Signature
Name (print, please)
Student No.

ECE 2300 Circuit Analysis
Summer 2009
July 2, 2009

Exam 1

DO NOT OPEN THIS EXAM BOOKLET UNTIL INSTRUCTED TO DO SO

This exam has 10 pages including this cover page. If you are missing any pages, raise your hand. You have 90 minutes to complete the exam.

Notes

1. Be sure your name and signature appear above.
2. The quiz is closed-book. You may have a calculator and one 8 ½ " x 11" crib sheet.
3. To receive full credit for a problem, you must:
   - Show all work necessary to solve the problem;
   - Define all variables and parameters and label them on circuit diagrams;
   - Use the proper notation for all variables.
   - Show all units explicitly in intermediate and final results;
   - Indicate clearly whether power being calculated is absorbed or delivered;

1. ____________/25

2. ____________/25

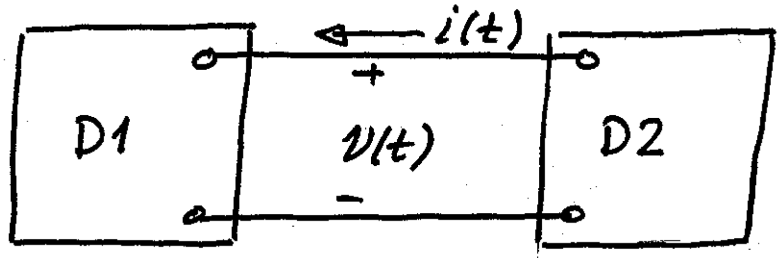
3. ____________/25

4. ____________/25

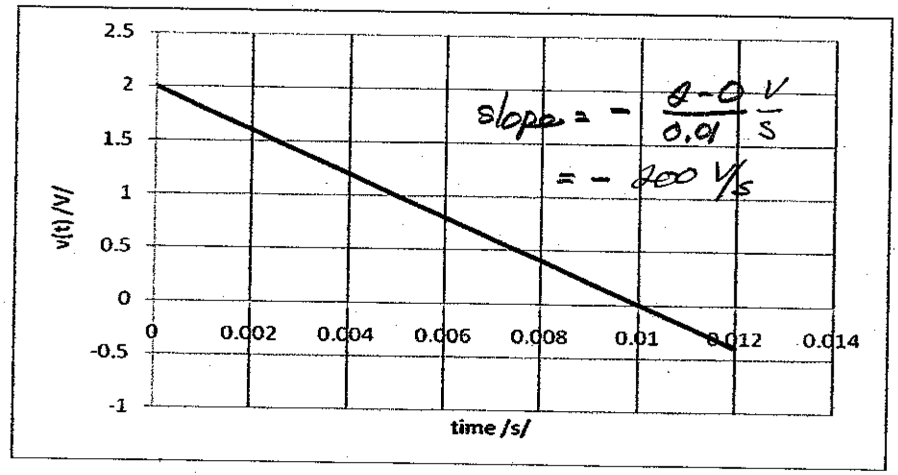
Total__________/100

1. (25 points) The box labeled D1 can be modeled using a current source in parallel with a resistance. The box labeled D2 can be modeled using a voltage source in series with a resistance. The voltage v(t) and current i(t) labeled in the figure have the time dependence shown in the graphs below the figure.

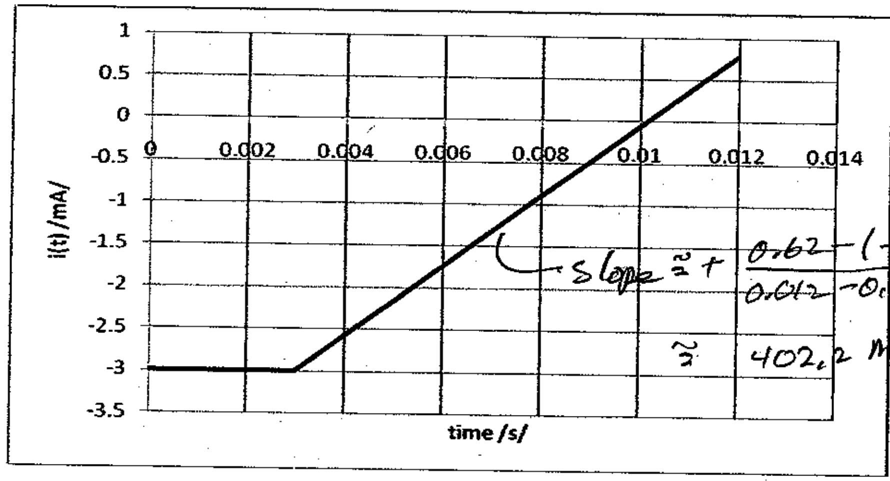
a) Find the power being delivered by D2 to D1 at t = 10 /ms/.
b) Find the total energy delivered by D2 to D1 between 0 and 10 /ms/.



v(t) →



i(t) →



Approximate slope:

$\frac{0.62-(-3)}{0.012-0.003}\ \frac{mA}{s}$

$\cong 402.2\ mA/s = 0.402\ A/s$

over →

Room for extra work

a) We will need to work out the linear functions representing $v(t)$ and $i(t)$:

$$v(t) = m_v t + b_v \qquad b_v = 2\ [V] \text{ by inspection}$$

$$m_v = \frac{2-0}{0-0.01} = -200\ V/s$$

+4 $\therefore$ $v(t) = 2 - 200t\ [V] \qquad t \text{ in } [s].$

+2 $i(t) = +3\ [mA] \qquad 0 \le t \le 3\ [ms]$

$$i(t) = m_i t + b_i \qquad 3 \le 12\ [ms]$$

$$-0.0005 = m_i(0.009) + b_i$$

$$-0.0030 = m_i(0.003) + b_i$$

$$\Rightarrow m_i = 0.4167\ [A/s] \quad b_i = -4.25\ [mA]$$

+4 $\therefore$ $i(t) = 0.4167t - 4.25 \times 10^{-3}\ [A] \qquad t \text{ in } [s]$

+4

Now it is clear that $t = 10\ [ms] \Rightarrow v(t) = 0$

so $P_{abs,D1} = 0$ at $t = 10\ [ms]$

Room for extra work

b) $$W_{del\ by\ D2} = \int_0^{0.01\,[s]} P_{del\ by\ D2}\, dt = +\int_0^{0.01\,[s]} v(t)\cdot i(t)\, dt$$

$$v(t)\cdot i(t) = -0.003(2-200t)\ [W] = -0.006 + 0.6t\ [W] \quad \{ 0 \le t \le 0.003\ [s]$$

$$v(t)\cdot i(t) = (-4.25\times10^{-3} + 0.4167t)\cdot(2-200t)\ [W] = -8.34\times10^{1}t^2 + 1.684t - 8.5\times10^{-3}\ [W] \quad \{ 0.003 \le t \le 0.01\ [s]$$

$$\therefore W_{del\ by\ D2} = \int_0^{0.003\,[s]} (-0.006 + 0.6t)\,dt + \int_{0.003}^{0.01\,[s]} (-83.4t^2 + 1.684t - 8.5\times10^{-3})\,dt$$

$$= -1.53\times10^{-5}\ [J] - 9.927\times10^{-6}\ [J]$$

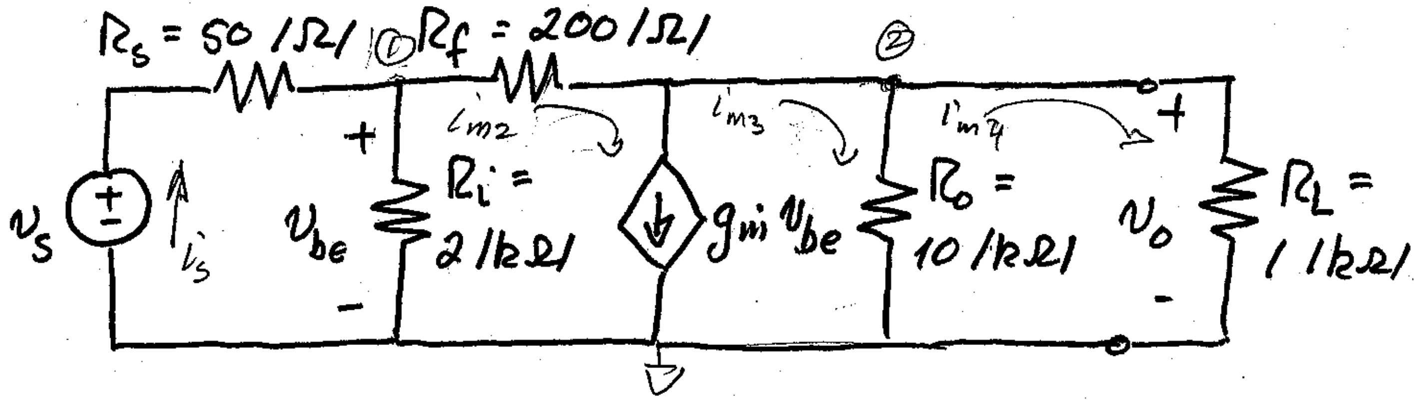
$$\boxed{W_{del\ by\ D2} = -2.523\times10^{-5}\ [J]}$$

formulation +6
sign +2
calculation +3
+11

divide ∫ limits +4

2. (25 points) The circuit below is a model for what is known as a *transconductance amplifier*. The voltage v_s represents an input signal; the load at the output is modeled by the resistor R_L. The parameter gm is the transconductance, and has a value of 150 /mS/.

a) If $v_s = 1$ /V/, find v_o.
b) Find the power delivered to the rest of the circuit by the signal voltage v_s.
c) Find the power delivered to the load R_L.



a) Looks like a job for node voltage... We have labeled the reference node as well as the non-ref. nodes ① and ②. We note that these node voltages are already labeled v_o and v_{be}. So...

Eqns: +15

① $$\frac{v_{be}-v_s}{R_s} + \frac{v_{be}}{R_i} + \frac{v_{be}-v_o}{R_f} = 0$$

② $$\frac{v_o-v_{be}}{R_f} + g_m v_{be} + \frac{v_o}{R_o} + \frac{v_o}{R_L} = 0$$

From here we will substitute numbers to make the going easier. We will also substitute $v_s = 1$ /V/.

$$\frac{v_{be}-1}{50} + \frac{v_{be}}{2000} + \frac{v_{be}-v_o}{200} = 0$$

$$\frac{v_o-v_{be}}{200} + 0.150\, v_{be} + \frac{v_o}{10000} + \frac{v_o}{1000} = 0$$

Room for extra work

Solving these equations gives

$v_{be} = 0.1386$ [V] $v_o = -3.293$ [V]

a) $v_o = -3.293$ [V]. +3

b) $P_{del\, v_s} = v_s \cdot i_s' = 1 \cdot (17.23 \times 10^{-3}) = 17.23$ [mW] +2 sign +3

$i_s' = \frac{v_s - v_{be}}{50} = \frac{1 - 0.1386}{50} = 17.23$ [mA]

c) $P_{abs\, R_L} = v_o^2 / R_L = \frac{(-3.293)^2}{1000} = 10.84$ [mW] +2

Alternative solution using mesh currents:

$v_s = 1$ [V]

① $-v_s + 50 i_s' + 2000 (i_s' - i_{m2}') = 0$

② $200 i_{m2}' + 10\,000 (i_{m3}' - i_{m4}') + 2000 (i_{m2}' - i_s') = 0$

③ $i_{m2}' - i_{m3}' = g_m \cdot 2000 (i_s' - i_{m2}')$ $g_m = 0.150$ [S]

④ $1000 i_{m4}' + 10\,000 (i_{m4}' - i_{m3}') = 0$

Soln: $i_{m4}' = -3.29 \times 10^{-3}$ [A] $i_s' = 1.72 \times 10^{-2}$ [A]

$i_{m2}' = 1.72 \times 10^{-2}$ [A] $i_{m3}' = -3.62 \times 10^{-3}$ [A]

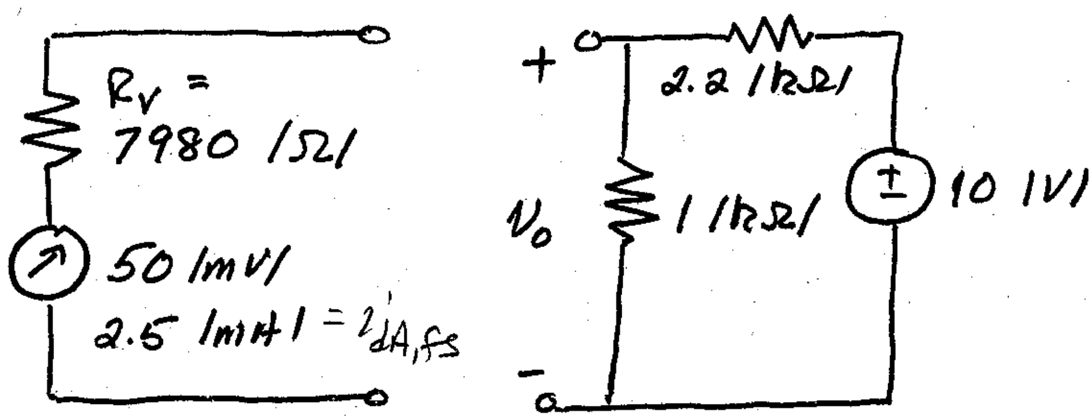
$\Rightarrow v_o = -3.29$ [V] so same answers.

3. (25 points) Depending on the value of R_V, the voltmeter shown below can be set to full-scale readings of 5, 10, 20, 25, 50, 80, 100, 125, 150, and 200 /V/. In the figure, R_V has been chosen so that the full-scale reading is 20 /V/.

+10 a) If the voltmeter is used on the 20 /V/ full scale range to measure the voltage v_o in the circuit on the right, what is the percent error in the reading? Assume that the "true" voltage $v_{o,true}$ is the value of v_o when the meter is not connected to the circuit, and that the percent error E is given by

$$E = 100\left(\frac{v_{o,measured} - v_{o,true}}{v_{o,true}}\right).$$

+15 b) What is the smallest full scale voltage setting that can be used and still ensure that the magnitude of the percent error is no more than 2% ?



i) d'Arsonval meter resistance is $R_{dA} = \frac{50\,/mV/}{2.5\,/mA/} = 20\,/\Omega/$

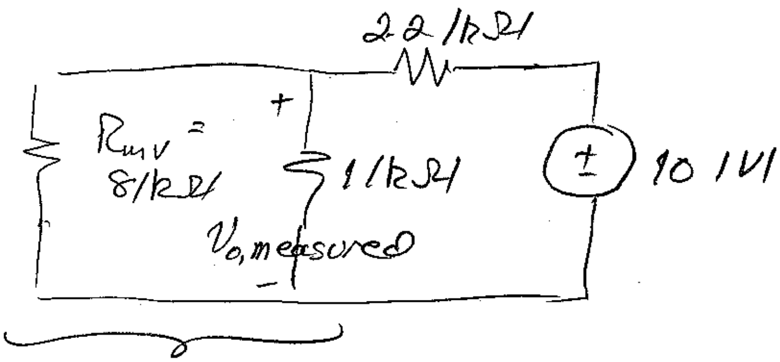
+4 meter resistance $\Rightarrow R_{mv} = 7980 + 20 = 8\,000\,/\Omega/.$

Thus full scale rating is $i_{dA,fs} \cdot R_{mv} = 20\,/V/$ (as stated)

+2 $v_{o,true} = 10 \cdot \frac{1\,/k\Omega/}{1\,/k\Omega/ + 2.2\,/k\Omega/} = 3.125\,/V/$

Room for Extra Work

with the meter attached, we have



$$R_{eq} = 8k \parallel 1k = 888.9\ \Omega$$

+4

$$V_{o,measured} = 10 \cdot \frac{888.9}{888.9 + 2200} = 2.878\ V$$

and $$E = \frac{2.878 - 3.125}{3.125} = -2.91\%$$

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ii) −2% error means that

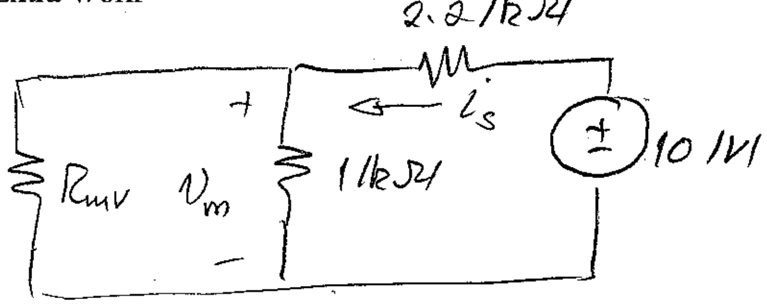
+6

$$\frac{V_{o,measured} - V_{o,true}}{V_{o,true}} = -0.02$$

$$\Rightarrow \quad V_{o,measured} = -0.02\, V_{o,true} + V_{o,true} = 3.0625\ V$$

If that's to be true, then we can find the required R_{mV}:

Room for Extra Work



$R_{eq} = R_{mV} \parallel 1\,k\Omega$

If v_m is 3.0625 V, then $i_s' = \frac{10 - 3.0625}{2200}$

$\Rightarrow i_s' = 3.153\ mA$. Thus

$$R_{eq} = \frac{3.0625\ V}{3.153\ mA} = 971.2\ \Omega$$

So then

$$\frac{R_{mV} \cdot 1\,k\Omega}{R_{mV} + 1\,k\Omega} = 971.2\ \Omega$$

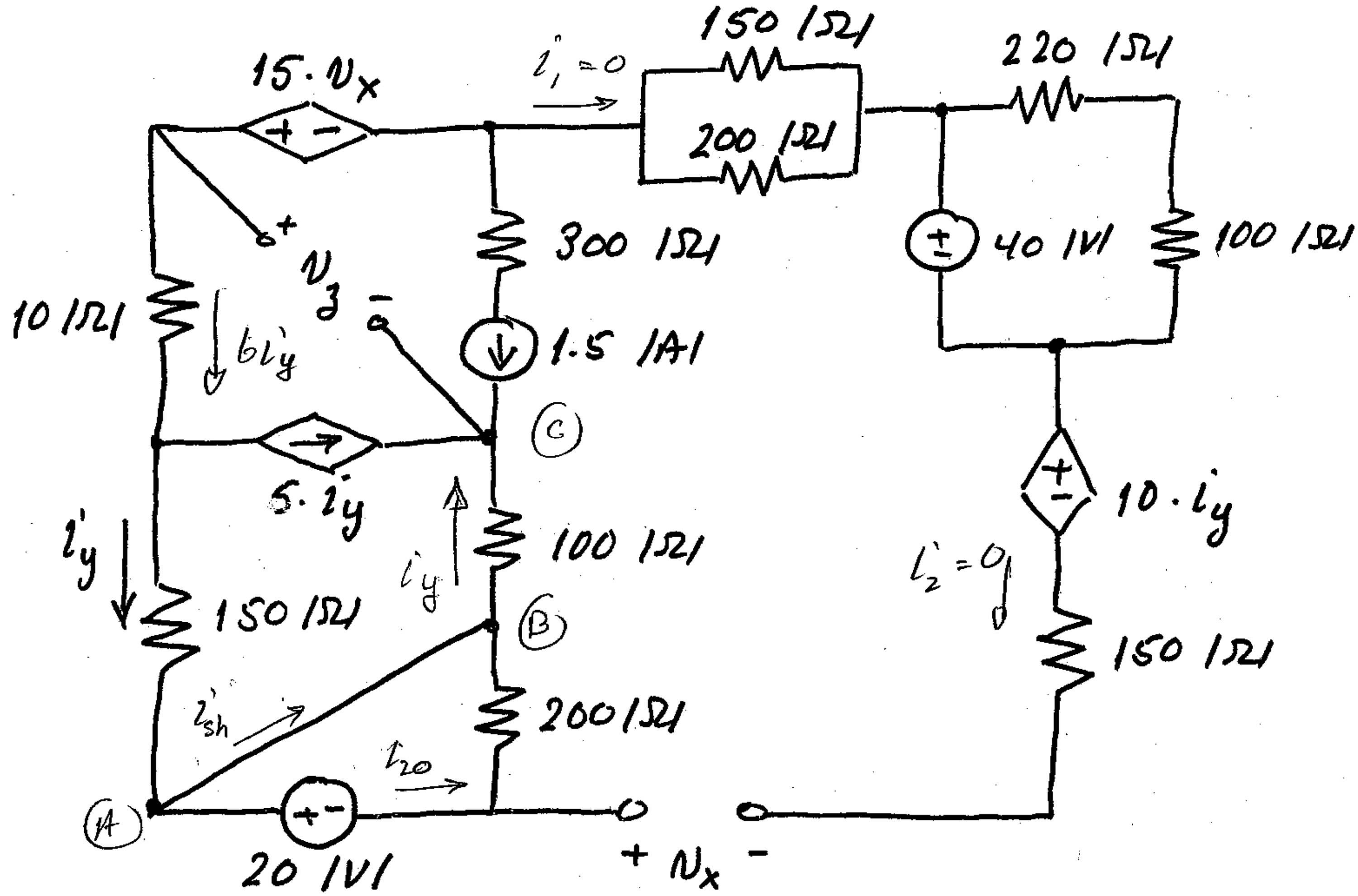
+7 $$\Rightarrow R_{mV} = \frac{0.9712}{1 - 0.9712}\ k\Omega = 33.72\ k\Omega$$

Finally, if the voltmeter has a full scale current of 2.5 mA and a resistance 33.72 kΩ, the full-scale voltage reading is

+2 $$V_{fs} = i_{dA,fs} \cdot R_{mV} = 84.30\ V$$

This is not available so we go to the next higher range, which is <u>100 V full-scale</u>.

4. (25 points) For the circuit below, find v_z.



The current in the 100 [Ω] resistor is i_y, as shown.

We can show this as follows:

KCL: $i_y = i_{sh} + i_{20}$ at (A)

But by KCL at (B), current in 100 [Ω] is also

$i_{sh} + i_{20} = i_y$.

Then at (C): $-i_y - 5 i_y - 1.5 [A] = 0$

$$\Rightarrow i_y = -\frac{1.5}{6} = -0.25 [A]$$

Room for extra work

Now the currents labeled i_1' and i_2' are 0, so KVL around the outside gives

$$15 v_x + 0 + 40 + 10 \cdot i_y' - v_x - 20$$
$$-150 i_y' - 6 i_y' \cdot 10 = 0$$

$$\therefore \quad 14 v_x - 200 i_y' = -20$$

$$\Rightarrow v_x = \frac{-20 + 200(-0.25)}{14}$$

$$v_x = -5.00 \text{ [V]}$$

(not asked)

For v_z:

$$v_z - 100 i_y' - 150 i_y' - 6 i_y' \times 10 = 0$$

$$v_z = 310 i_y' = -77.5 \text{ [V]}$$

KVL, KCL sign error -3

NV sign error -3

Correct eqns, wrong answer -3

leave out v across current source → missing eqn. } -6