Room for extra work

1. {30 Points} Use the **node-voltage method** to write a complete set of equations that could be used to solve this circuit. Do not simplify the circuit. Do not attempt to simplify or solve your equations. **Define all variables clearly.**



6−1 eq. + 4 auxiliary eq = 9 equations

A + D $\quad -3[A] + 7[S]v_X + \dfrac{v_A}{17[\Omega]} - 5i_Z + \dfrac{v_D - v_E}{11[\Omega]} = 0$

A + D $\quad v_D - v_A = 10[V]$

B + C $\quad v_B - v_C = 23[V]$

C $\quad v_C = 6[\Omega]i_P$

Room for extra work

(E) $$\frac{v_E - v_D}{11[\Omega]} + \frac{v_E}{12[\Omega]} + \frac{v_E - 2[V]}{25[V]} = 0$$

(v_X) $$v_X + \frac{4v_Y}{40[\Omega]} \cdot 22[\Omega] - v_D + 9[A] \cdot 8[\Omega] = 0$$

(v_Y) $$v_Y + 3[A] \cdot 19[\Omega] = 0$$

(i_Z) $$-7[S]v_X - \frac{v_B}{21[\Omega]} + \frac{0}{3[\Omega]} + i_Z + \frac{4v_Y}{40[\Omega]} - \frac{v_E}{12[\Omega]} + \dots$$

$$\dots + 5i_Z - \frac{v_A}{17[\Omega]} = 0$$

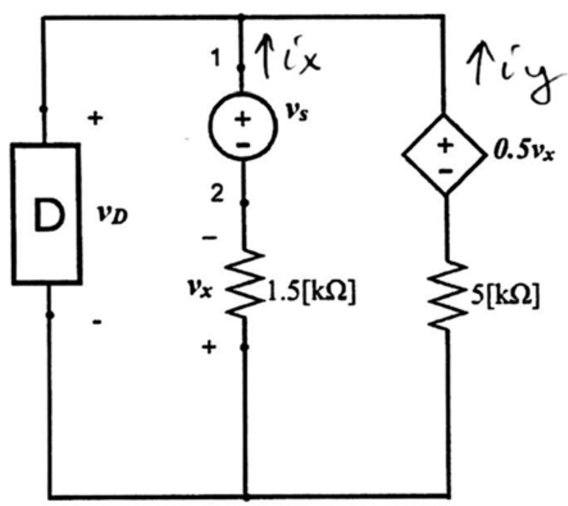
(i_P) $$-i_P - 3[A] + 7[S]v_X + \frac{v_A}{17[\Omega]} - 9[A] + \frac{0}{14[\Omega]} = 0$$

2. {30 Points} In the circuit shown below, positive charges move out from the independent voltage source v_s through terminal 1. Their number is given by

$$q_s(t) = 10e^{-20\left(\frac{1}{ms}\right)t}\ [\mu C]$$

The voltage on device D is constant v_D=100 [V].

a) Find an expression for power delivered by the voltage source v_s.
b) Find an expression for power delivered by the dependent voltage source $0.5v_x$.
c) Find the energy delivered by device D between 0 and 10 [s].



i_x direction is determined by the flow of ⊕ charges.

$$i_x(t) = \left|\frac{dQ_s(t)}{dt}\right|$$
$$= 20\cdot 10\left[\frac{\mu C}{ms}\right] e^{-20\left[\frac{1}{ms}\right]t}$$
$$= 200\, e^{-20{,}000\left[\frac{1}{s}\right]t}\ [mA]$$
$$= 0.2\, e^{-20{,}000\, t}\ [A]$$

units: $$\boxed{\frac{\mu C}{ms} = \frac{10^{-6}\, C}{\cancel{\mu C}} \cdot \frac{10^{3}\, \cancel{ms}}{s} \cdot \frac{\cancel{\mu C}}{\cancel{ms}} = 10^{-3}\,\frac{C}{s} = mA}$$

Find v_s

a) $$v_D = v_s - 1.5[k\Omega]\cdot i_x(t) = 100[V]$$
$$v_s = 100 + 300\, e^{-20{,}000\, t}\ [V]$$
$$P_{del,\, v_s} = v_s \cdot i_x(t)$$
$$= 20\, e^{-20{,}000 t} + 60 e^{-40{,}000 t}\ [W]$$

Room for extra work

b) Calculate $v_x \rightarrow i_y \rightarrow$ power

$$v_x = -100 + v_s = 300e^{-20{,}000t} \text{ [V]}$$

$$i_y = \frac{-100 + 0.5v_x}{5[k\Omega]} = -20 + 30e^{-20{,}000t} \text{ [mA]}$$

$$= -0.02 + 0.03e^{-20{,}000t} \text{ [A]}$$

$$P_{del, 0.5v_x} = 0.5v_x \cdot i_y$$

$$= 150e^{-20{,}000t} \text{ [V]} \cdot (-0.02 + 0.03e^{-20{,}000t})$$

$$= -3e^{-20{,}000t} + 4.5e^{-40{,}000t} \text{ [W]}$$

c) $P_{del, D} = -v_D(i_x + i_y)$

$$= -100 \cdot (0.2e^{-20{,}000t} - 0.02 + 0.03e^{-20{,}000t})$$

$$= -23e^{-20{,}000t} + 2 \text{ [W]}$$

$$W_{del, D} = -\int_0^{10[s]} 23e^{-20{,}000t}\,dt + \int_0^{10[s]} 2\,dt$$

$$= -\frac{23[W]}{-20{,}000[\frac{1}{s}]} e^{-20{,}000[\frac{1}{s}]t} \Big|_0^{10[s]} + 2t \Big|_0^{10[s]}$$

$$= -1.15[mJ] + 20[J] \cong 20[J]$$

3. {35 Points} A device can be modeled as a voltage source in series with a resistance. This device is shown in Figure 1. The relationship between the voltage across the device and the current through the device is shown in Figure 2. This device is connected to the circuit as shown in Figure 3.

a) Find a device model that would be valid for the current range, $i_D \geq -1[\text{mA}]$ and draw it, showing terminals **A** and **B**.

b) Find the power absorbed by the **dependent current source**.

c) Are the electrons moving through the dependent current source losing or gaining energy? Briefly explain your answer.

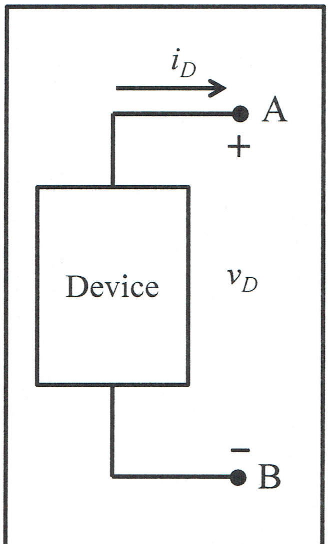


Figure 1

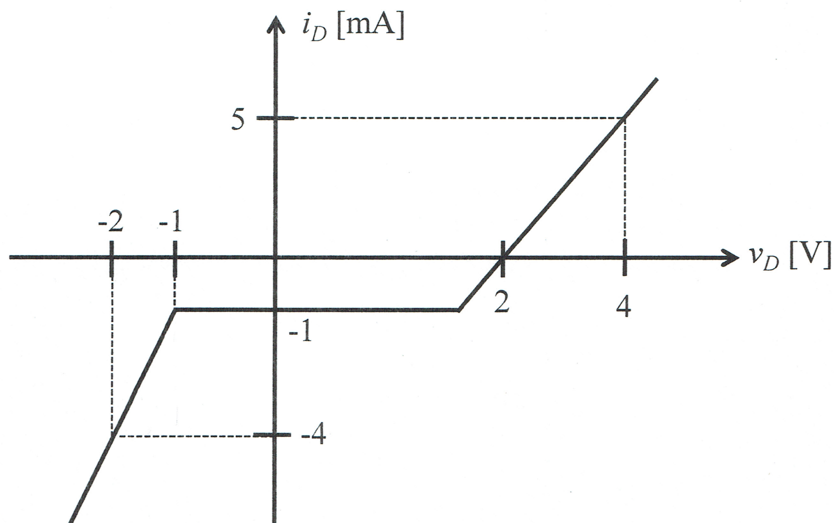


Figure 2

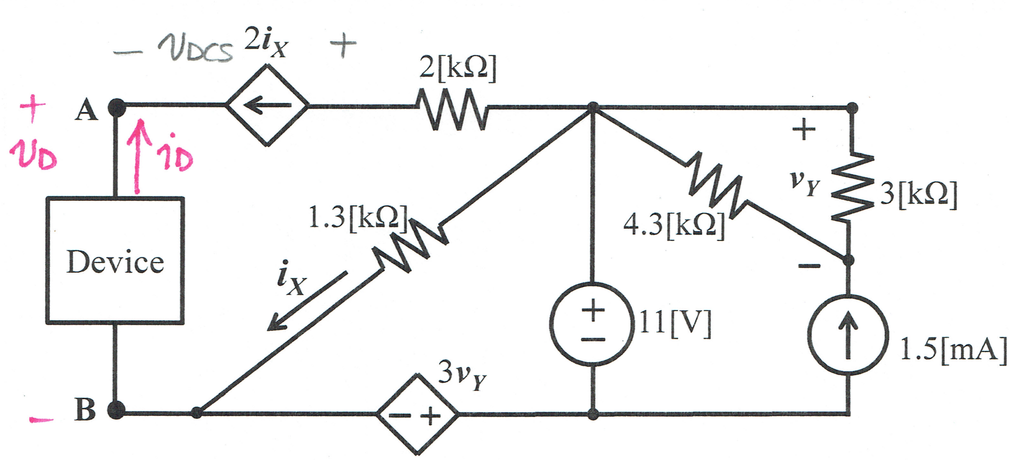
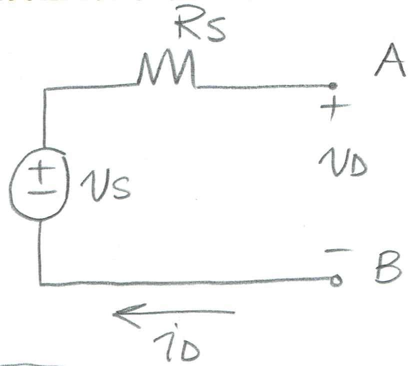


Figure 3

Room for extra work

a)



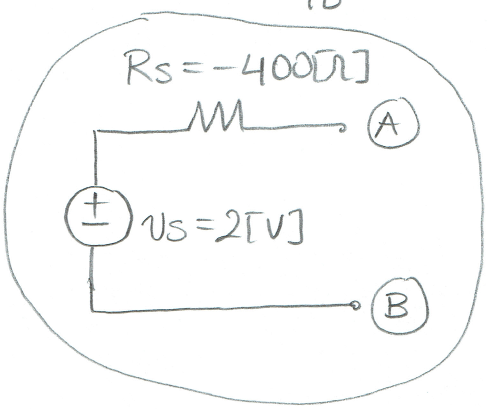
$-v_S + i_D R_S + v_D = 0$

For $i_D \geq -1[mA]$

$-v_S + 2[V] = 0 \Rightarrow v_S = 2[V]$

$-v_S + 5[mA] R_S + 4[V] = 0$

$\Rightarrow R_S = -400[\Omega]$



b) $v_Y = -1.5[mA] \cdot (3[k\Omega] // 4.3[k\Omega])$

$v_Y = -2.65[V]$

$-i_X \cdot 1.3[k\Omega] + 11[V] + 3 v_Y = 0 \Rightarrow i_X = 2.35[mA]$

$i_D = -2 i_X = -4.7[mA] \rightarrow$ Since $i_D < -1[mA]$, the device model found in part (a) is NOT valid.

$i_D = 3\left[\frac{mA}{V}\right] \cdot v_D + 2[mA]$ for $i_D \leq -1[mA]$

For $i_D = -4.7[mA]$, $v_D = -2.23[V]$

$-v_{DCS} - 2 i_X \cdot 2[k\Omega] + i_X \cdot 1.3[k\Omega] - v_D = 0$

$\Rightarrow v_{DCS} = -4.115[V]$

$p_{ABS,DCS} = 2 i_X \cdot v_{DCS} = \boxed{-19.34[mA]}$

c) Since $p_{ABS,DCS} < 0$, dependent current source delivers power $\rightarrow$ e^-s moving through it gain energy.

4. {35 Points} **DO NOT** use the Node-Voltage Method or Mesh-Current Method to solve this problem.

a) Find the Norton equivalent seen at terminals A and B in Figure 1, by applying **ONLY** 3 source transformations and combining parallel and series components together.

b) Find the Norton equivalent seen at terminals **C** and **D** in Figure 2, by using equivalent resistances.

c) Attach the circuit in Figure 2 to the circuit in Figure 1 by connecting terminal **C to** terminal **B** and terminal **D to** terminal **A**. Find the power delivered by i_S.

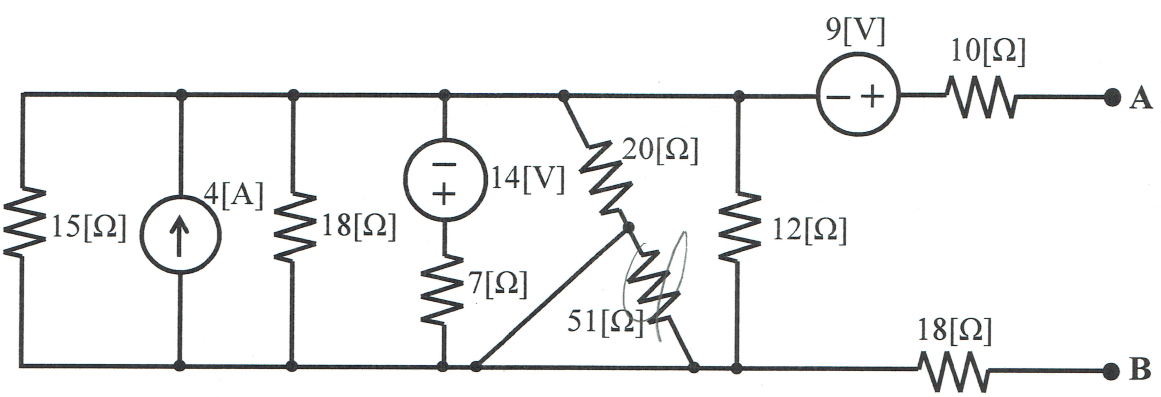


Figure1

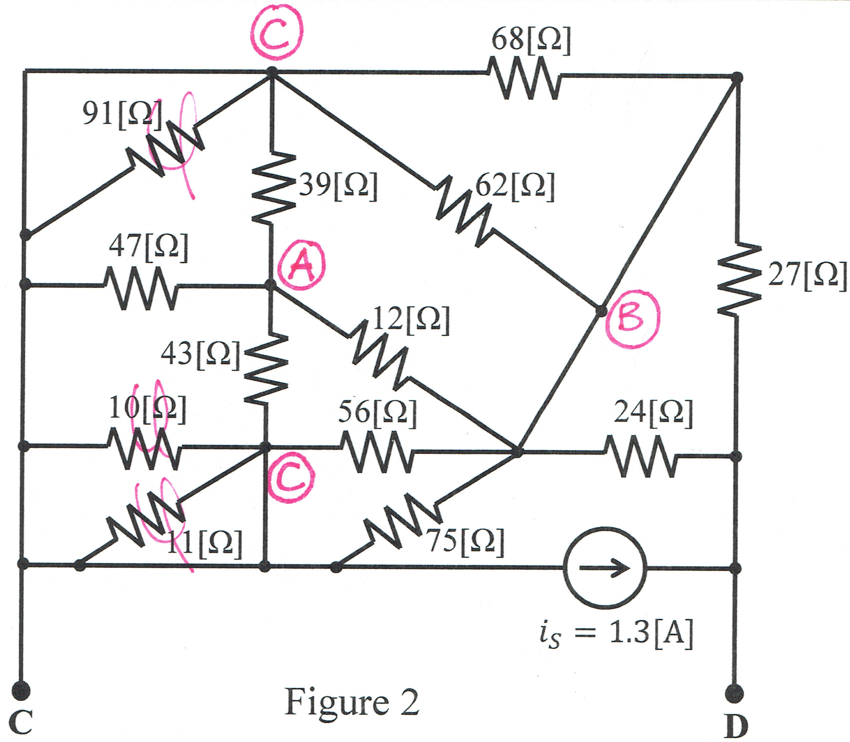
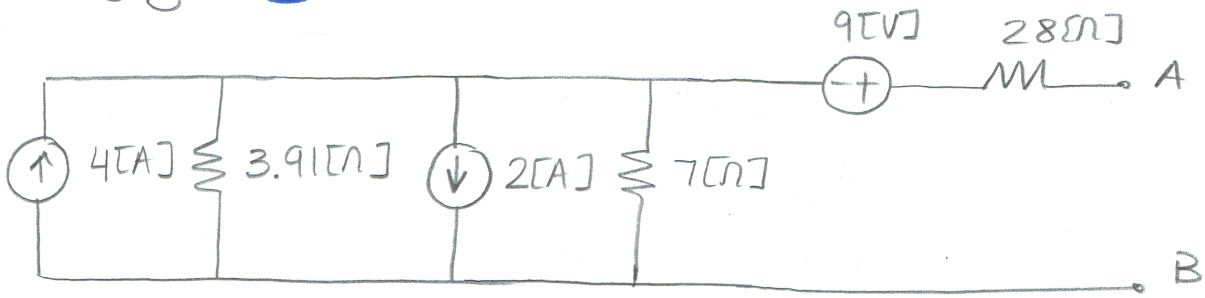


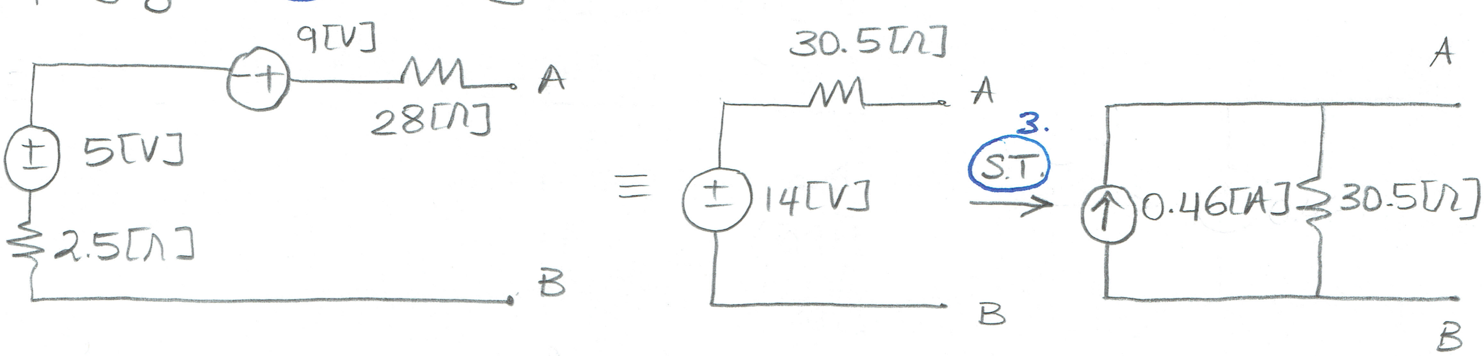
Figure 2

Room for extra work

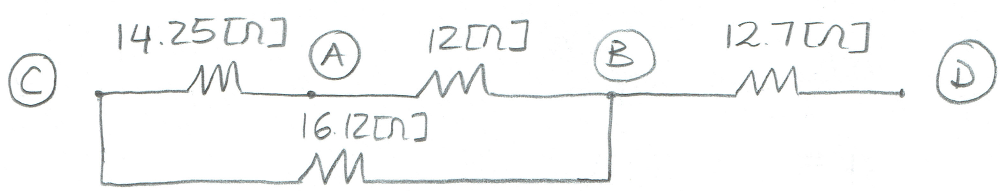
a) 51[Ω] is shorted   15[Ω] // 18[Ω] // 20[Ω] // 12[Ω] = 3.91[Ω]
Applying S.T. 1. to 14[V] in series w/ 7[Ω] and redrawing:



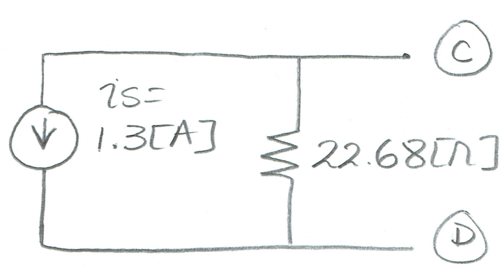
Combining parallel current sources and resistors, and then applying S.T. 2. we get: (3.91[Ω] // 7[Ω] = 2.5[Ω])



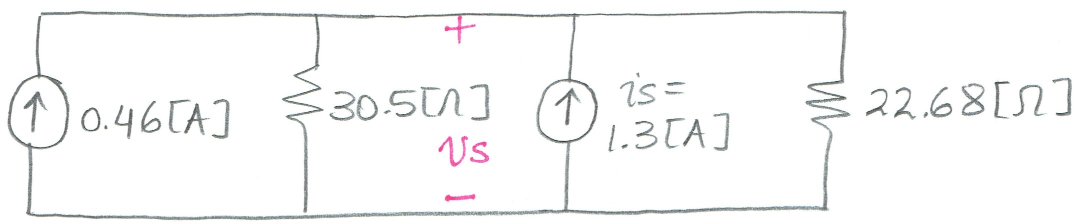
b) 39[Ω] // 47[Ω] // 43[Ω] = 14.25[Ω]
68[Ω] // 62[Ω] // 56[Ω] // 75[Ω] = 16.12[Ω]
27[Ω] // 24[Ω] = 12.7[Ω]



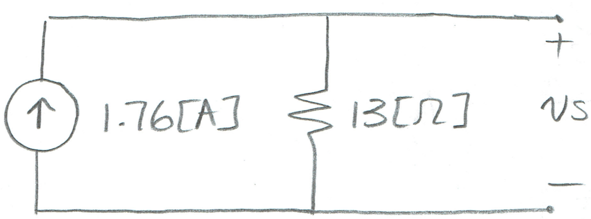
$R_{CD} = [(14.25[\Omega] + 12[\Omega]) \parallel 16.12[\Omega]] + 12.7[\Omega] = 22.68[\Omega]$



c) Attaching the circuits from a) and b) together, we get:



$22.68[\Omega] \,//\, 30.5[\Omega] = 13[\Omega]$



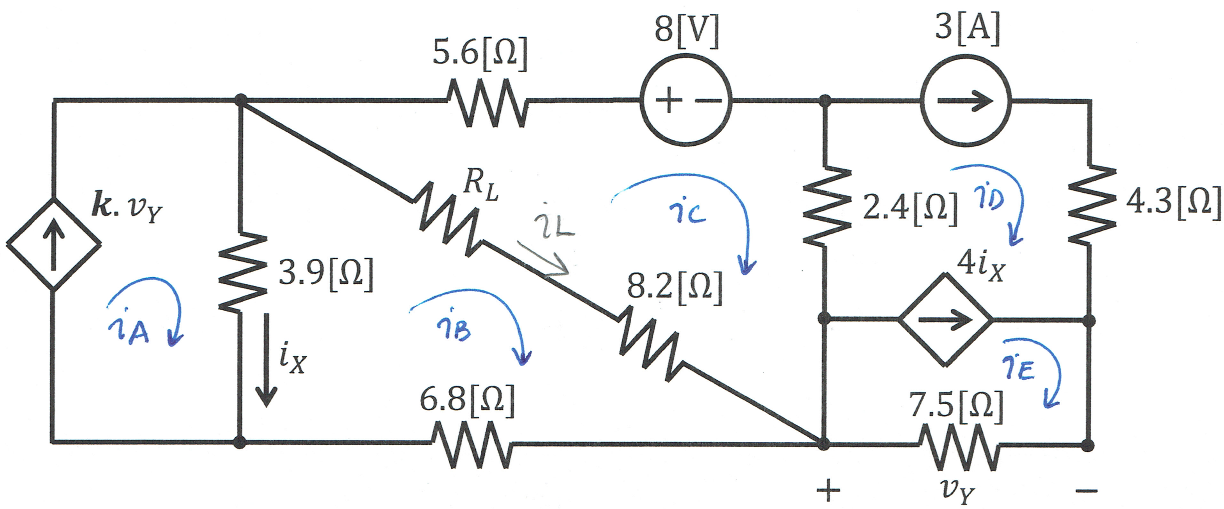
$v_s = 22.88\,[V]$

$p_{DEL,i_s} = v_s \cdot i_s = 22.88[V] \cdot 1.3[A] = \boxed{29.744\,[W]}$

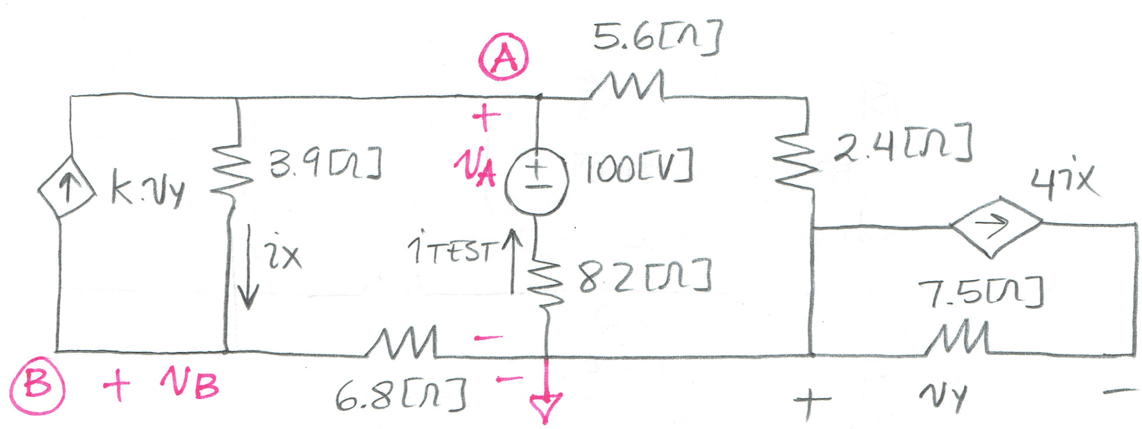
5. {35 Points} In the following circuit, the value of R_L must be chosen to be 13.2[Ω] in order to deliver maximum power to R_L.

a) Find $\boldsymbol{k}$.

b) Find the power delivered to R_L.



a) To deliver max power to R_L, R_L must be set equal to R_{TH} seen by R_L. To find R_{TH} seen by R_L, kill ind. sources and apply test source method.



$$R_{TH} = 13.2[\Omega] = \frac{100[V]}{i_{TEST}} \Rightarrow i_{TEST} = 7.57[A]$$

Applying NVM:

$$(A) \quad -kv_Y + \frac{v_A - v_B}{3.9[\Omega]} + \frac{v_A - 100[V]}{8.2[\Omega]} + \frac{v_A}{8[\Omega]} = 0$$

Room for extra work

B $\quad k \cdot v_Y + \dfrac{v_B - v_A}{3.9[\Omega]} + \dfrac{v_B}{6.8[\Omega]} = 0$ $\qquad$ i_X $\quad \dfrac{v_A - v_B}{3.9[\Omega]} = i_X$

v_Y $\quad v_Y = -4 i_X \cdot 7.5[\Omega] = -30 i_X$

i_{TEST} $\quad \dfrac{v_A - 100[V]}{8.2[\Omega]} = -i_{TEST} = -7.57[A]$

Solving, we get $\boxed{k = -13.65[mS]}$

b) $p_{ABS,R_L} = R_L \cdot i_L^2$ $\quad$ To find i_L, MCM is applied. Mesh currents are defined on the circuit.

i_A $\quad i_A = k \cdot v_Y$

i_B $\quad 3.9[\Omega](i_B - i_A) + (R_L + 8.2[\Omega])(i_B - i_C) + 6.8[\Omega] i_B = 0$

i_C $\quad 5.6[\Omega] i_C + 8[V] + 2.4[\Omega](i_C - i_D) + (R_L + 8.2[\Omega])(i_C - i_B) = 0$

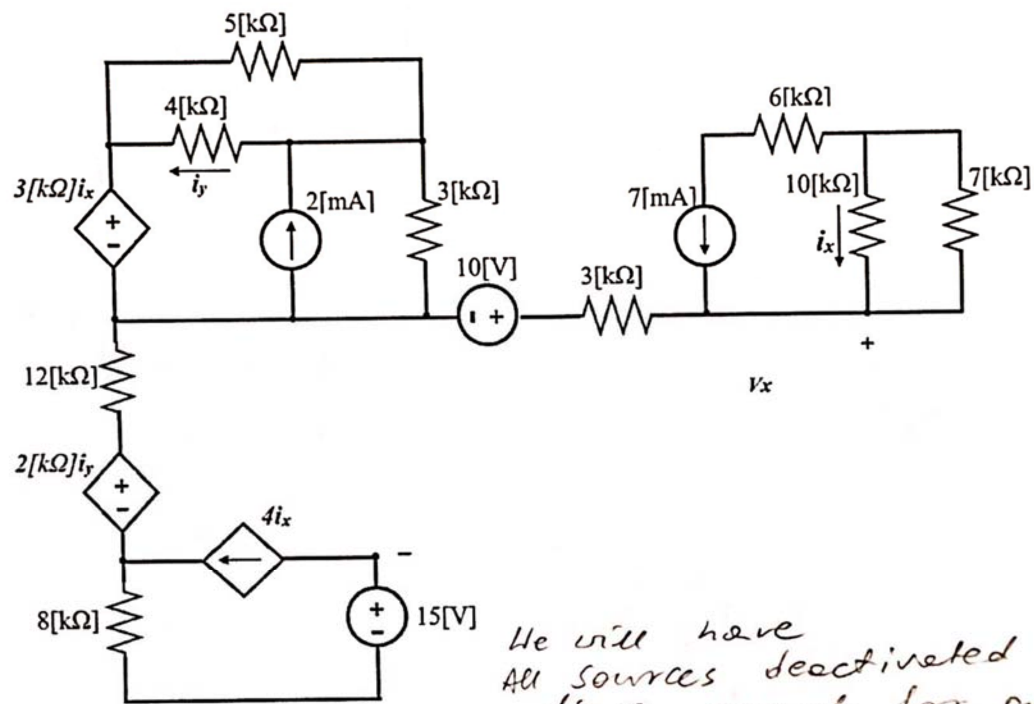
i_D $\quad i_D = 3[A]$ $\qquad$ $i_D + i_E$ $\quad i_E - i_D = 4 i_X$

v_Y $\quad v_Y = -7.5[\Omega] i_E$ $\qquad$ i_X $\quad i_X = i_A - i_B$ $\qquad$ i_L $\quad i_L = i_B - i_C$

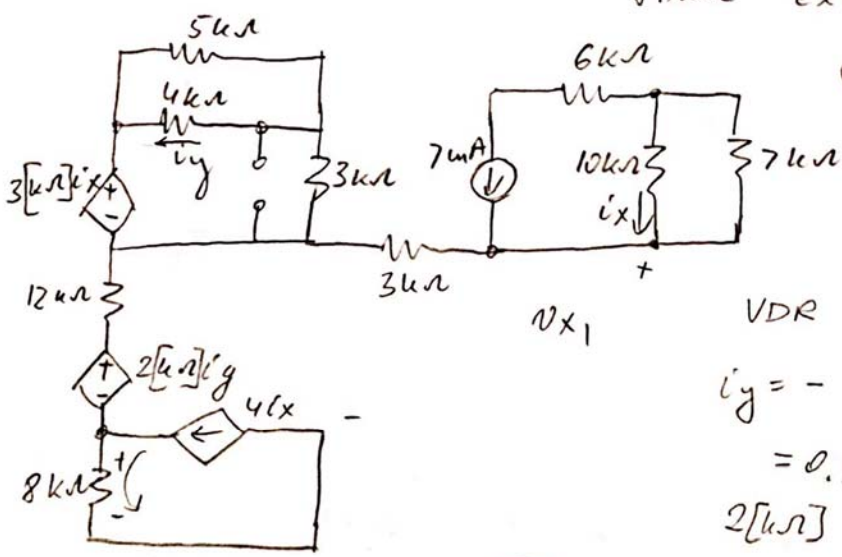
Using $k = -13.65[mS]$ and $R_L = 13.2[\Omega]$ and solving,

we get: $\quad i_L = 47.67[mA]$ $\quad \boxed{p_{ABS,R_L} = 30[mW]}$

6. {35 Points} Use the circuit shown below to solve this problem.
   a) Use superposition to find the voltage v_x.
   b) Find the power delivered by the dependent voltage source $3[k\Omega]i_x$.



We will have All sources deactivated @ a time except for one:



7mA

$$i_x = -7[mA]\cdot\frac{10\|7}{10} = -2.88[mA]$$

VDR for i_y

$$i_y = -\frac{3[k\Omega]\cdot(-2.88[mA])}{3+\underbrace{4\|5}_{2.22k\Omega}}\cdot 4\|5\cdot\frac{1}{4[k\Omega]}$$

$$= 0.92[mA]$$

$$2[k\Omega]\cdot i_y = 1.84[V]$$

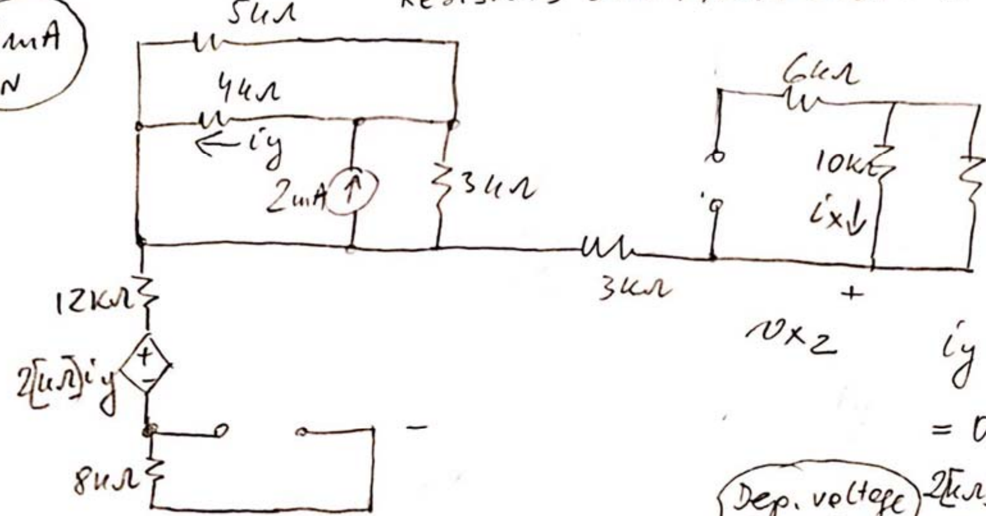
First $v_{x_1} = 4\cdot(-2.58[mA])\cdot 8[k\Omega] + 2[k\Omega]\cdot i_y$

$$= -90.32[V]$$

Room for extra work

Resistors $3k\Omega \parallel 4k\Omega \parallel 5k\Omega = 1.277k\Omega$

2mA ON



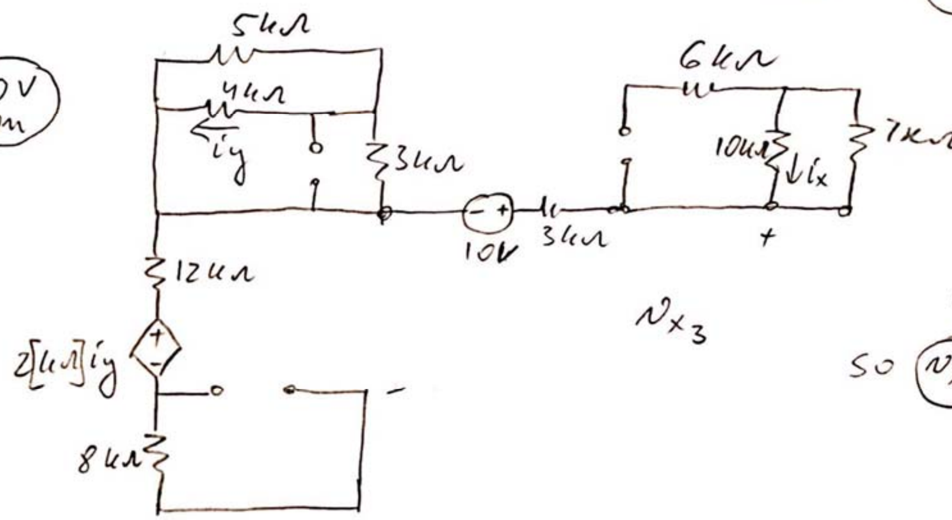
$$i_y = 2[mA] \cdot \frac{3\parallel 4 \parallel 5}{4[k\Omega]}$$

$$= 0.64[mA]$$

Dep. voltage source $2[k\Omega] \cdot i_y = 1.277[V]$

$v_{x2} = +1.277[V]$

10V on

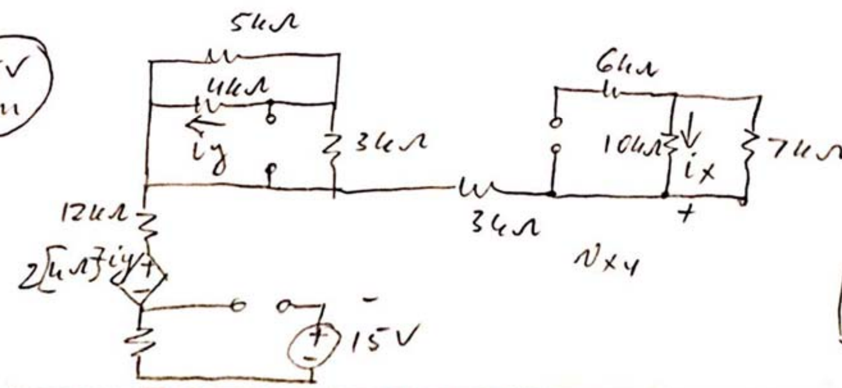


$i_x = 0$

$i_y = 0$

so $v_{x3} = +10[V]$

15V on



$i_x = 0$

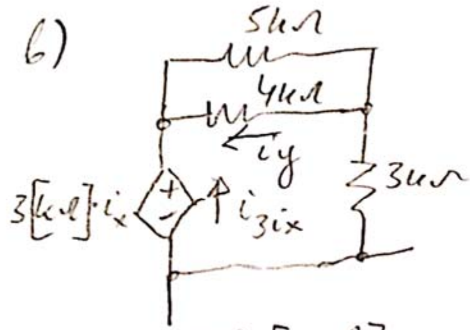
$i_y = 0$

so $v_{x4} = -15[V]$

$\boxed{V_x = -94.04[V]}$

Room for extra work

b)



From a) with 7mA current source active we need current i'_{3ix}

$R_{eq} = 5k\Omega \parallel 4k\Omega + 3k\Omega = 5.22[k\Omega]$

$i'_{3ix} = \dfrac{+3[k\Omega]\cdot i'_x}{R_{eq}} = \dfrac{-8.64\,[V]}{5.22\,[k\Omega]} \simeq -1.66\,[mA]$

$i'_x = -2.88\,[mA]$

$P_{del,3ix} = 3[k\Omega]\cdot i'_x \cdot i'_{3ix} = 14.3\ [W]$