Chapter 1: Circuit Variables

- Objectives
  - Understand the use of circuit schematics in circuit modeling
    - BJT Circuit
  - Understand basic concepts of voltage and current
  - Understand sign conventions in voltage and current
  - Be able to do dc power calculations and correctly interpret signs

- Homework/Quiz/Exam Prep
  - Units and labeling; homework format
  - Math requirements
    - Trig functions: sin, cos
    - Equations of a straight line

- Presentation
  - Present BJT schematic and introduce
    - Modeling concept
    - Labeling of voltage and current
    - Intro to reference polarities

Activity: students label their own diagrams and exchange with neighbors to check their work

  - Definitions of voltage and current
    - A simple circuit
    - Water analogy
  - Ideal Circuit Elements: the "box"
    - Reference vs. actual polarities
    - Direction of charge flow
    - Voltage drop and voltage rise
  - Power and Energy
    - Delivered, absorbed
    - Relationship to charge flow
  - Active and Passive Sign Conventions

Activity: simple source/resistor circuit with given current; find power delivered or absorbed by source

Chapter 1: Circuit Variables

1.1 Electrical Engineering: An Overview

Electrical Engineers are concerned with the design, analysis, and operation of systems involving electrical signals. Examples:

- Communications/signal engineering
- Computer systems
- Control systems/robotics
- Power systems
- Microelectronics

Theoretical Basis for Electrical Engineering: Electromagnetics and Maxwell's Equations. We will not deal with this topic here. Instead we will talk about a specialization of electromagnetics…

Circuit Theory is an important special case of electromagnetics. It is appropriate where the spatial dimensions of the electrical system are small compared with the wavelength of an electromagnetic signal, i.e., where the shape of the circuit does not matter.

1.2 Units

We will use the International System of units:

Length:	meters	[m]
Mass:	kilogram	[kg]
Time:	second	[s]
Current:	Ampere	[A]
Temperature:	Kelvin	[K]

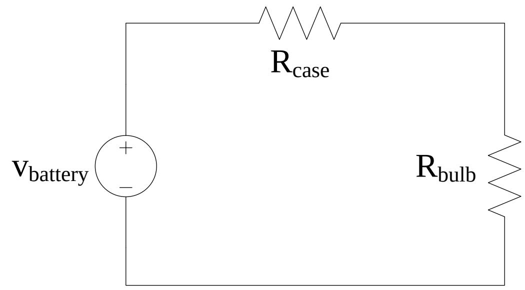
Also: 1 ***Ampere*** is the current that, maintained in two straight, parallel, infinite conductors of negligible circular cross section placed 1 m apart in vacuum, would produce a force between the conductors of 2×10^{-7} [N/m].

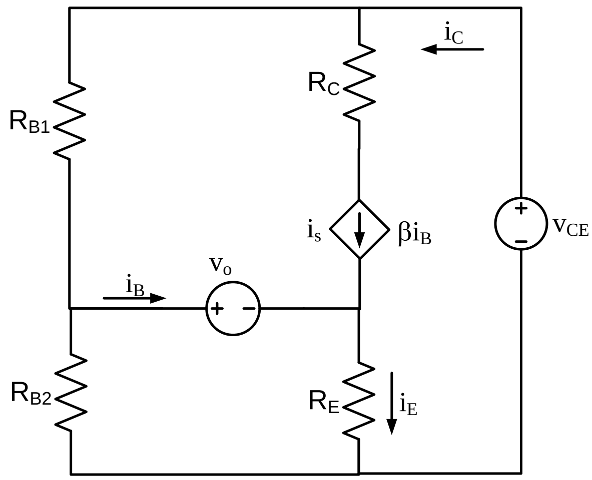
The ***Coulomb*** is a unit of charge derived from the Ampere; more below.

1.3 Circuit Analysis: An Overview

Idea: We want to be able to make quantitative predictions about electrical circuit behavior. We do this using idealized connections among circuit elements called *circuit models*.

Modeling We can use the five basic circuit elements to construct models of any electrical system. These models help us to analyze and design real electrical systems. They tell us how the system will behave if we make changes to it, for example. The diagram to the right is a circuit model for a flashlight; we will be using it in the next chapter to discuss basic circuit analysis concepts.





The circuit model to the left was taken from Nilsson and Reidel, 8ed. It models a transistor amplifier. We will have a few things to say about it in class…

1.4 Charge, Voltage and Current

Voltage and current are the important variables for electrical circuits. We start with a more fundamental concept:

Charge is a basic property of matter. The smallest "piece" of charge is the charge on an electron, with magnitude $q = 1.6 \times 10^{-19}$ Coulombs [Coul]. Also, we find that...

- There are two types of charge: positive and negative;
- Like charges (i.e., both positive or both negative) repel one another, while opposites attract.

Electrons are considered to have a negative charge.

Voltage and current arise from charge. To separate charges from one another requires that we exert energy, that is, do work. This work is related to potential, or voltage.

Separation of charge ⇒ voltage

If charge is moving, we have a current.

Movement of charge ⇒ current

Formal Definitions

Potential and Voltage: To move a hypothetical positive test charge from point A to point B in a region where electric forces are present requires work (energy). The work required is the difference in ***electric potential energy*** between point A and point B. ***Electric potential*** is the work per unit charge required to move the test charge. It is defined in the limit of a vanishingly small test charge (i.e., as q becomes 0):

$$v_{BA} = v_B - v_A \equiv \frac{W_{AB}}{q} \Big| q \to 0$$

Here, v_B - v_A is the difference in **potential**, or **voltage** v_{BA}, between A and B; W_{AB} is the work done in moving the charge from A to B; q is the charge in Coulombs. In differential form,

$$\boxed{v = \frac{dw}{dq}.}$$

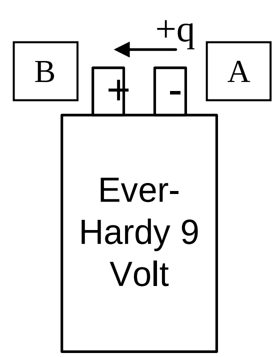
Units:

1 Volt = 1 Joule/Coulomb

1 Joule = 1 [kg m^2/s^2]
1 Coulomb = 1 [Ampere] . 1 [s].

Important: ***voltage***, like potential energy, is defined as a ***difference*** in potential. It is not a force, and it is not energy.

Simple Example Application



Imagine a 9 [V] battery: If I move a positive test charge +q from terminal A to terminal B, I need to do positive work on the test charge. (Basic electrostatics tells me that the test charge doesn't want to be at the positive battery terminal because like charges repel, so I will have to push it there, which means I am doing work on it.) So in this case, the work done in moving from A to B, i.e., W_{AB}, is positive.

If W_{AB} is positive, then by the equation above, v_{BA} is positive. This is as it should be: v_{BA} = + 9 [V] for our battery.

Current: When charge flows in a conducting material, a current exists. Current is the defined as the rate at which charge moves past an imaginary plane in a device, or in a wire. Formally,

$$\boxed{i = \frac{dq}{dt}}$$

In this equation, *i* is the current; *q* is the charge; t is time.

<u>Units:</u>

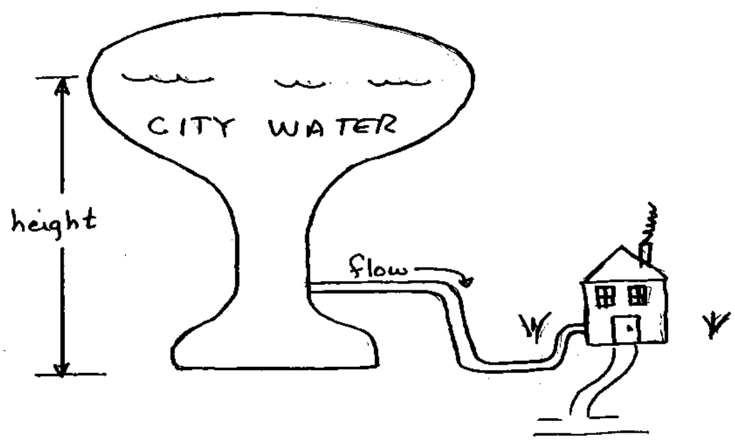
$$\boxed{\text{1 Ampere = 1 Coulomb/second}}$$

We will not be concerned with the details of how current flows. We usually think of it as a flow of electrons in a wire, and that will be good enough for us here.

The Water Analogy

Voltage (electric potential) ⇒ height of water (gravitational potential)

Current ⇒ water flowing in pipes



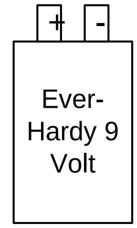
Summary

So what is voltage? Voltage is a ***difference in potential***. It describes the ability of the system to move charge through a wire, just as gravity has the ability to move water through pipes.

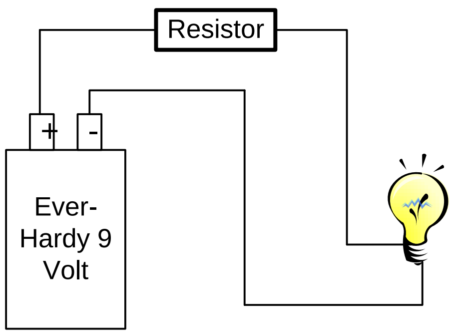
What about current? Current is the ***flow of charge***. If charge is moving, we have a current. For circuit analysis, we don't need to worry about what exactly is moving, or how it is moving.

A simple circuit

The battery on the right provides a voltage of 9 [V], with "positive" and "negative" terminals as indicated. Chemical forces in the battery maintain an electric potential difference between the terminals. We can think of the terminals as being analogous to "up" and "down" in the water tower.



The battery doesn't "do" anything until we include it in a complete circuit. Example:



What's going on here? When wires and a light bulb are connected to the battery, the battery voltage causes a current to flow. The current is causing the light to glow. A resistor has been inserted since we may need to limit the current so the bulb doesn't burn up!

Note that we need a connection to *and* from the bulb for the circuit to work; that is, we need a complete path for current to flow.

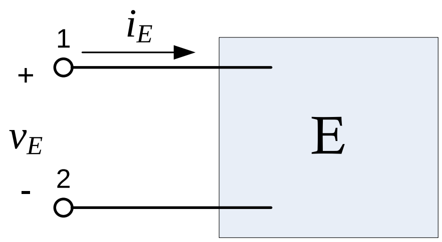
How does this work? We can go back to our Physics text to find out that the battery exerts a force on the charges in the wire and causes them to move, provided the circuit is complete. We do not need to know about these forces to do circuit analysis. Instead of forces, we talk about voltage.

1.5 The Ideal Basic Circuit Elements

We will use a box like the one below to represent several things. It can be just one or perhaps many electrical components, or even an entire circuit or electrical system. For now, we assume it represents an ***ideal basic circuit element***. Ideal basic circuit elements are the building blocks of circuit models.

The ideal basic circuit elements are:

- Voltage source
- Resistor
- Inductor
- Capacitor
- Inductor



The box contains a basic circuit element 'E'. It is connected to something at terminals 1 and 2. What it's connected to is not shown and for now doesn't matter.

We can model any circuit, no matter how complex, with a combination of these basic circuit elements.

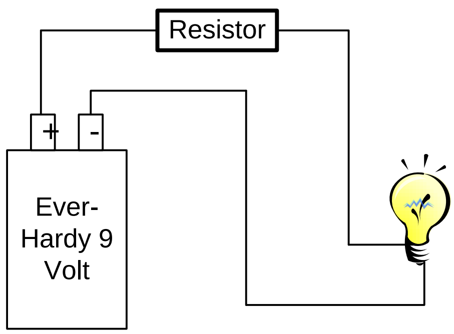
Ideal We are calling these things "ideal" circuit elements. This means that we should not expect real voltage sources, a battery for example, to act like ideal voltage sources. The same is true for the other elements. But as we will see, we can model non-ideal behavior using these basic circuit elements in combination.

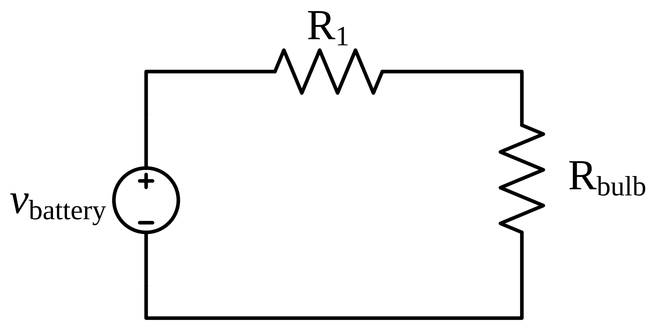
Properties of the ideal basic circuit element:

- It has two terminals (labeled "1" and "2").
- It can be described mathematically in terms of a voltage v and/or a current i.
- It cannot be subdivided into other circuit elements, hence it is *basic*.
- It is ideal in the sense that it has idealized properties that do not necessarily hold for real circuit elements.

Our simple light bulb, battery, and resistor circuit can be modeled using basic circuit elements… We will of course use a voltage source for the battery, and a resistor for the resistor! But what about the bulb?

The bulb absorbs electrical energy from the battery, and gives off light and heat as a result. We will not try to model the light and heat, but we will model the absorption of electrical energy. This can be done using a resistor to model the bulb as well.





The circuit model on the right represents a “schematic” of the light bulb circuit on the left. Using techniques of circuit analysis, we can predict the behavior of the circuit, at least in terms of electrical properties, from this schematic.

Activity

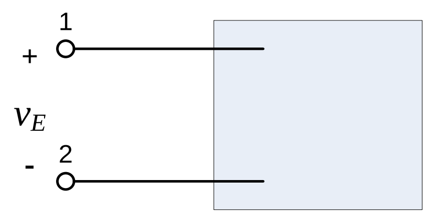
We will build this simple circuit in the classroom. We will also add another light bulb on the other side of the resistor and ask whether both bulbs shine equally brightly, or whether one is brighter than the other.

Labeling Voltage: Reference and Actual Polarities

Because voltage v_E across a circuit element is a potential ***difference***, one side of the circuit element has a higher potential than the other. We need a way to indicate this. **But**: we won't always know which side is the higher potential until we calculate or measure the voltage. So we also need a way to handle that issue.

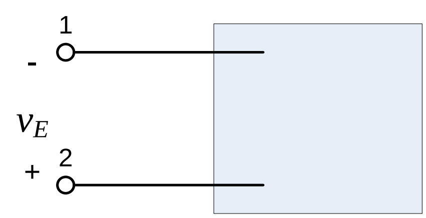
The '+' and '-' associated with v_E in the boxes below indicate the ***reference polarity*** for the voltage. These are analogous to labels on an x-y graph, where arrows indicate "positive x" and "positive y". The first box below is labeled as if the higher potential is at terminal 1. We can assign that polarity without knowing whether terminal 1 is in fact the higher potential; in other words, without knowing the ***actual polarity***. The voltage across the first box may in fact be more positive at terminal 2. To handle this, we need to know both a *magnitude* and a *sign* (positive or negative) for v_E. Both of these are needed if we want to know the ***actual*** potential difference between terminals 1 and 2.

Consider the circuit element to the right. My *reference* polarity, v_E, shows the higher potential at terminal 1, but I don't yet know the *actual* polarity.



a) If I measure v_E and find that it is +5 [V], then terminal 1 is 5 [V] higher in potential than terminal 2. The *actual* polarity is the same as the *reference* polarity.
b) If I measure v_E and find that it is -5 [V], then terminal 2 is 5 [V] higher in potential than terminal 1. The *actual* polarity is opposite to the *reference* polarity.

If I like, I can change the reference polarity….



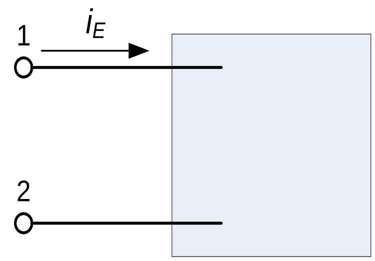
In this case, the measurements I made above will give the opposite sign. If terminal 1 is 5 [V] higher in potential than terminal 2, v_E will be negative. If terminal 2 is higher in potential than terminal 1, v_E will be positive.

Bottom line: The '+' and '-' signs tell us the ***reference polarity***. We don't know the ***actual polarity*** until we are measure (or calculate, or we are given) the sign of v_E. Often we will have to label a voltage before we know what the actual polarity is.

But…how do I "measure" v_E?? To measure v_E, I put the red lead of my voltmeter at the positive terminal of v_E, and the black lead at the negative terminal.

Notation: **Use a lower case 'v' with a subscript (v_E) to indicate a voltage. Always label the polarity with + and - .**

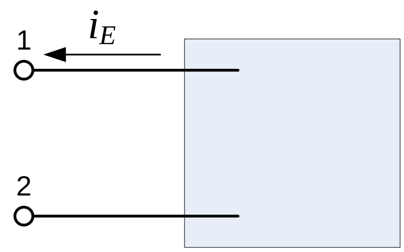
Labeling Current: Reference and Actual Polarities



We can think about the same kind of thing for current. Current flowing through a circuit element, like the box at the right, may be flowing from top to bottom (terminal 1 to terminal 2), or the other way around.

The arrow shown on the box to the right indicates the ***reference current polarity (*** or ***reference current direction)***. It assumes that current is entering terminal 1 and leaving terminal 2.

a) If I measure i_E and find that it is +30 [mA], then 30 [mA] of current is entering terminal 1 and leaving terminal 2. The *actual* current direction and the *reference* current direction are the same.
b) If I measure i_E and find that it is -30 [mA], then 30 [mA] of current is entering terminal 2 and leaving terminal 1. The *actual* and *reference* current directions are now opposite to each other.



If I change the reference direction, then the measurements above will give the opposite signs.

If current is entering terminal 1, i_E will be negative; if it is leaving terminal 1, i_E will be positive.

Bottom Line: The arrow indicates a ***reference current direction***. We don't know the ***actual current direction*** until we know the sign of i_E, which we get by measurement or calculation. We will have to label currents even though we don't know which way they are going. These labels will need to include reference directions.

But…how do I "measure" i_E?? To measure i_E, I put the red lead at the tail of the arrow, and the black lead at the head of the arrow.

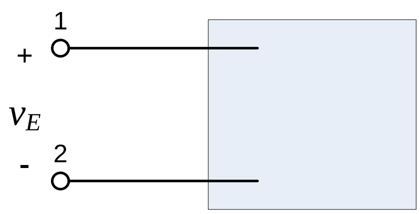
Notation: **Use a lower case 'i' with a subscript (i_E) to indicate a current. Always label the polarity (direction) with an arrow.**

Direction of charge flow

Current is defined as the direction in which positive charge carriers are moving. In other words, if the current in the last figure above (second example) is positive, it means either that positive charges are *leaving* the box through terminal 1, or else that negative charges are *entering* the box through terminal 1.

Voltage Drop and Voltage Rise

We give here a couple of definitions that will be useful later. Referring to our basic circuit elements: when the voltage at terminal 1 is higher than the voltage at terminal 2, we say there is a ***voltage drop*** from terminal 1 to terminal 2. If the terminal 1 voltage is *lower* than the terminal 2 voltage, there is a ***voltage rise*** from terminal 1 to terminal 2. Some examples:



If v_E = 5 [V], there is a voltage drop (of 5 [V]) from terminal 1 to terminal 2. We could also say there is a voltage rise from terminal 2 to terminal 1.

If v_E = -3 [V], there is voltage rise from terminal 1 to terminal 2. We could also say there is a voltage drop from terminal 2 to terminal 1.

So the term "voltage drop" depends on how we are looking at the diagram.

1.6 Power and Energy

Usually, we will be talking about electrical energy, as opposed to sound or light energy. Some important ideas:

- Electrical energy can be either *delivered* or *absorbed*. For example:
  - A glowing flashlight bulb is absorbing electrical energy. (It is giving off (delivering) light and heat energy.)
  - A flashlight battery is delivering energy to the bulb (assuming the flashlight is on).
  - Rechargeable batteries can be re-charged, during which time the battery is absorbing electrical energy.

- Power is the rate at which energy is delivered or absorbed. That is,

$$p = \frac{dw}{dt}$$

  Units: *w* is the energy in Joules, t is the time in seconds, and *p* is the power in Watts.

- Since electrical energy can be delivered or absorbed, electrical power can be delivered or absorbed. It is very important that we keep track of whether a circuit element is delivering or absorbing power and energy.

We can relate electrical power to voltage and current.

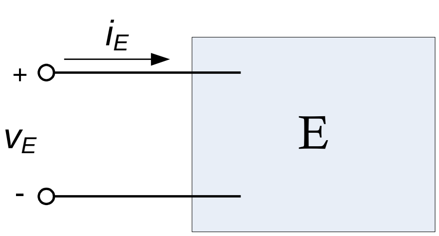
$$p = \frac{dw}{dt} = \frac{dw}{dq}\frac{dq}{dt} = v.i$$

Power delivered vs. power absorbed

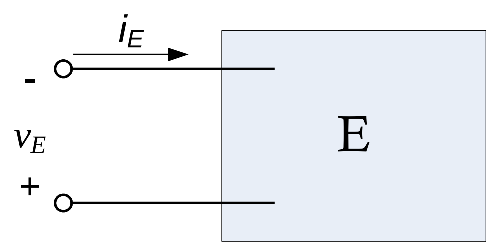
Electrical power is obtained by multiplying voltage and current. But how can we tell whether power is being delivered or absorbed? And what if the voltage is negative or the current is negative? To get all of that right, we need a rule for signs.

To illustrate, we will calculate the power absorbed by our circuit element. Here is the rule: In the diagram on the left, current is entering the positive terminal and leaving the negative terminal. So ***for the thing inside the box***, the current is in the direction of the voltage drop. In that case, we write $p_{abs,E} = v_E.i_E$. Here "$p_{abs,E}$" means "the power being absorbed by element E".

For the diagram on the right, where the current is in the direction of the voltage rise (for the thing inside the box), we write $p_{abs,E} = - v_E.i_E$. We need to use the appropriate sign, which we get by looking at the diagram, and at which way the current is going relative to the voltage drop.



$$p_{abs,E} = v_E i_E$$



$$p_{abs,E} = -v_E i_E$$

Some examples:

For the case on the left above:

If v_E = 3 [V] and i_E = 25 [mA], then $p_{abs.E} = v_E.i_E$ = (3) (0.025) = 0.075 [W] = 75 [mW].
If v_E = -2 [V] and i_E = 10 [mA], then $p_{abs.E} = v_E.i_E$ = (-2) (0.010) = - 20 [mW].

For the case on the right:

If v_E = 2 [V] and i_E = - 250 [mA], $p_{abs,E} = - v_E.i_E$ = - (2) (-0.250) = 0.5 [W].
If v_E = - 10 [V] and i_E = - 35 [mA], $p_{abs} = - v_E.i_E$ = - (-10) (-0.035) = - 350 [mW].

But wait: what does it mean that the absorbed power is negative? We interpret a negative absorbed power to mean that power is in fact being *delivered*, not absorbed. We could also have calculated delivered power as $p_{del} = - p_{abs}$.

Flow of charge and its relation to energy:

In the diagram on the left above, the current is entering at the top, and the voltage is positive at the top. Let's assume, as we did in the first case, that the current and the voltage are both positive. Let's also assume that positive charge is the current carrier. In that case, positive charge is flowing into the box at the top, and flowing out of it at a lower potential. This means that the positive charge is losing energy. Why? Imagine a ball rolling down a hill and think about its potential energy when it gets to the bottom – the positive charge moving from higher to lower potential is the same situation, energy-wise.

Now if the positive charge is losing energy, where is the energy going? It is going to whatever is inside the box, i.e., the circuit element, which means the circuit element must be absorbing electrical energy. The same thing holds if we assume current is due to electron flow. In that case, electron flow is from bottom to top, so the electrons are going from a more negative to a more positive potential, and they too are losing energy.

Important notes:

- Our choice of formula ($p_{\text{abs,E}} = v_{\text{E}}.i_{\text{E}}$ or $p_{\text{abs,E}} = - v_{\text{E}}.i_{\text{E}}$) depends only on the reference polarities for voltage and current in the figure. It does not depend on actual polarities. So all I need to do is look at the figure.
- Regardless of which formula I use, when I substitute values for v and i, I must keep track of the signs, and include them in the calculation.
- I may not know before hand whether power is being absorbed or delivered, but it doesn't matter. I can calculate the power absorbed anyway, and if the answer is negative, I conclude that power is being delivered.

Notation:

- It is **very, very important** to keep track of whether power is being delivered or absorbed. Therefore every time you calculate power, I want you to indicate whether you are calculating absorbed power or delivered power, and for which element. Do this by writing either $p_{\text{abs,E}}$ or $p_{\text{del,E}}$, or some other convenient and clear notation. The same goes for energy: tell me what you are calculating using proper notation.

- I am using a lower case p for power. I want you to do the same thing.

Summary

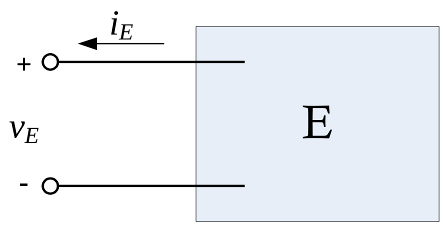
To calculate power…

- Choose whether to calculate absorbed power or delivered power; this is your choice, even if you have no idea which it "actually" is. Then, use the appropriate formula, which is based on a diagram showing reference current direction and reference voltage polarity.

- Plug voltage and current into the appropriate formula, including the signs of v and i.

- Decide whether power is actually being absorbed or delivered based on the sign of the result.

Example



Given: v_E = - 12 [V]; i_E = - 0.2 [A].

Problem: Calculate the power absorbed by the circuit element E.

Solution: At the box, the current is in the direction of the voltage rise, so we write

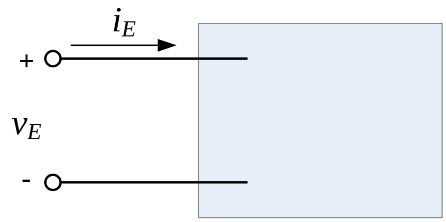
$$p_{abs,E} = - v_E . i_E = - (-12)\ (- 0.2) = - 2.4\ [W].$$

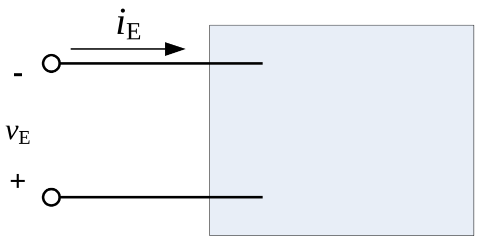
So power is in fact being delivered. Thus, the delivered power is 2.4 [W]; the absorbed power is -2.4 [W].

Sign Relationships: Active and Passive Sign Convention

We finish with a definition. Whether or not the reference direction for the current is in the direction of the voltage drop is an issue that will come up in future discussions, so we need a way to refer to it.

Passive Sign Convention: When the reference current direction is in the direction of the reference voltage drop, as it is in the figure to the right, we say that we are using the ***passive sign convention***.





Active Sign Convention: The alternative is a situation in which the reference current direction is in the direction of the reference voltage rise, as it is n the figure to the left. This is the ***active sign convention***.

The passive and active sign conventions provide names for dealing with these two cases.