Chapter 2: Circuit Elements

- Objectives
  - Understand concept of basic circuit elements
    - Resistor, voltage source, current source, capacitor, inductor
  - Learn to apply KVL, KCL, Ohm’s Law
    - Sign conventions

- Homework/Quiz/Exam Prep
  - When do we have enough equations?
  - Math requirements
    - Simultaneous linear algebraic equations

- Presentation
  - Ideal Basic Circuit Elements
    - Sources: Dependent and Independent
    - R, L, C
  - Resistors
    - Ohm’s Law
    - Power
  - Flashlight Circuit
    - KVL, KCL, Ohm
    - KVL shortcut
  - Special Cases
    - Short circuit; Open circuit
    - What happens to resistors that are shorted/opened?

Activity: worksheet on KVL, KCL, Ohm
Activity: sample problems

Chapter 2: Circuit Elements

There are five *Ideal Basic Circuit Elements*. We listed these in the previous chapter; we discuss them further here. The Ideal Basic Circuit Elements are as follows.

- Voltage source
- Current source
- Resistor
- Capacitor
- Inductor

These circuit elements are used to model electrical systems, as we discussed in Chapter 1. They are available in the laboratory, but the ones in the lab are not “ideal”; they are “real”. When we draw circuit models on the board or in quizzes and exams, we assume that ideal elements are intended, unless otherwise stated.

To solve circuits involving capacitors and inductors, we require differential equations. Therefore we will postpone these circuit elements until later in the course and deal for the moment with sources and resistors only.

2.1 Sources

Source: a device capable of conversion between non-electrical energy and electrical energy. Examples…

Generator: mechanical $\Rightarrow$ electrical
Motor: electrical $\Rightarrow$ mechanical

Important: energy and power can be either delivered or absorbed, depending on the circuit element and how it is connected into the circuit.

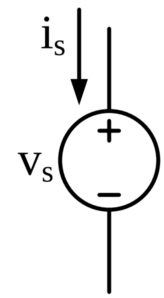
We will deal with two kinds of sources: ***Dependent*** and ***Independent***

Dependent Source: a source whose value depends on a voltage or current elsewhere in the circuit.

Independent Source: a source whose value does is independent of any other voltages or currents in the circuit.

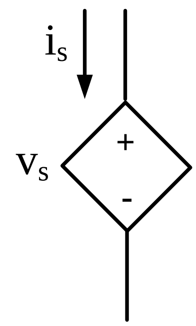
Voltage Sources: In general, there will be a current flowing through a voltage source. That current can be positive, negative, or zero, depending on how the source is connected into the circuit.

Ideal Independent Voltage Source



The ***ideal independent voltage source*** maintains a fixed voltage across its terminals regardless of the current through it.

Ideal Dependent Voltage Source

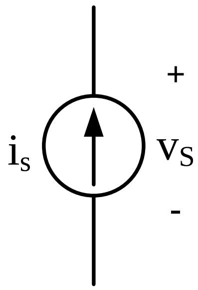


The ideal dependent (or controlled) voltage source maintains a voltage across its terminals that depends on either a voltage or current elsewhere in the circuit. Thus

$$v_s = \mu v_x \text{ or } v_s = \rho i_x$$

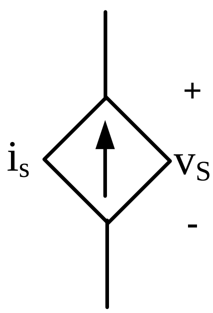
where v_x (i_x) is a voltage (current) somewhere else in the circuit, and μ and ρ are constants. Also note that μ is dimensionless but ρ has dimensions [Volts/Amp] (it multiplies a current but the resultant unit must be Volts).

Current Sources: In general, there will be a voltage across a current source. That voltage can be positive, negative, or 0 depending on how it is connected into the circuit.



Ideal Independent Current Source

The ideal independent current source maintains a fixed current through its terminals regardless of the voltage across it.



Ideal Dependent Current Source

The ideal dependent (or controlled) current source maintains a current through its terminals that depends on either a voltage or current elsewhere in the circuit. Thus

$$i_s = \alpha v_x \text{ or } i_s = \beta i_x$$

where v_x (i_x) is a voltage (current) somewhere else in the circuit, and α and β are constants. Also note that β is dimensionless but α has dimensions [Amps/Volt] (it multiplies a voltage but the resultant unit must be Amps).

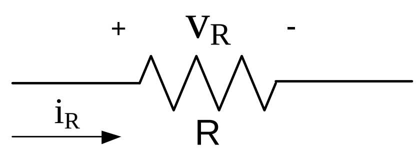
2.2 Electrical Resistance; Ohm's Law

Resistance: The characteristic of a material by which it impedes the flow of current.

Resistor: A basic circuit element that models resistance; it is characterized by its resistance R in Ohms [Ω]. For an ideal resistor, R is constant regardless of the current through or voltage across it.

The voltage across a resistor is related to the current through it. The relationship is called ***Ohm's Law***, which states that **for a resistor**, ***the ratio of the voltage to the current is the resistance***. In other words

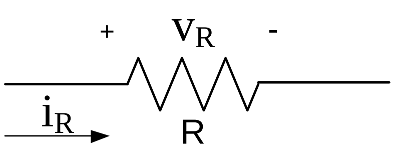
$$\boxed{R = \frac{v_R}{i_R}}$$



and R is constant.

We generally assume that R is positive. But remember that the voltage and current polarities labeled in the figure above are simply reference polarities: the voltage v_R and current i_R can be either positive or negative. To make sure R is positive, we have to have a sign convention…

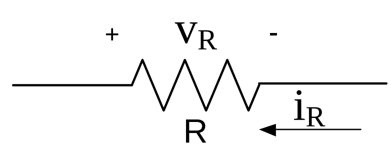
Sign Conventions



In the drawing to the left, the current reference is in the direction of the voltage reference drop (passive sign convention). For this case, we write Ohm's Law as

$$v_R = i_R\, R.$$

In the drawing to the right, the current reference is in the direction of the voltage reference rise (active sign convention). In that case, Ohm's Law becomes



$$v_R = -i_R\, R.$$

Some very important notes:

- Q: **<u>Why do we need two of Ohm's Law?</u>** Can't we just have $v = iR$?
  A: We are free to label the resistor voltage and current in either the active or the passive convention. But the value of R is assumed to be positive, so we have to "adjust" Ohm's law with a sign, to avoid having the ratio of the voltage to the current come out negative.

- **<u>Ohm's Law holds only for resistors.</u>** It is a common error to assume that Ohm's Law holds for voltage sources and current sources. Students try to find a "resistance" for a voltage source and then figure out the current. **<u>This cannot be done</u>**; there is no such relationship for voltage sources. Similarly **<u>you cannot define a resistance</u>** for a current source using Ohm's law, because Ohm's law does not hold for current sources.

- **<u>A Common Error Concerning Voltage Sources</u>** Sometimes a student will assume that the current through a voltage source ***must*** be going one way or another because of the source polarity. This is wrong-thinking. The current through a voltage source can go in either direction, or it could be 0.

- **<u>A Common Error Concerning Current Sources</u>** Another common error is to that there is no voltage across a current source. The voltage across a current source ***might*** be zero, but in general it is not. Only Kirchhoff's Voltage Law can tell us what it is, as we shall see later on.

Conductance

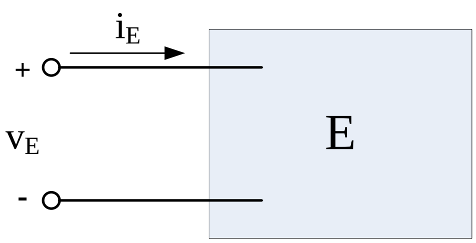
We often find it convenient to define the conductance, which is the reciprocal of resistance.

$$G = {}^{1}\!/_{R}\,.$$

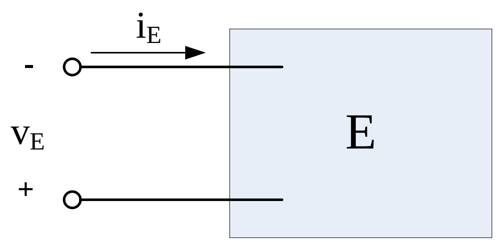
The units of conductance are *Siemens*, or *Mho's* (1/Ω).

Power for a resistor

Recall from Chapter 1 the rules for getting the equation for absorbed power. Here is a circuit element 'E'.



$$p_{abs,E} = v_E i_E$$



$$p_{abs,E} = -v_E i_E$$

Now imagine that the circuit element is a resistor with resistance R. Then for the box on the left, we have, with $v_E = v_R$ and $i_E = i_R$

$$p_{abs,R} = v_R\, i_R \quad \text{and from Ohm's Law:} \quad v_R = i_R\, R.$$

If we use the second equation to eliminate v_R from the first one, we get

$$p_{abs,R} = i_R^2 R.$$

If we use second equation to eliminate i from the first one, we get

$$p_{abs,R} = {v_R^2}/{R}.$$

In either case, note that the absorbed power $p_{abs,R}$ is always positive, as it should be for a resistor.

If we consider the box on the right (previous page), we have

$$p_{abs,R} = -v_R\, i_R \qquad v_R = -i_R R$$

In this case, substitution of the second equation into the first gives the same results as before, as it should. This is an example of how the rules for dealing with signs lead us to the correct results.

2.3 Construction of a Circuit Model

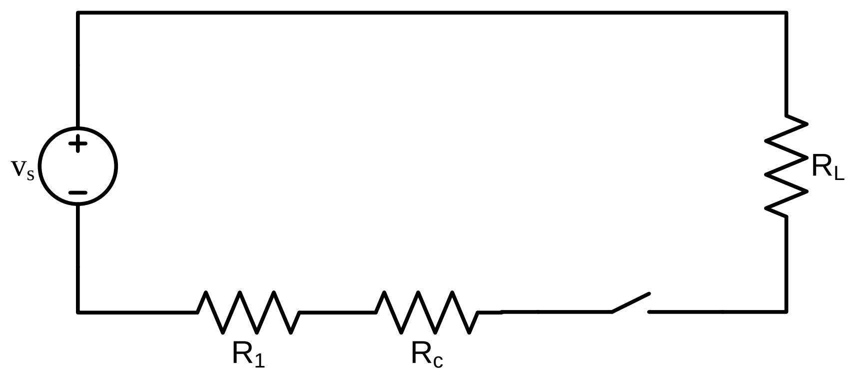
Suppose we want to construct a circuit model for a flashlight. What are the important components?

- Batteries: v_s
- Filament (lamp): R_L
- Case: R_c
- Switch
- Coil: R_1

We can model the batteries with a voltage source. The other components can be modeled with resistors, except for the switch. The switch is nothing more than a connection that can be either open (off) or closed (on).

The filament is more than a resistor: it gives off light and heat. However, we are interested in only electrical behavior, so we will ignore those properties and stick with the resistor as a model.

How do we connect these elements together? An examination of a flashlight shows that these things are connected in series (more on that later), which is to say they are connected end-to-end. The following diagram results.



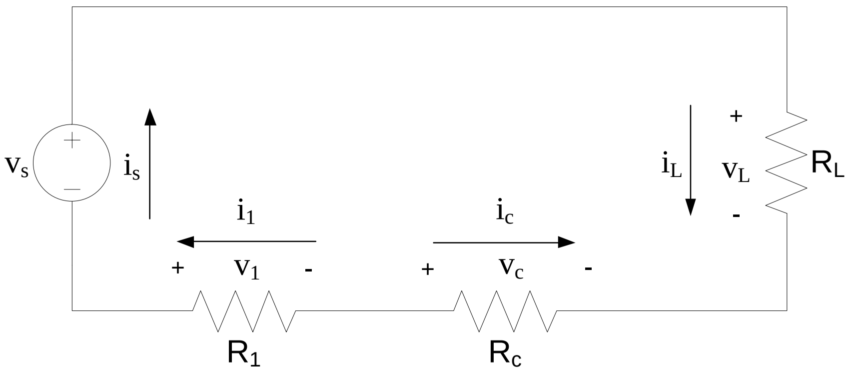
This is a circuit model for a flashlight. We have made some assumptions here…

- The switch is ideal and has no resistance when it is closed, and an infinite resistance when it is open. Real switches have some small resistance when closed, and a large but finite resistance when open.
- The batteries are ideal: the voltage does not depend on the fact that current is flowing through them, which is not the case for real batteries.
- We use a resistor, which has constant R, to model the lamp. In fact, the resistance of a real flashlight lamp changes a little when current flows through it. We will ignore that detail here.

Now what? We want to "solve" this circuit. That means we want to know all the voltages and currents associated with all the circuit elements. To do that, we need to know something more about the voltages and currents in this circuit. That information comes from Kirchhoff's Laws.

2.4 Kirchhoff's Laws

We now assume the switch on the flashlight is closed, so the flashlight is on. Then each of the circuit elements will have a voltage across it and a current through it, which we need to find. The voltages and currents are labeled below. Note that the voltage polarities and current directions are arbitrary; I can label them any way I like



How many unknowns do we have? There are seven: i_1, i_c, i_L, i_s, v_1, v_c, and v_L (v_s and the resistances assumed to be known). That means we need seven equations. We can use Ohm's Law to get three equations, but we need more.

$$v_1 = -i_1 R_1 \quad v_c = i_c R_c \quad v_L = i_L R_L \, .$$

Activity

Students will do this at their desks with TA assistance.

Kirchhoff's Current Law

Kirchhoff's Current Law (KCL) puts constraints on the currents in a circuit. Before we can state it we need a definition:

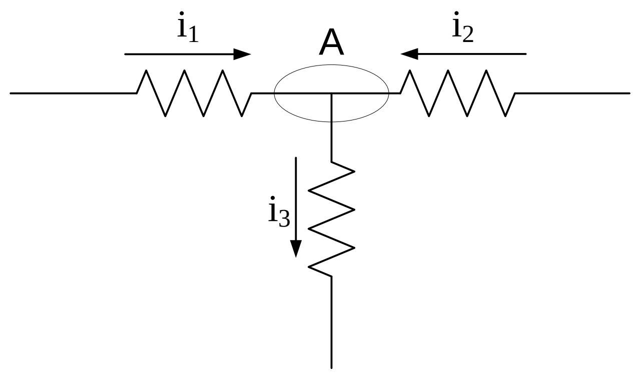
> **Definition:** *A **node** is a place on the circuit where two or more circuit elements join.*

Kirchhoff's Current Law states the following.

> *The algebraic sum of all currents at any node in a circuit is equal to zero.*

The word "algebraic" means that we must associate the proper sign with each of the currents. That means we need a rule for assigning signs: <u>A current entering a node will get a negative sign when used in KCL; a current leaving a node will get a positive sign.</u>

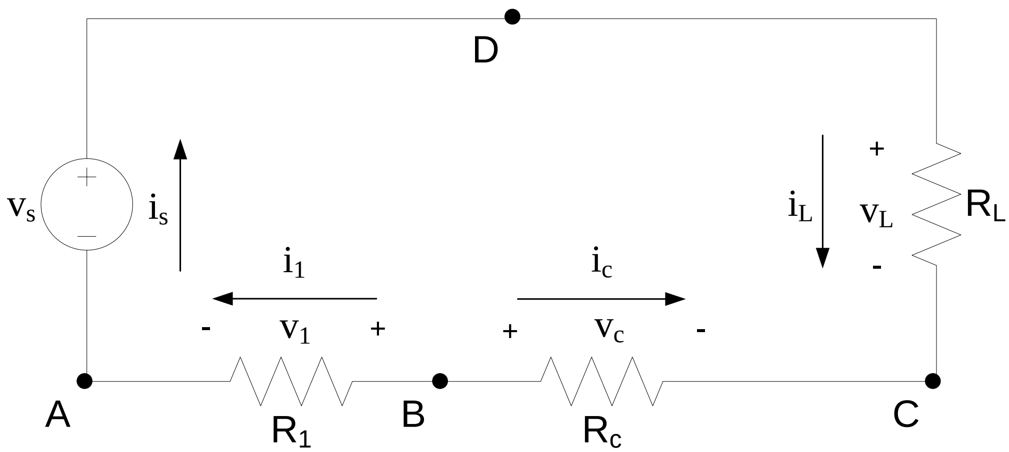
You can find more on the definition of nodes in Dr. Dave's Project: Module 1 Part IV.



Simple example: In the circuit to the right, the node is circled and labeled A. We have three currents either entering or leaving A. They're labeled arbitrarily; we do not know the actual current directions. For the purposes of writing KCL, we note that **i_1 and i_2 are entering** the node **and i_3 is leaving**. We therefore write

$$-i_1 - i_2 + i_3 = 0.$$

We now apply KCL to our flashlight circuit. The nodes in that circuit each have only two circuit elements connected to them, so the equations are simple. Note that node D includes the entire connection between the voltage source and the resistor R_L.



$$
\begin{aligned}
A&:\ i_s - i_1 = 0 \\
B&:\ i_1 + i_c = 0 \\
C&:-i_c - i_L = 0 \\
D&:\ -i_s + i_L = 0
\end{aligned}
$$

If we examine the KCL equations, we find that any one of them can be obtained from the other three. For example, adding the equations for nodes A and B and C gives the equation for node D. So they are not independent equations, and we can only use three of them.

This will always be the case: when you have **n** *nodes in a circuit, you can only write* **n-1** *independent KCL equations.*

So far we have six equations (three Ohm's Law and three KCL) so we need one more to solve for seven variables.

Kirchhoff's Voltage Law

Kirchhoff's Voltage Law (KVL) puts constraints on the voltages in a circuit. Before we can state it we need a definition.

> **Definition:** *A **closed path** or **loop** is a path traced through circuit elements that begins and ends at the same place but does not go through any node more than once.*

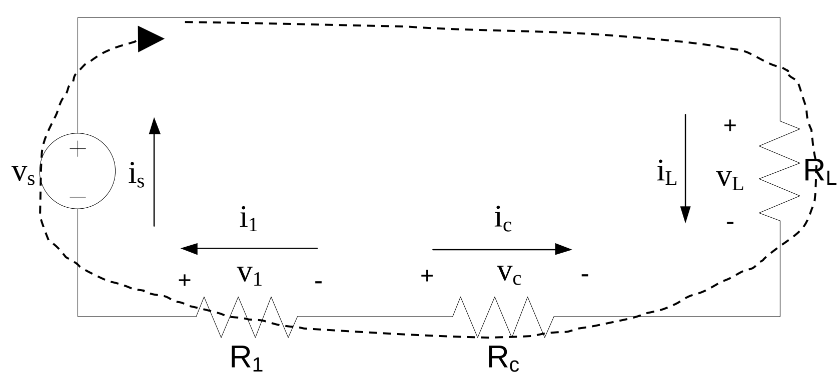
You might want to check out Dr. Dave's Project: Module 1 Part IV.

Kirchhoff's Voltage Law states the following.

> *The algebraic sum of the all voltages around any closed path equals zero.*

The word "algebraic" means that we must associate the proper sign with each of the voltages if we are going to apply KVL correctly. That means we need a rule for assigning signs. We will adopt the following. <u>In traversing our closed path, if we encounter a voltage drop, we will use a positive sign in KVL; if we encounter a voltage rise, we will use a negative sign.</u>

In the flashlight circuit, we will traverse a path going around the circuit from the top right corner in a clockwise direction. This is an arbitrary choice; I can start anywhere and go in either direction. The clockwise path is shown in the circuit to the right.



We have the following KVL equation based on the path shown in the figure.

$$v_L - v_c - v_1 - v_s = 0.$$

Note that whether a voltage (v_L, for example) gets a positive sign or a negative sign in our KVL depends on which direction we are going in. If we go counter-clockwise, we encounter a voltage rise at R_L and v_L would have entered the KVL with a negative sign. In fact, if we go counter-clockwise, all the voltage signs change, which means the equation would be algebraically the same as the one we get going clockwise.

We can now solve for all the currents and voltages in our circuit, assuming that v_s and the resistances are known. But: In a circuit with only one path, there is only one current. We might have guessed this, but in any case it follows from KCL. When we solve our KCL equations together we find

$$i_s = i_1 = -i_c = i_L \ .$$

Now applying Ohm's Law to each of our flashlight resistors gives

$$\begin{aligned} v_1 &= -i_1 R_1 = -i_s R_1 \\ v_c &= i_c R_c = -i_s R_c \\ v_L &= i_L R_L = i_s R_L \end{aligned}$$

I have chosen to express all my currents in terms of i_s. We can "solve" the circuit by finding the current i_s from KVL. If we substitute the equations above into KVL we get

$$v_L - v_c - v_1 - v_s = 0 \ \Rightarrow \ i_s R_L - \left(-i_s R_c\right) - (-i_s R_1) - v_s = 0 .$$

Finally, we have the current:

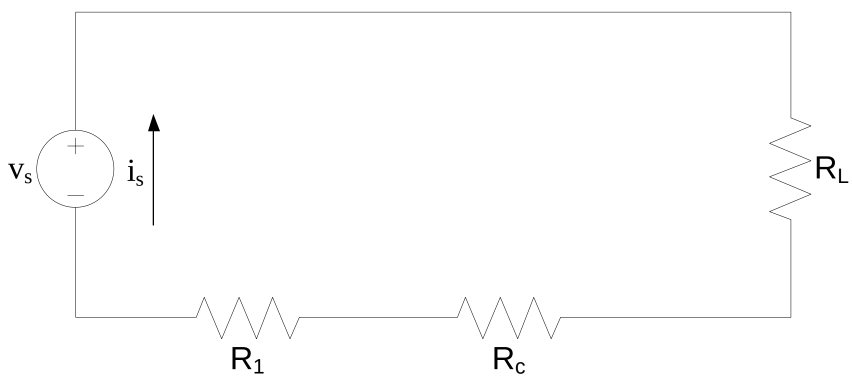
$$i_s = \frac{v_s}{R_1 + R_c + R_L} .$$

Shortcut for KVL equations

Note that the last step before solving for current in the flashlight circuit was to substitute Ohm's Law into the KVL equations. We can get the results of that substitution in one step: When traversing the KVL path, instead of writing "v" for each resistor, write "iR". Be sure to use the correct sign from the following rules:

- When traversing a resistor in the same direction as the KVL path, use a positive sign for the "iR" term; when traversing a resistor in the opposite direction as the KVL path, use a negative sign.

- When going through a voltage source in the direction of the voltage drop, use a positive sign; when going through a voltage source in the direction of the voltage rise, use a negative sign.



Let's rework the flashlight problem. Since we will substitute Ohm's Law directly, we don't need labels on the resistor voltages. Also we will use the fact that there is only one current in the circuit (i_S). Now if go clockwise…

$$-v_S + i_S R_L + i_S R_C + i_S R_1 = 0\ .$$

Our KVL path is in the same direction as the current, so all our "iR" terms are positive.

If we go counterclockwise…

$$v_S - i_S R_L - i_S R_C - i_S R_1 = 0\ .$$

Now the KVL path is opposite the direction of the current, so all our "iR" terms are negative. But we go through the voltage source the other way too, so that sign changes and the equation is the same as before.

In class we will apply KVL, KCL, and Ohm's Law to circuits that are considerably more complex than the one above. After doing that, we will be in a position to understand the following definitions.

Definitions: Open Circuit and Short Circuit

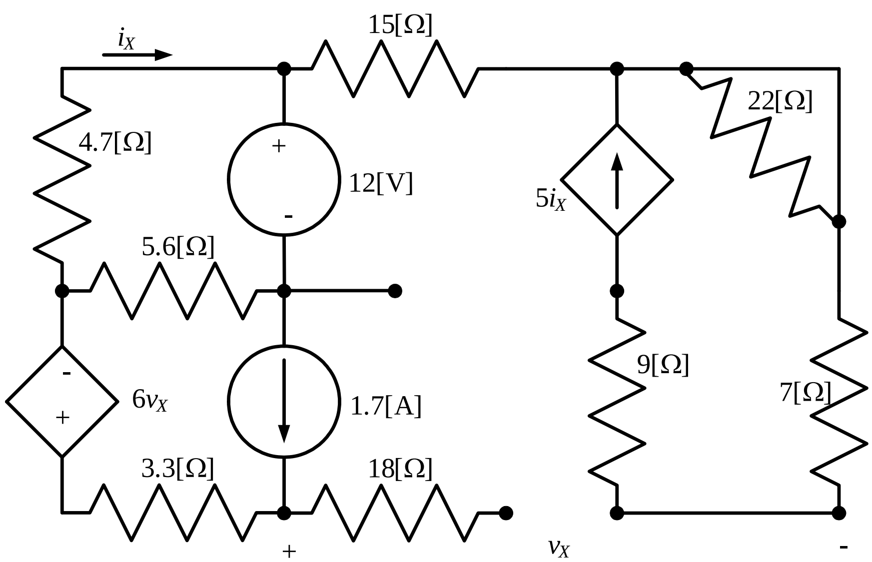
When looking at two terminals somewhere in a circuit, we are often interested in certain special conditions at those terminals:

- ***Open Circuit***: When there is nothing attached to the terminals; the circuit is ***open*** there. Open circuit means $R_L = \infty$. The voltage across the terminals in this case is the ***open circuit voltage***.

- ***Short Circuit***: In this condition, there is a wire connected between the terminals; in other words, $R_L = 0$. The current flowing through the wire is the ***short circuit current***.

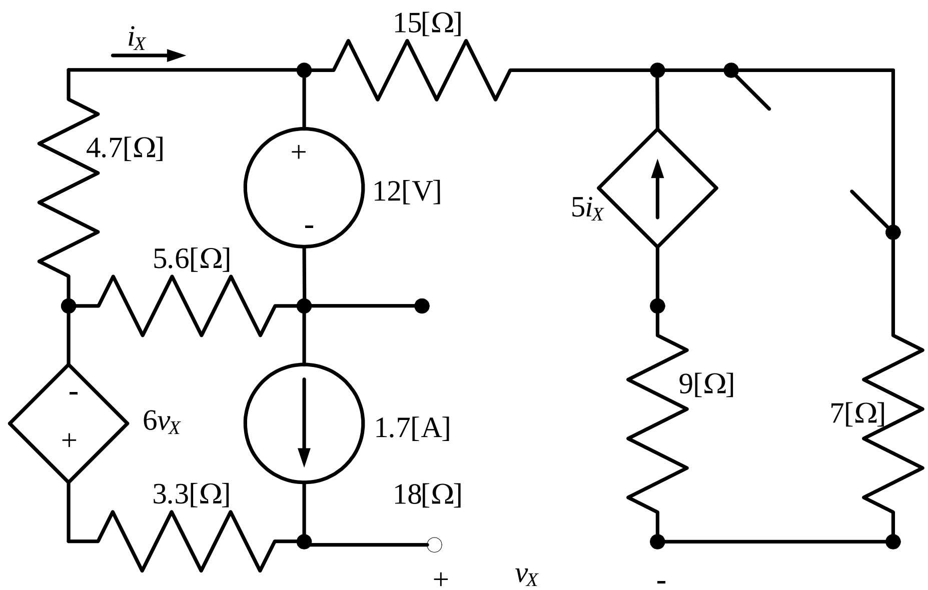
Example: Short-Circuited and Open-Circuited Resistors

It's instructive to see what happens if a resistor is short-circuited or open-circuited. Let's look at the circuit below.



Notice the 22 [Ω] resistor: there is a wire connected across it. Since there is no voltage drop across a wire, we have forced the voltage drop across the resistor to be zero. By Ohm's Law, the current in that resistor must be 0. That means we can replace the resistor with an open circuit (because an open circuit has zero current through it). We have done that in the next figure.

Now look at the 18 [Ω] resistor. It's hanging in the air – there is nothing connected to it. That means there is no current in it (since there's no place for current to go when it gets to the end of that resistor). So in this case we have forced the current in the resistor to be 0, which mans by Ohm's Law that the voltage across it must be 0. So we can replace this resistor with a short circuit (because there's no voltage across a short-circuit). We have done that in the figure below as well. (We have also taken the liberty of re-drawing the voltage v_x in a way that makes it more obvious what that voltage is.)



Bottom line: if there is a wire across a resistor, the current in it is 0 and we can replace it with an open circuit (i.e., remove it). If the resistor is hanging with nothing connected to it, the voltage across it is 0 and we can replace it with a short circuit.