Name: ______________________________ (please print)

Signature: ______________________________

ECE 2201 – Final Exam
May 4, 2023
Keep this exam closed and face up until you are told to begin.

1. This exam is closed book, closed notes. You may use any calculator. You may **not** use a cell phone, tablet computer, nor laptop computer. You may have a crib sheet in the form of one 8 ½" x 11" piece of paper, with material written on both sides.
2. Print your name, and provide your signature above.
3. Show all work on these pages. Show all work necessary to complete the problem. A solution without the appropriate work shown will receive no credit. A solution which is not given in a reasonable order will lose credit. You may separate the pages as you work.
4. Show all units in solutions, intermediate results, and figures. Units in the exam will be included between square brackets.
5. If the grader has difficulty following your work because it is messy or disorganized, you will lose credit.
6. Do not use red ink. Do not use red pencil.
7. You will have 160 minutes to work on this exam.

1.	______________/25
2.	______________/30
3.	______________/30
4.	______________/40
5.	______________/40
6.	______________/35

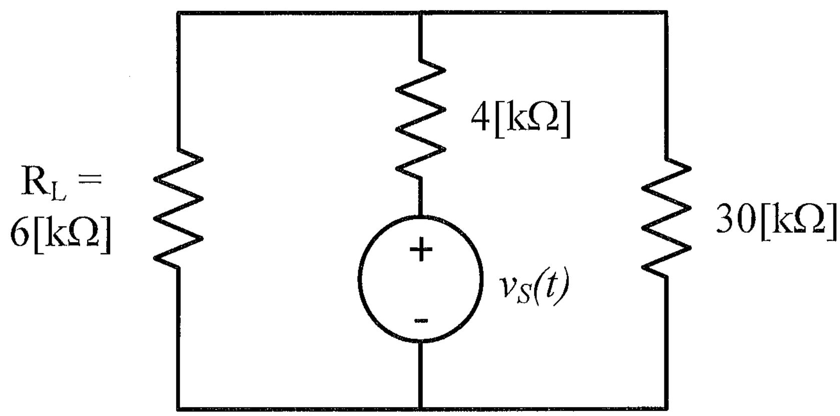
Total = 200

Room for extra work

1. (25 points) In the circuit below, the voltage source has the following value.

$$v_S(t) = 20[kV]e^{-0.1\left[\frac{1}{ms}\right]t}$$

Find the energy delivered to the load resistor R_L between 0 and 3.5[ms].

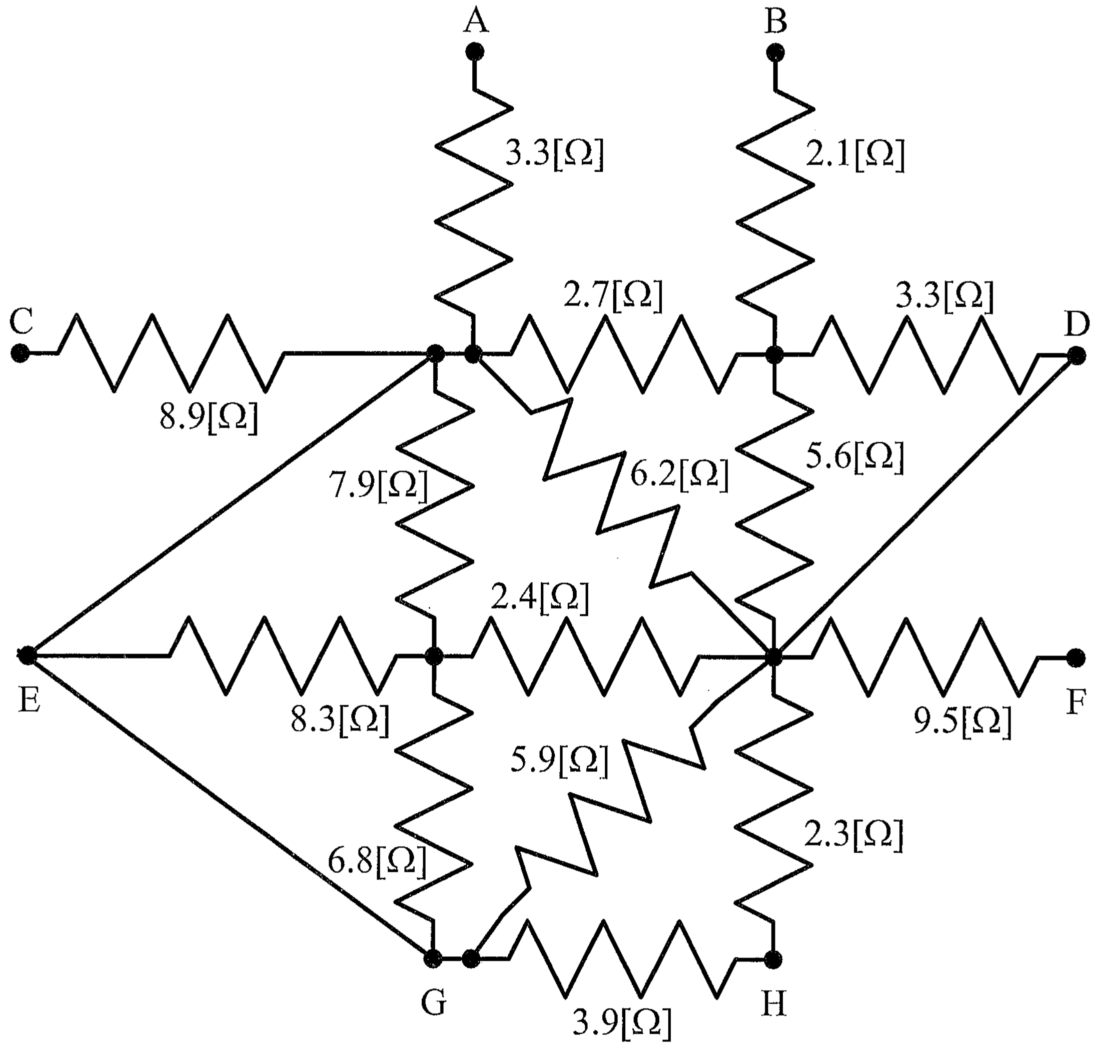


Room for extra work

2. (30 points) Use the circuit given below to solve.

a) Find the equivalent resistance as seen by terminals A and B.

b) Find the equivalent resistance with respect to terminals D and E.



Room for extra work

3. (30 points) Device D shown in Figure 1 can be modeled by a Norton equivalent. The terminal voltage v_D and current i_D for device D are shown in the graph in Figure 2. Two identical copies of device D are inserted into the circuit shown in Figure 3, and connected at terminals a, b as indicated in Figure 3.

Find the voltage v_A in Figure 3.

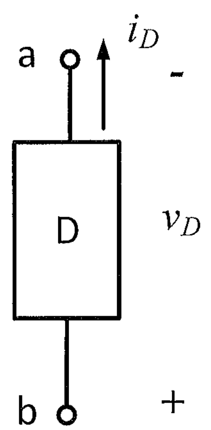


Figure 1

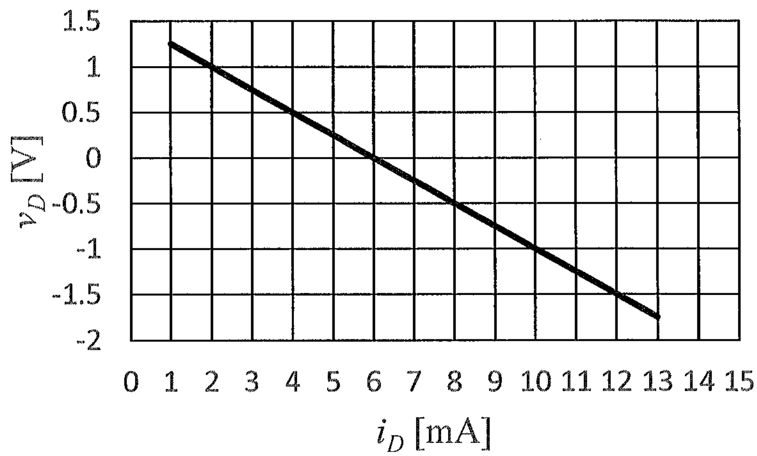


Figure 2

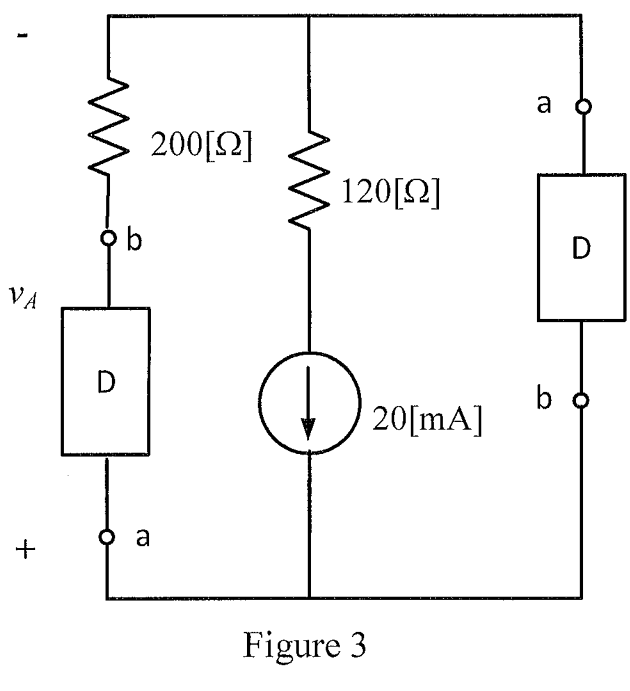
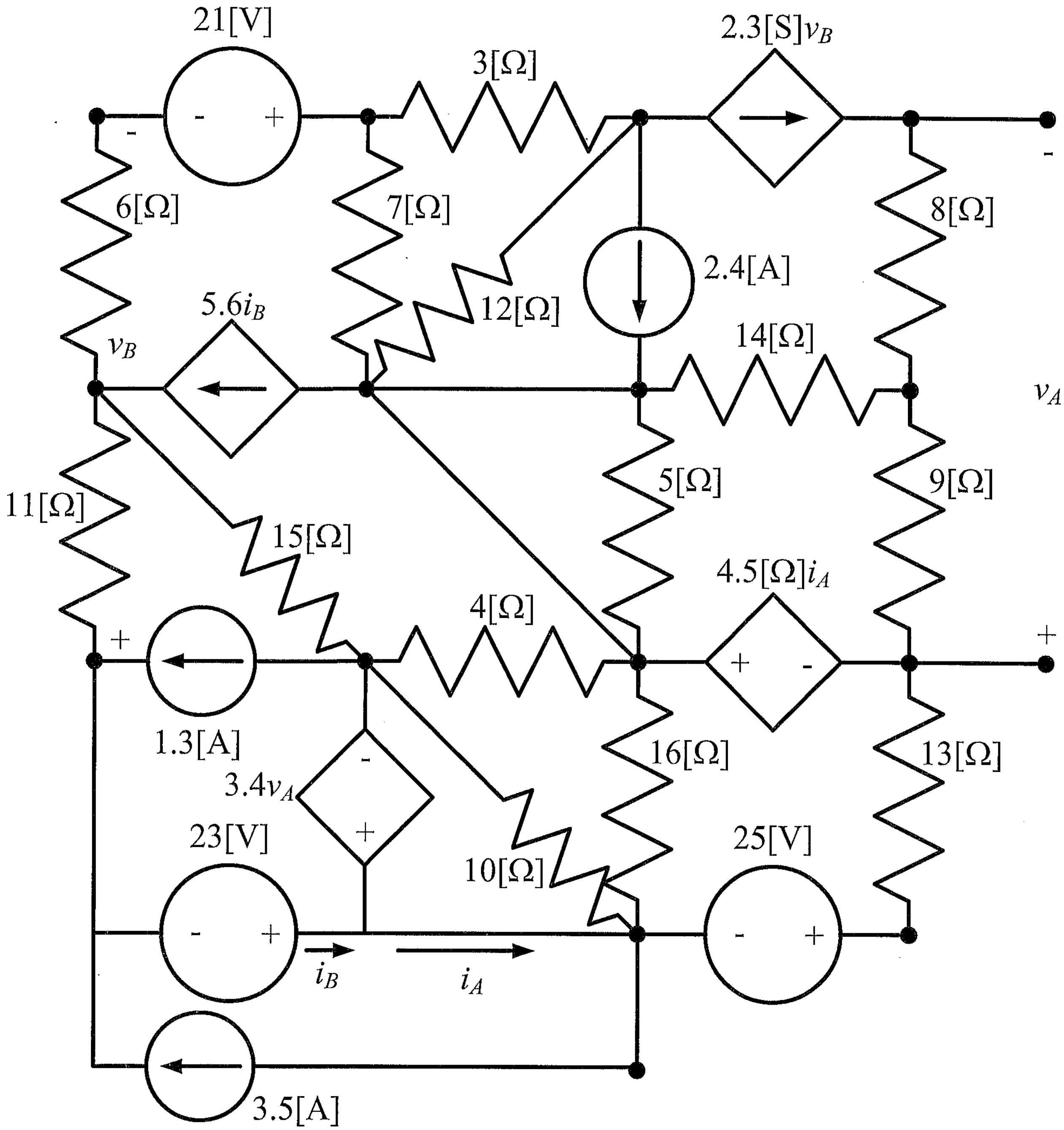


Figure 3

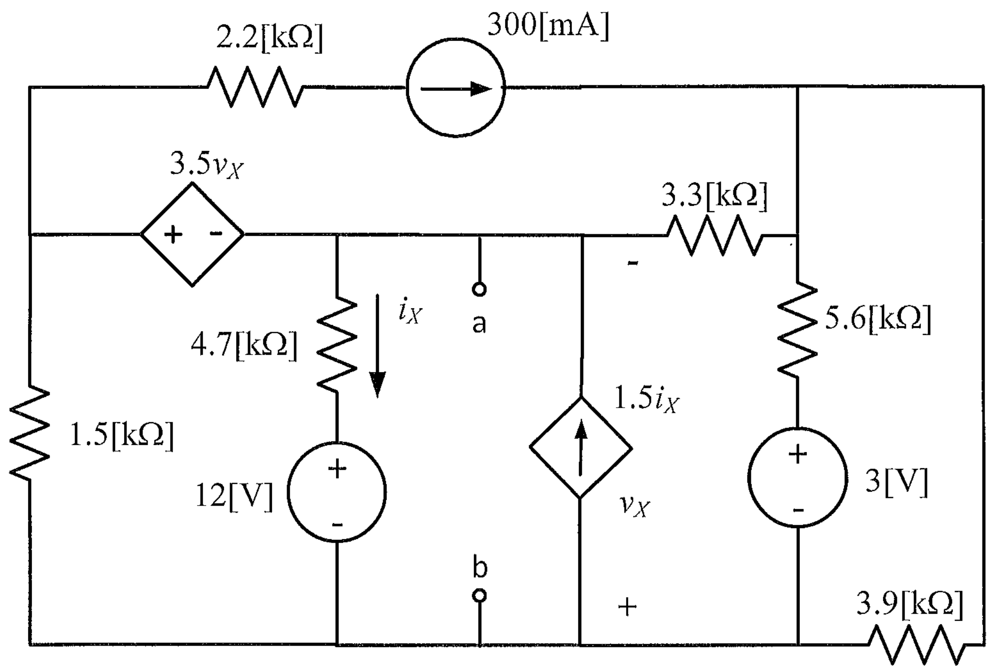
Room for extra work

4. (40 points) Use the node-voltage method to write a complete set of equations that could be used to solve this circuit. Do not simplify the circuit. Do not attempt to solve or simplify your equations. Define all variables appropriately.



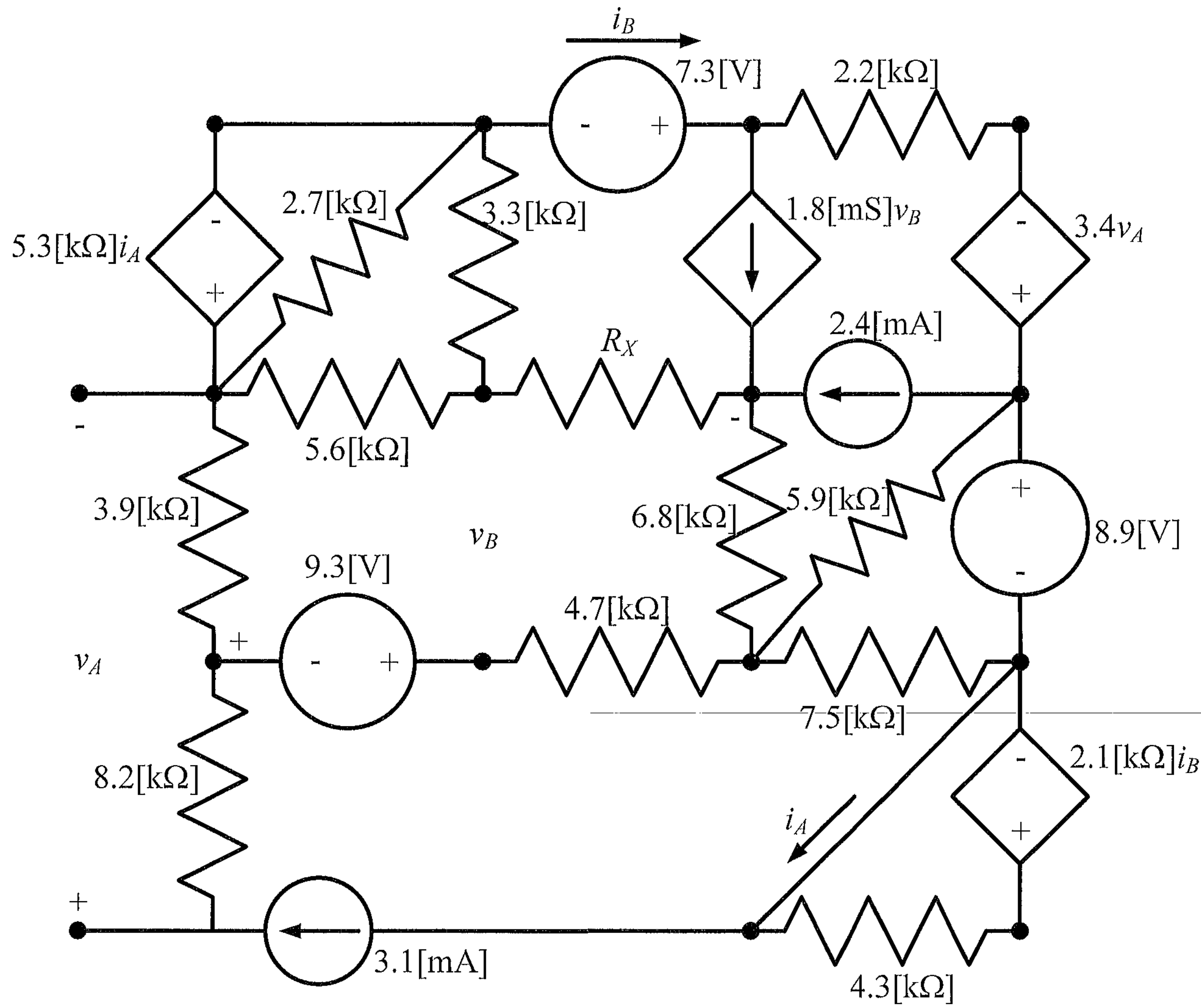
Room for extra work

5. (40 points) For the circuit shown below, find the Norton equivalent at terminals a, b. Draw the Norton equivalent, clearly showing the parameter values and the terminals a, b.



Room for extra work

6. (35 points) For the circuit given below, the resistor R_X is equal to 1[kΩ]. Find the Thevenin equivalent resistance as seen by R_X.

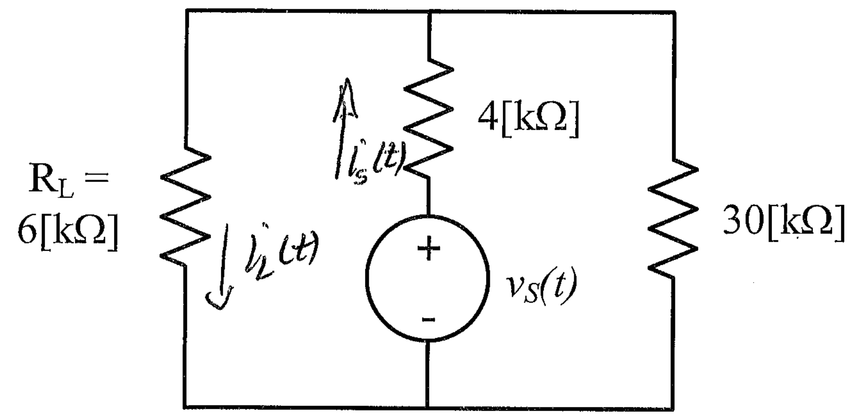


Room for extra work

1. (25 points) In the circuit below, the voltage source has the following value.

$$v_S(t) = 20[kV]e^{-0.1\left[\frac{1}{ms}\right]t}$$

Find the energy delivered to the load resistor R_L between 0 and 3.5[ms].



We can find $i_S(t)$ if we combine $30[k\Omega]$ and $6[k\Omega]$:
$30[k\Omega] \parallel 6[k\Omega] = 5[k\Omega]$.

Then $i_S(t) = \dfrac{v_S(t)}{5000 + 4000} = \dfrac{v_S(t)}{9000\,[\Omega]}$

Now $i_L(t) = i_S(t)\dfrac{30}{30+6} = 9.259 \times 10^{-5}\left[\frac{A}{V}\right] v_S(t)$

So power absorbed by (delivered to) R_L is

$P_{abs\ by\ R_L} = \left(9.259 \times 10^{-5}\left[\frac{A}{V}\right] v_S(t)\right)^2 \cdot R_L$

+15 $\quad = 2.058 \times 10^4\,[W]\, e^{-0.2\left[\frac{1}{ms}\right]t}$

Finally,

$W_{abs\ by\ R_L} = \int_0^{t_0} (2.058 \times 10^4\,[W])\, e^{-0.2\left[\frac{1}{ms}\right]t}\, dt$

Room for extra work

For t_0 in [ms] :

+5 integral

$$W_{abs\ by\ R_L} = \int_0^{3.5[ms]} 2.058\times10^4 [W]\, e^{-0.2[\frac{1}{ms}]t}\, \underbrace{dt}_{[ms]}$$

+15 calculation

$$\boxed{= 5.179\times10^4 [mJ]}$$

For t_0 in [s] :

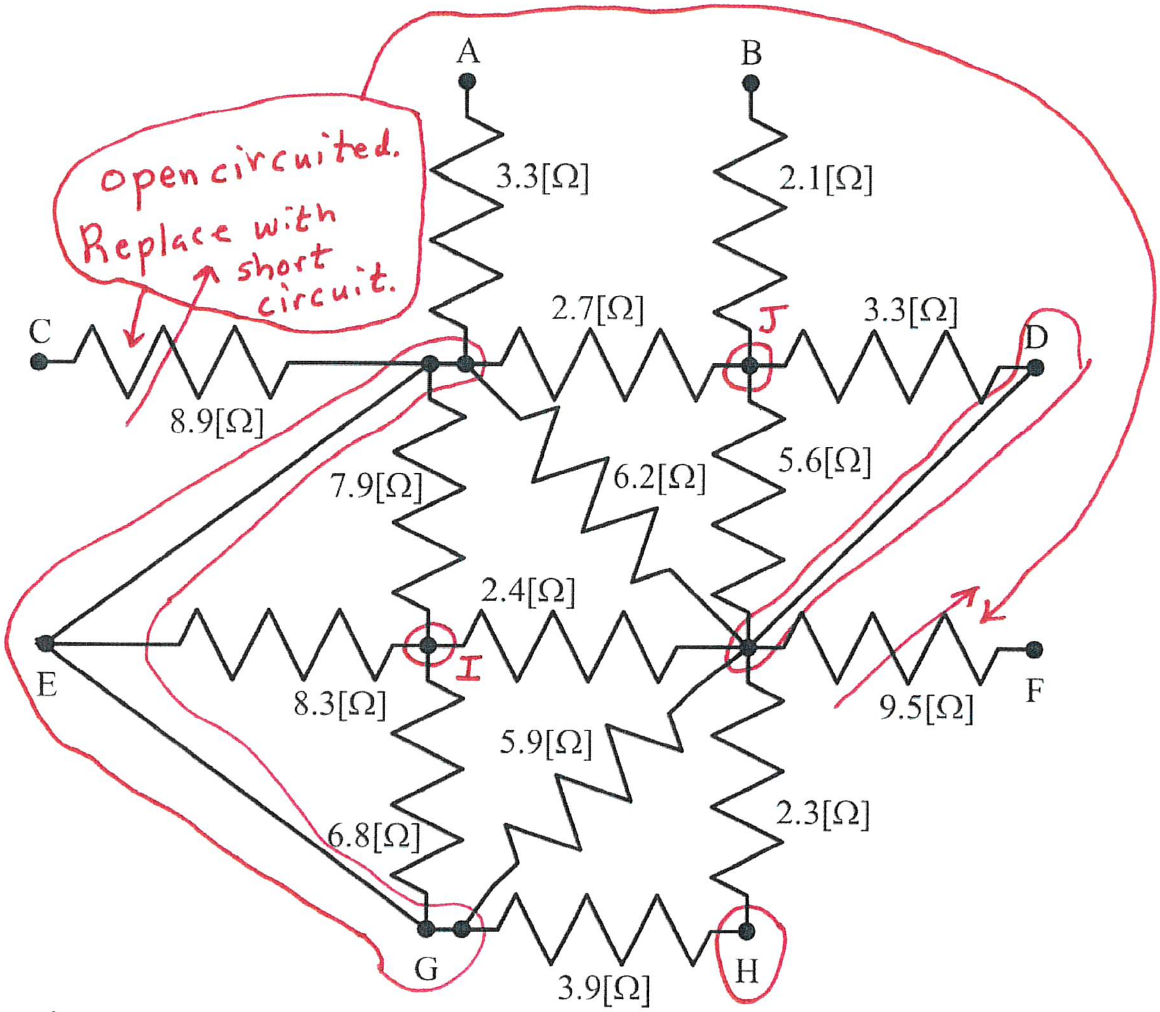
$$W_{abs\ by\ R_L} = \int_0^{0.0035} 2.058\times10^4 [W]\, e^{-200[\frac{1}{s}]t}\, \underbrace{dt}_{[s]}$$

$$\boxed{= 5.179\times10^1 [J]}$$

wrong units [J] vs [mJ] −3

W in units of [W] −5

2. (30 points) Use the circuit given below to solve.
a) Find the equivalent resistance as seen by terminals A and B.
b) Find the equivalent resistance with respect to terminals D and E.



We began by identifying the nodes, and naming those that did not have names.

We can make the following simplifications:

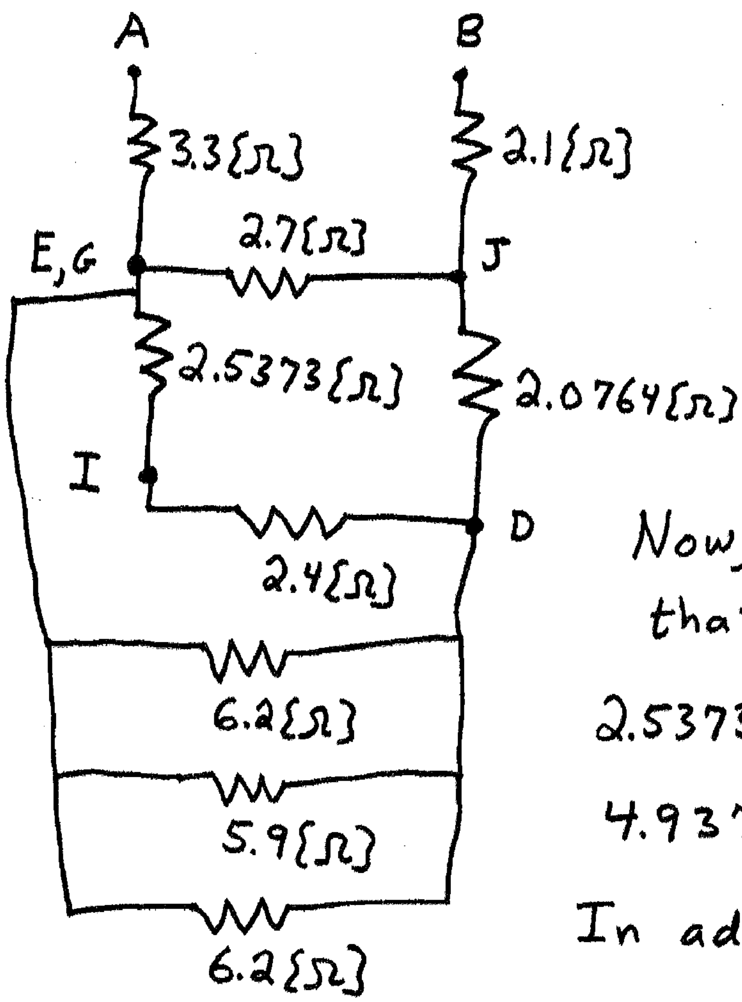
$$7.9\{\Omega\} \,||\, 8.3\{\Omega\} \,||\, 6.8\{\Omega\} = 2.5373\{\Omega\}$$

$$3.3\{\Omega\} \,||\, 5.6\{\Omega\} = 2.0764\{\Omega\}$$

$$3.9\{\Omega\} + 2.3\{\Omega\} = 6.2\{\Omega\}$$

See next Page

With these simplifications, we redraw.



Now, we can see that

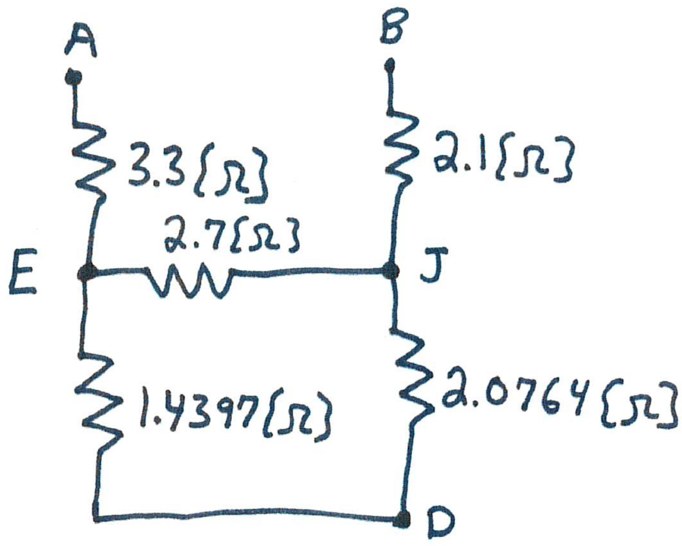
$2.5373\{\Omega\} + 2.4\{\Omega\} = 4.9373\{\Omega\}$.

In addition, we have

$$4.9373\{\Omega\} \| 6.2\{\Omega\} \| 5.9\{\Omega\} \| 6.2\{\Omega\} = 1.4397\{\Omega\}.$$

With these simplifications, we can redraw again.

See next page



a) Seen from A and B,

$$\left(1.4397\{\Omega\} + 2.0764\{\Omega\}\right) \parallel 2.7\{\Omega\} = 1.5272\{\Omega\}$$

$$R_{AB} = 3.3\{\Omega\} + 1.5272\{\Omega\} + 2.1\{\Omega\}$$

$$\boxed{R_{AB} = 6.9272\{\Omega\}}$$

b) Seen from E and D,

$$2.7\{\Omega\} + 2.0764\{\Omega\} = 4.7764\{\Omega\}$$

$$R_{DE} = 1.4397\{\Omega\} \parallel 4.7764\{\Omega\}$$

$$\boxed{R_{DE} = 1.1063\{\Omega\}}$$

3. (30 points) Device D shown in Figure 1 can be modeled by a Norton equivalent. The terminal voltage v_D and current i_D for device D are shown in the graph in Figure 2. Two identical copies of device D are inserted into the circuit shown in Figure 3, and connected at terminals a, b as indicated in Figure 3.

Find the voltage v_A in Figure 3.

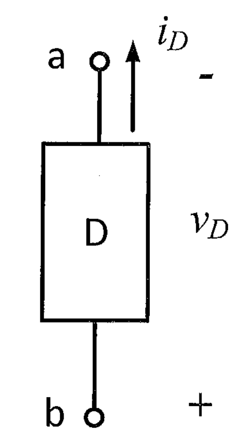


Figure 1

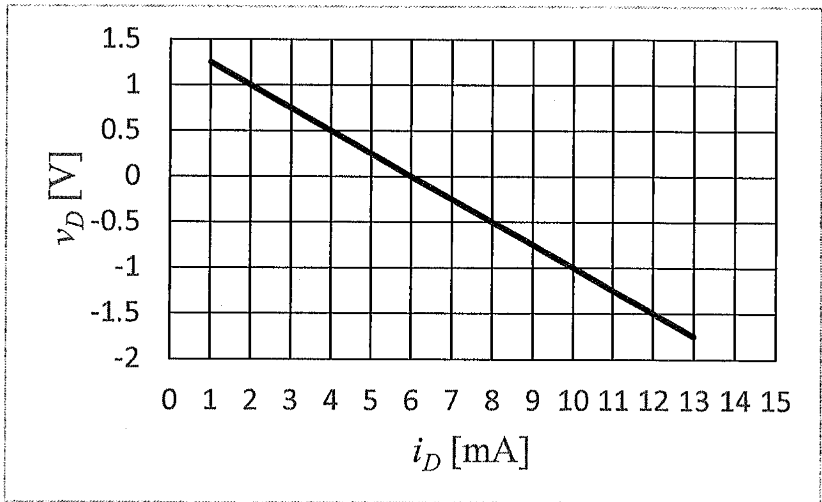


Figure 2

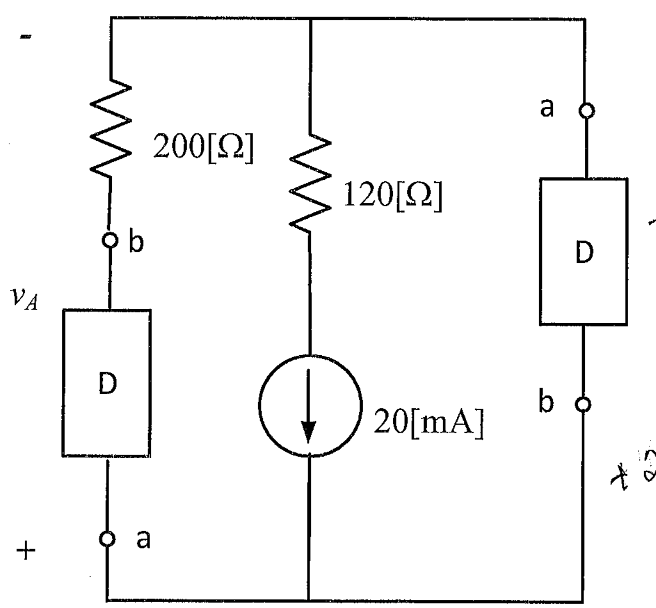
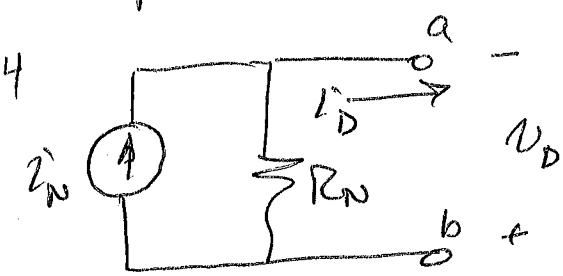


Figure 3

+4

+2

We first need a model for device D:



$$-i_N - \frac{v_D}{R_N} + i_D = 0$$

+2

$v_D = 0 \Rightarrow i_D = 0.006$ [A] $\Rightarrow i_N = i_D = 0.006$ [A]

+8

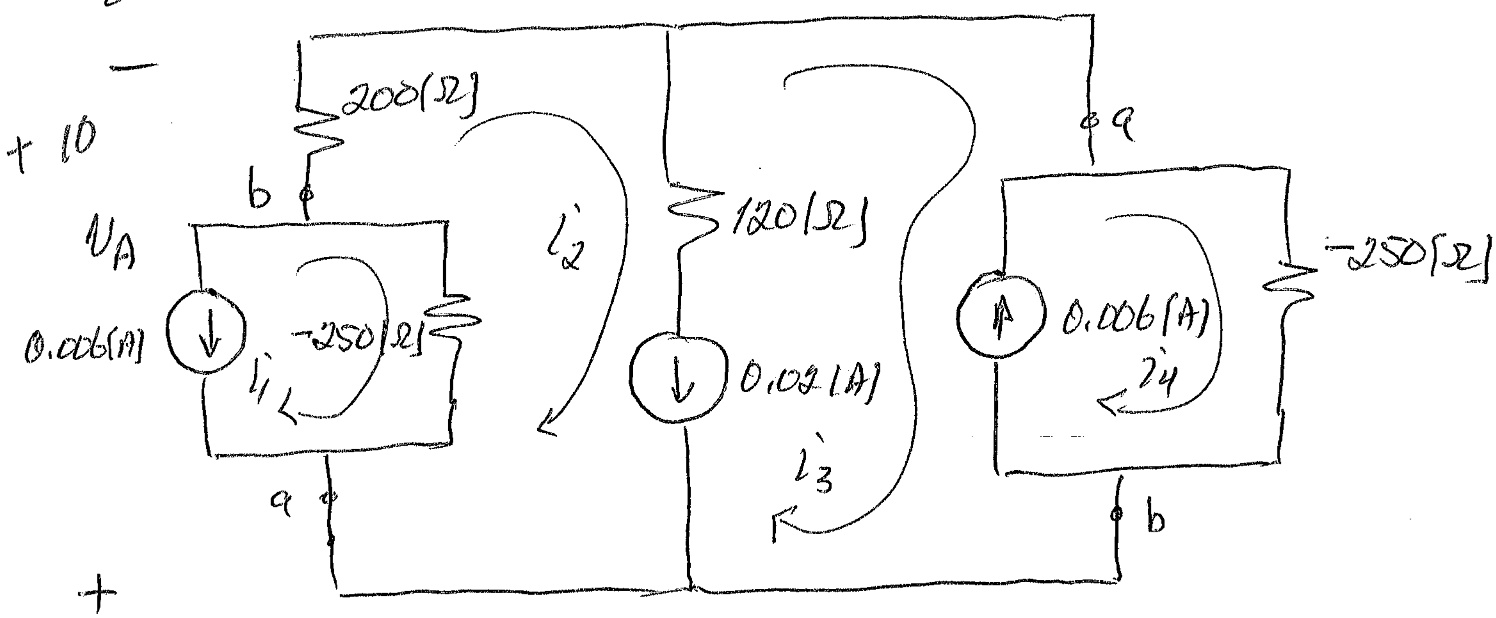
$v_D = -1.5$ [V] $\Rightarrow -0.006 + \frac{1.5}{R_N} + 0.012 = 0 \Rightarrow R_N = -250$ [Ω]

$i_D = 0.012$ (~~m~~A)

Room for extra work

We will insert this model into the circuit, noting that the device on left has terminal b facing upward, but the device on the right has it facing downward.

Note also that the circuit is easier to solve if we do a source transformation to a Thevenin equivalent. We will do it both ways.



+7

$$i_1' = -0.006\,[A] \qquad i_4' - i_3' = 0.006\,[A]$$

$$200\, i_2' + (-250)\, i_4' + (-250(i_2' - i_1')) = 0$$

$$i_2' - i_3' = 0.02$$

$$i_2' = 6.667\,[mA]$$

$$i_3' = -13.333\,[mA]$$

$$i_4' = -7.333\,[mA]$$

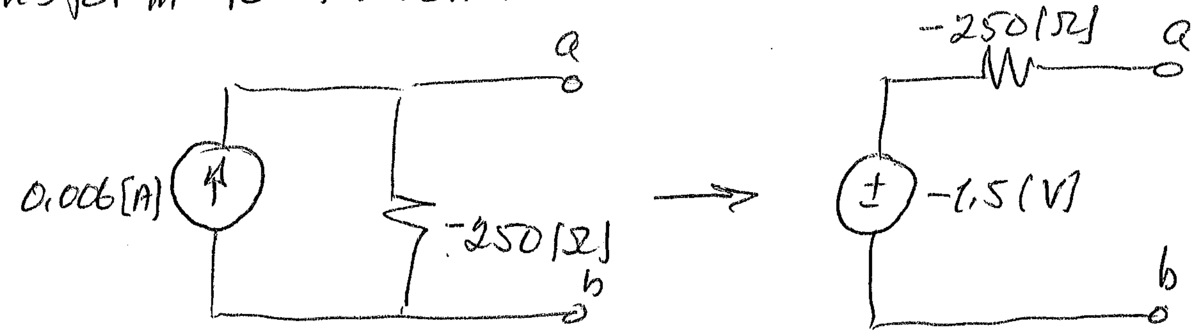
+13

$$\boxed{v_A = 200\, i_2' - 250\,(i_2' - i_1') = -1.833\,[V]}$$

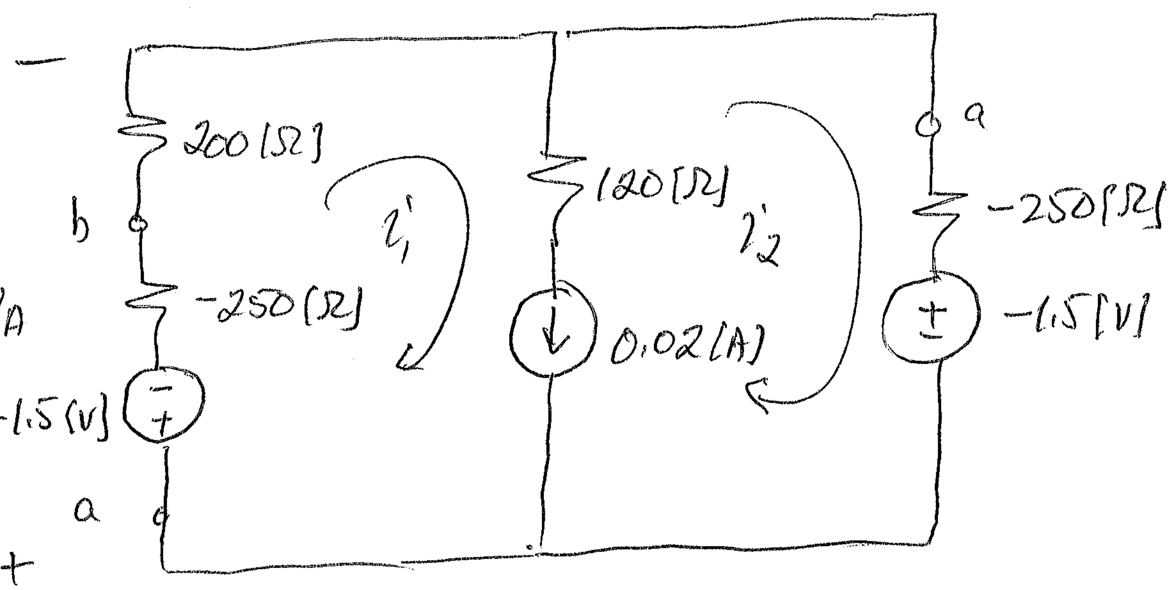
Room for extra work

Transform to Thevenin:

+2



+10



+6

$$200 i_1' - 250 i_2' - 1.5 - 1.5 - 250 i_1' = 0$$

$$i_1' - i_2' = 0.02$$

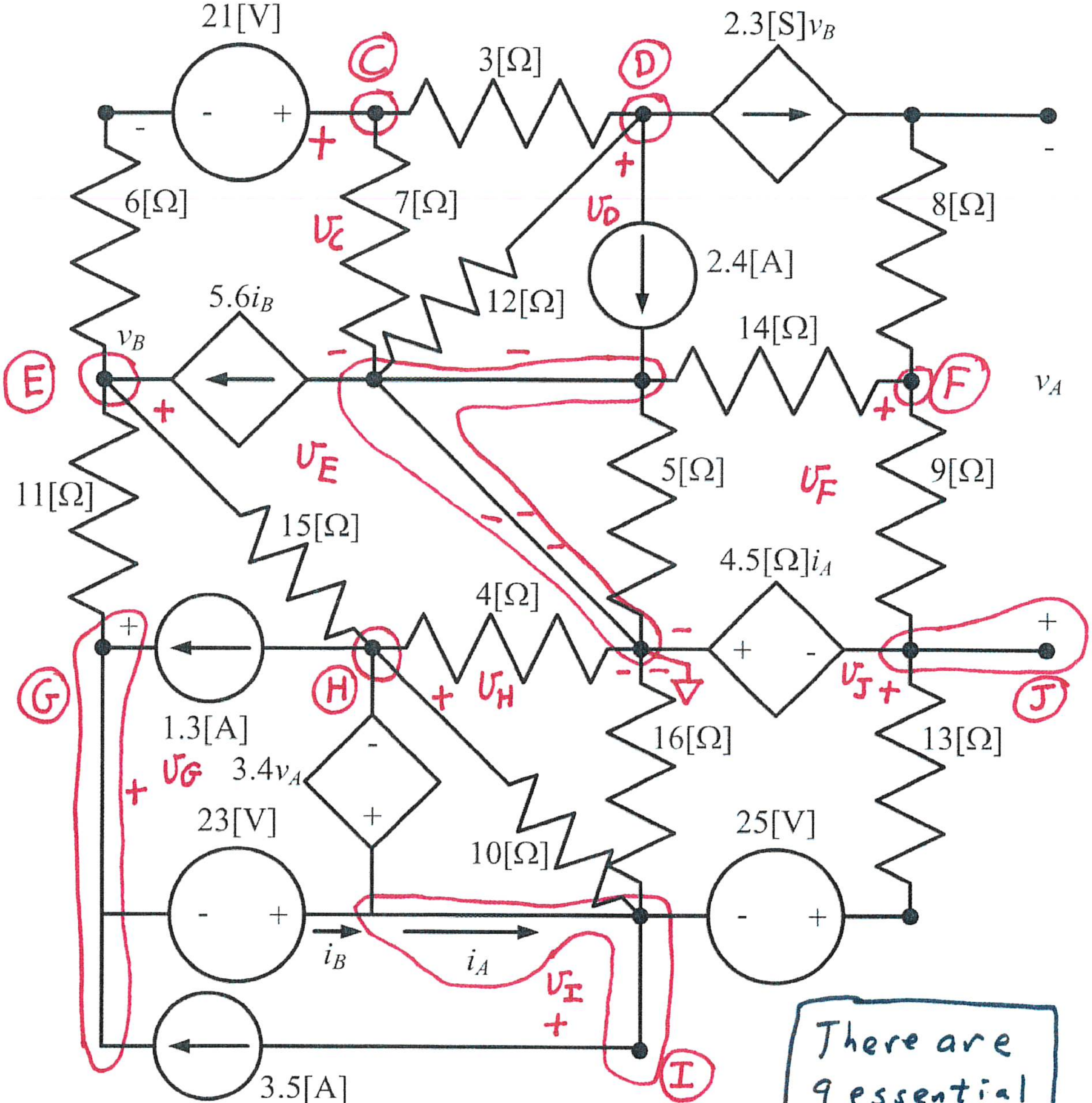
$i_1' = 6.667$ [mA]

$i_2' = -13.333$ [mA]

+4

$$\boxed{v_A = -1.5 - 250 i_1' + 200 i_1' = -1.833 \text{ [V]}}$$

4. (40 points) Use the node-voltage method to write a complete set of equations that could be used to solve this circuit. Do not simplify the circuit. Do not attempt to solve or simplify your equations. Define all variables appropriately.



There are 9 essential nodes.

We will need to write

$9-1+4 = 12$ equations.

See next page

C $\quad \dfrac{v_C}{7[\Omega]} + \dfrac{v_C - 21[V] - v_E}{6[\Omega]} + \dfrac{v_C - v_D}{3[\Omega]} = 0$

D $\quad 2.4[A] + \dfrac{v_D}{12[\Omega]} + \dfrac{v_D - v_C}{3[\Omega]} + 2.3[S]\,v_B = 0$

E $\quad \dfrac{v_E - v_G}{11[\Omega]} + \dfrac{v_E + 21[V] - v_C}{6[\Omega]} - 5.6\,i_B + \dfrac{v_E - v_H}{15[\Omega]} = 0$

F $\quad \dfrac{v_F}{14[\Omega]} - 2.3[S]\,v_B + \dfrac{v_F - v_J}{9[\Omega]} = 0$

G+H+I $\quad \dfrac{v_G - v_E}{11[\Omega]} + \dfrac{v_H - v_E}{15[\Omega]} + \dfrac{v_H}{4[\Omega]} + \dfrac{v_I}{16[\Omega]} +$

$$+ \frac{v_I + 25[V] - v_J}{13[\Omega]} = 0$$

G+I $\quad v_I - v_G = 23[V]$

H+I $\quad v_I - v_H = 3.4\,v_A$

See next page

(J) $v_J = -4.5[\Omega]\, i_A$

(i_A) $-i_A + \dfrac{v_I - v_H}{10[\Omega]} + \dfrac{v_I}{16[\Omega]} + \dfrac{v_I + 25[V] - v_J}{13[\Omega]} + 3.5[A] = 0$

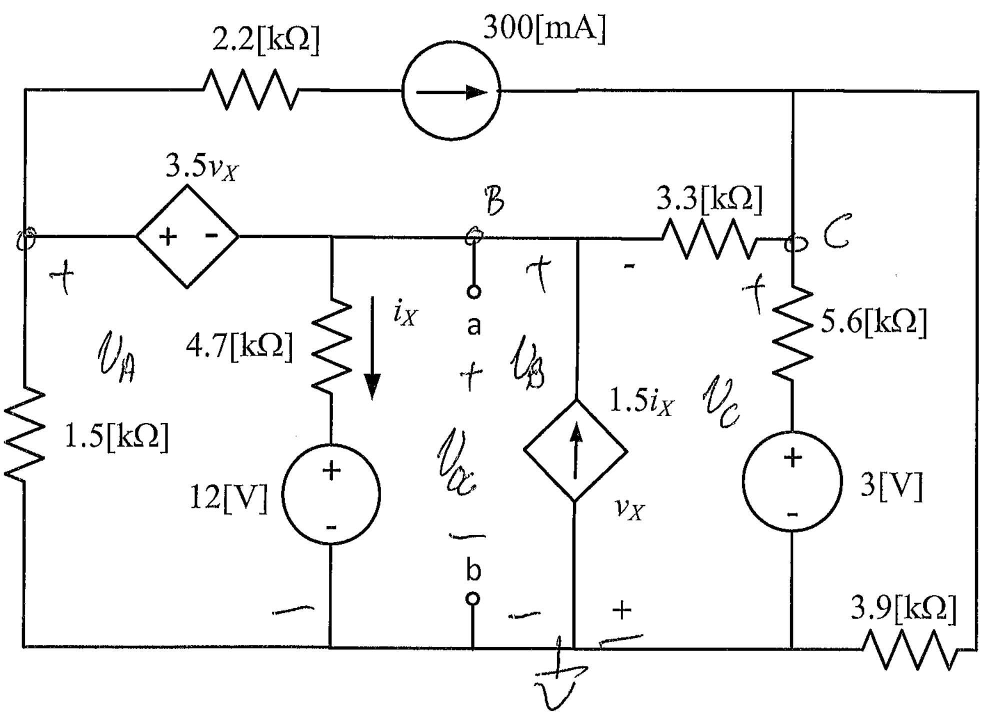
(i_B) $i_B - 3.5[A] + \dfrac{v_G - v_E}{11[\Omega]} - 1.3[A] = 0$

(v_A) $-v_A + v_J - v_F - (2.3[S]\, v_B)(8[\Omega]) = 0$

(v_B) $v_B - 21[V] + v_C - v_G = 0$

5. (40 points) For the circuit shown below, find the Norton equivalent at terminals a, b. Draw the Norton equivalent, clearly showing the parameter values and the terminals a, b.

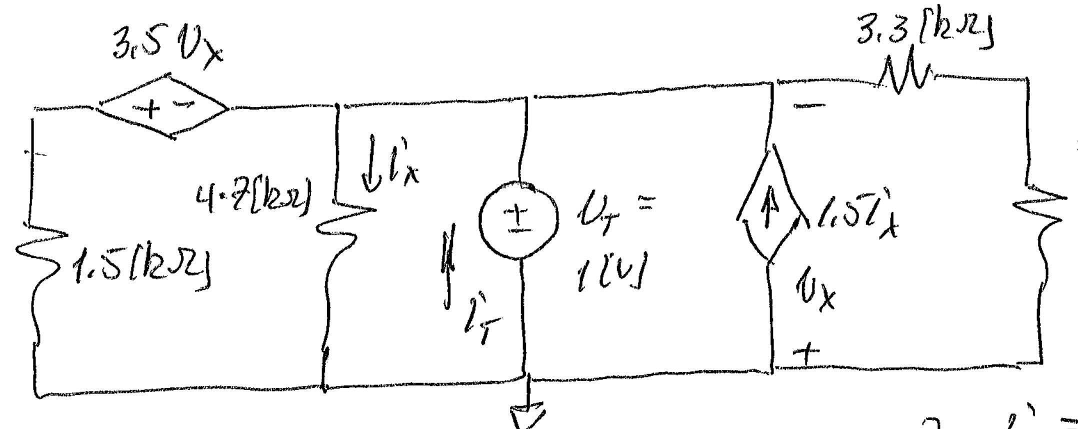
labelling is for calculation of v_{oc} done later



It seems clear that a test source and short-circuit current are the best choices, and that v_{oc} is the most difficult of the calculations needed here. But we will do all three.

TEST SOURCE

+ 10



$3900 \| 5600 = 2.299\,[k\Omega]$

+ 6

$$i_T' = -1.5\, i_x' + i_x' + \frac{1}{3300+2299} + \frac{1+3.5v_X}{1500} = 0$$

$$i_x' = \frac{1}{4700} \qquad v_X = -1\ [V]$$

+ 2

$i_T' = -1.594$ [mA] $\Rightarrow$

$\boxed{R_N = -627.2\,[\Omega]}$

Room for extra work

OPEN-CIRCUIT VOLTAGE

On the original diagram, node voltages v_A, v_B, v_C are labeled. $v_B = v_{OC}$

Supernode A, B: $-1.5 i_x + 1 i_x + \frac{v_B - v_C}{3300} + \frac{v_A}{1500} + 0.3 = 0$

+14

$$v_A - v_B = 3.5\, v_x$$

C: $\frac{v_C - v_B}{3300} - 0.3 + \frac{v_C - 3}{5600} + \frac{v_C}{3900} = 0$

$i_x = \frac{v_B - 12}{4700}$ $\quad v_x + 4700 i_x + 12 = 0$

$v_A = -278.9$ [V] $\qquad i_x = 21.183$ [mA]

$v_B = 111.56$ [V] $\qquad v_x = -111.56$ [V] $= v_B$

$v_C = 457.0$ [V]

$v_{OC} = v_B = 111.56$ [V]

+4

From Norton equivalent,

$$v_{OC} = i_N R_N = (-0.1779)(-627.2) = 111.58 \text{ [V]}$$

round-off error

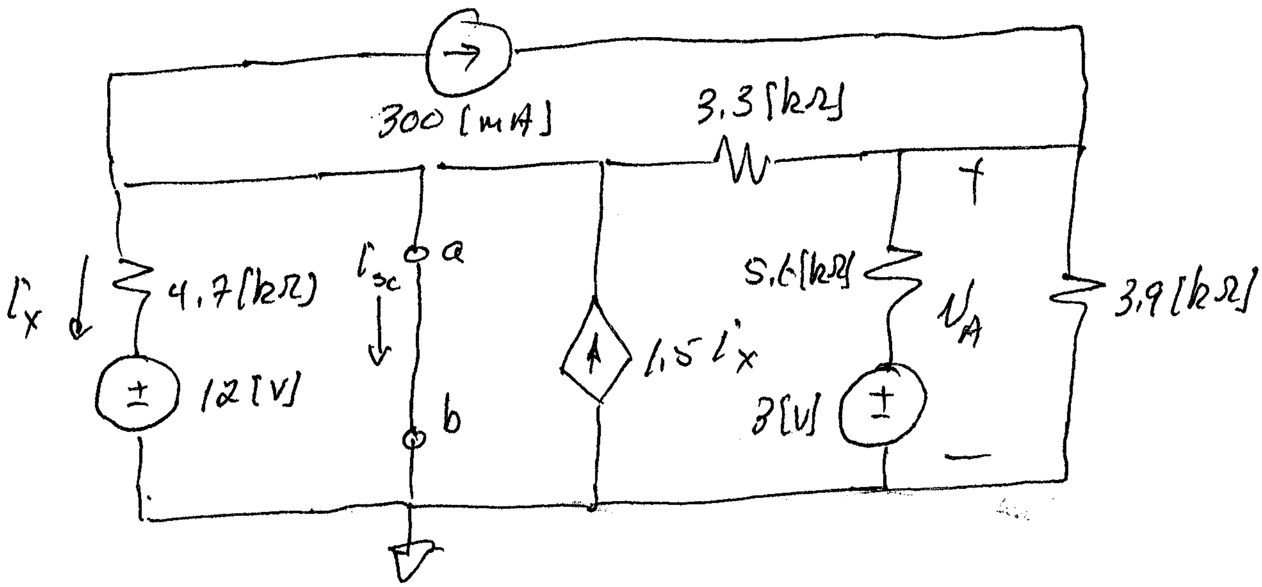
so this ✓'s!

Room for extra work

SHORT-CIRCUIT CURREN

With a, b shorted, $v_X = 0$, $3.5v_X = 0$ (short), so 1500 [Ω] is shorted.

+8



+8

$$i_X = \frac{-12}{4700} \qquad \frac{v_A}{3900} + \frac{v_A}{3300} + \frac{v_A - 3}{5600} - 0.3 = 0$$

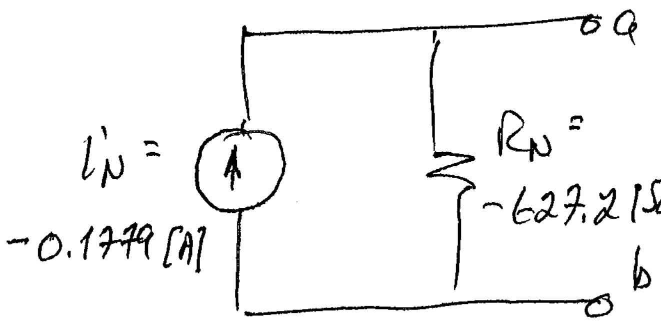
$$\Rightarrow v_A = 407.2\,[V]$$

$$i_{sc} + i_X - 1.5 i_X - \frac{v_A}{3300} + 0.3 = 0$$

+2 $i_{sc} = -0.1779\,[A] = i_N$

So we have the Norton equivalent:

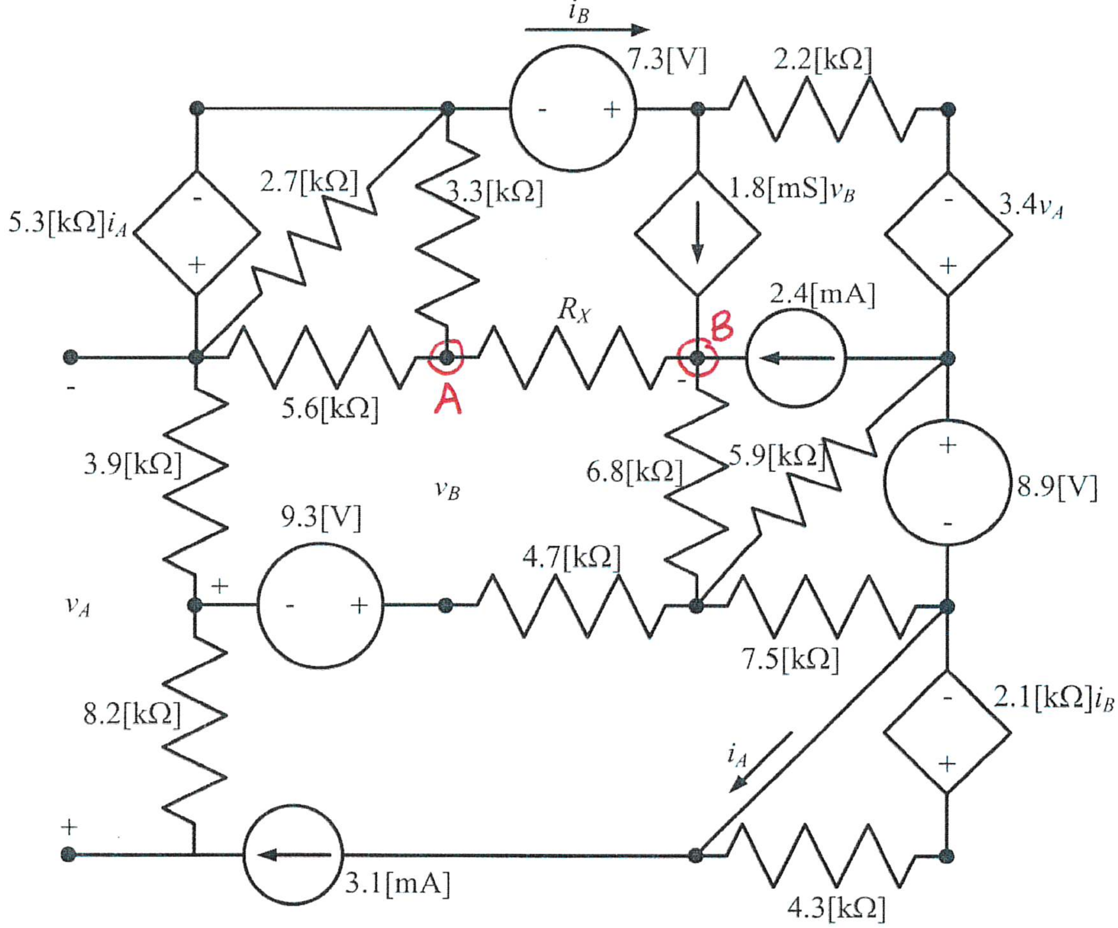
+4



Let's check v_{oc} ...

pg 12a

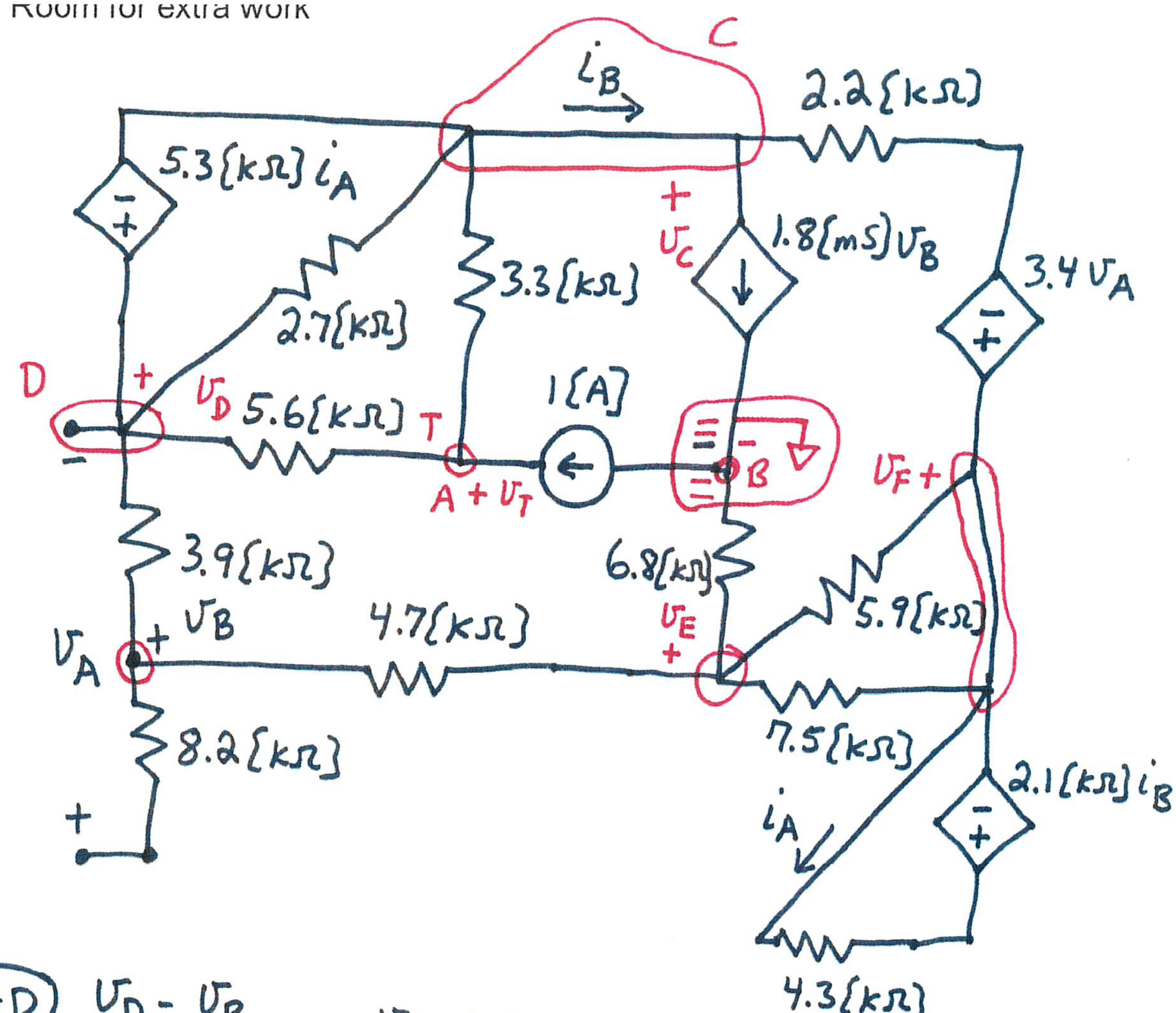
6. (35 points) For the circuit given below, the resistor R_X is equal to 1[kΩ]. Find the Thevenin equivalent resistance as seen by R_X.



We begin by naming the terminals of R_X in the circuit above. Then, to get the Thevenin resistance, we remove R_X, set all independent sources equal to zero, and the apply a test source. Here, we choose a 1[A] current source.

See next page.

Room for extra work



(C+D) $$\frac{v_D - v_B}{3.9\{k\Omega\}} + \frac{v_C + 3.4 v_A - v_F}{2.2\{k\Omega\}} + 1.8\{mS\} v_B + \frac{v_C - v_T}{3.3\{k\Omega\}} + \frac{v_D - v_T}{5.6\{k\Omega\}} = 0$$

(C+D) $$\underline{v_D - v_C} = 5.3\{k\Omega\}\, i_A$$

(B) $$\frac{v_B - v_D}{3.9\{k\Omega\}} + \frac{v_B - v_E}{4.7\{k\Omega\}} = 0$$

See next page

E $$\frac{v_E}{6.8\{k\Omega\}} + \frac{v_E - v_F}{5.9\{k\Omega\}} + \frac{v_E - v_F}{7.5\{k\Omega\}} + \frac{v_E - v_B}{4.7\{k\Omega\}} = 0$$

F $$\frac{v_F - v_E}{7.5\{k\Omega\}} + \frac{v_F - v_E}{5.9\{k\Omega\}} + \frac{v_F - 3.4v_A - v_C}{2.2\{k\Omega\}} = 0$$

T $$\frac{v_T - v_D}{5.6\{k\Omega\}} + \frac{v_T - v_C}{3.3\{k\Omega\}} - 1\{A\} = 0$$

i_A $$i_A = \frac{-2.1\{k\Omega\}\, i_B}{4.3\{k\Omega\}}$$

i_B $$-i_B + \frac{v_C + 3.4v_A - v_F}{2.2\{k\Omega\}} + 1.8\{mS\}\, v_B = 0$$

v_A $$v_A = v_B - v_D$$

R_{EQ} $$R_{EQ} = \frac{v_T}{1\{A\}}$$

$i_A = -1.088\{A\}$
$i_B = 2.2278\{A\}$
$v_F = 12.743\{kV\}$
$v_E = 5.8485\{kV\}$
$v_D = -4.7107\{kV\}$
$v_C = 1.0557\{kV\}$
$v_B = 77.74\{V\}$
$v_A = 4.7885\{kV\}$

$$\boxed{R_{EQ} = 993.98\{\Omega\}}$$

$v_T = 993.98\{V\}$