Chapter 3: Simple Resistive Circuits

- Objectives
  - Understand resistor combinations
    - Meaning of *series* and *parallel*
    - VDR and CDR and their proper application
- Device Modeling
  - Foreshadowing of Thevenin and Norton

- Presentation
  - Series and parallel circuit elements
  - VDR, CDR
    - Paying attention to the derivation so that we know where these apply
  - Wheatstone Bridge and Delta-Wye connections
  - Device Modeling

Activity: worksheet on R combinations
Activity: sample problems

Chapter 3: Simple Resistive Circuits

This chapter presents techniques that are useful in solving and thinking about circuits. You should think of these things as "tools" in a toolbox; keep them handy and take them out when you need them. Be very careful, however, that you apply them correctly. It is a very common problem that students apply rules where they are not valid. To avoid that problem, make sure you understand the derivation of each of these rules; if you don't, you run the risk of misusing them.

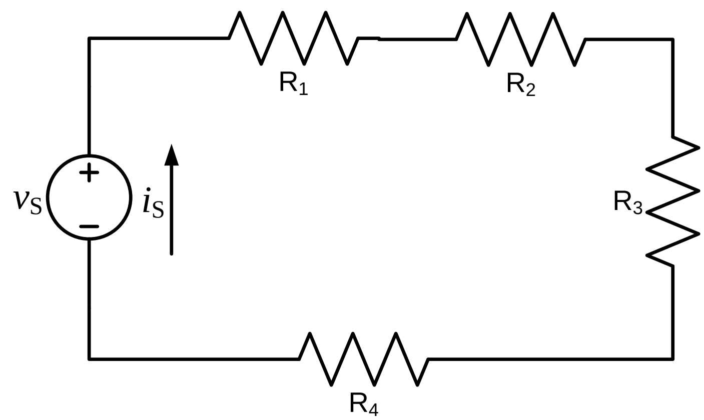
3.1 Resistors in Series

<u>Circuit elements are in ***series*** if the same current flows through them.</u>

Two circuit elements are in series if there is nothing else connected between them - in other words, if there is no junction connecting additional circuit elements.

Simplification:

The resistors in the circuit to the right are in series, so the current in them is the same. In that case, by KVL:

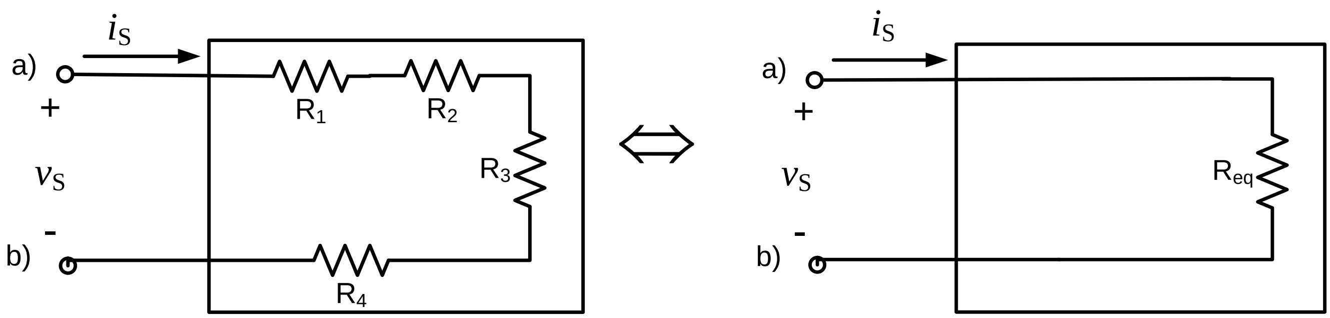


$$v_s = i_s\left(R_1 + R_2 + R_3 + R_4\right).$$

This equation says that, given v_s, the current will stay the same if we replace the four resistors with an "equivalent" resistor $R_{eq} = R_1 + R_2 + R_3 + R_4$. In other words

$$v_s = i_s\left(R_{eq}\right) \Rightarrow R_{eq} = R_1 + R_2 + R_3 + R_4$$

What "equivalence" means: If the resistors were placed in a box, anything connected to the box at terminals a, b would not know the difference between the four original resistors and the equivalent resistor.



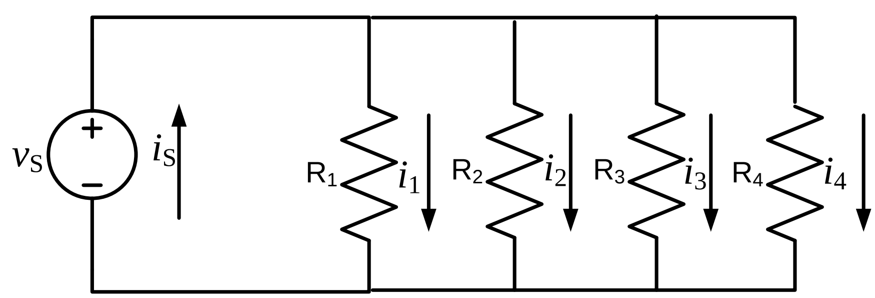
More generally, for 'k' resistors in series, we can define an equivalent given by

$$R_{eq} = R_1 + R_2 + R_3 + \cdots = \sum_1^k R_k \ .$$

3.2 Resistors in Parallel

<u>Circuit elements are in ***parallel*** if the same voltage is across them</u>.

Circuit elements are in parallel if the voltage across them is the same – that is, if they are connected together at both ends.



Simplification:

The four resistors in the figure to the right are in parallel with each other, and with the voltage source.

Since the same voltage across each resistor is the same, we have

$$i_1 = \frac{v_s}{R_1} \qquad i_2 = \frac{v_s}{R_2} \qquad i_3 = \frac{v_s}{R_3} \qquad i_4 = \frac{v_s}{R_4}$$

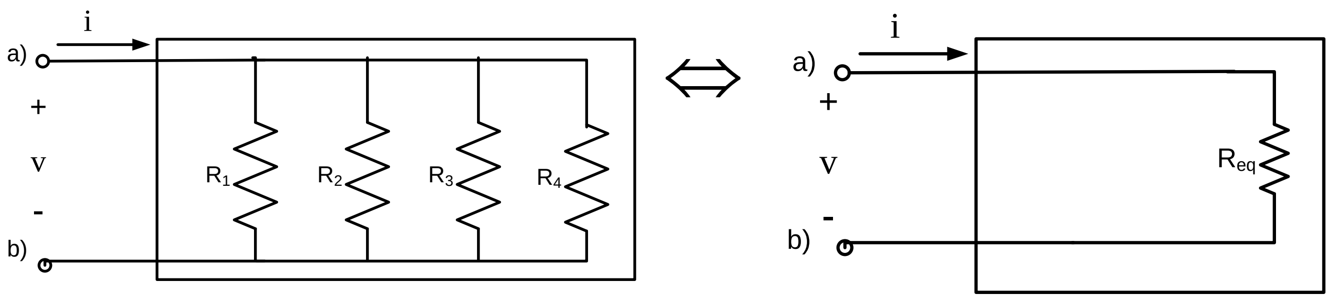
But these currents add to the source current, so $i_s = i_1 + i_2 + i_3 + i_4$. Then

$$i_s = \frac{v_s}{R_1} + \frac{v_s}{R_2} + \frac{v_s}{R_3} + \frac{v_s}{R_4} = v_s\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}\right)$$

This last equation says that, given v_s, the current will stay the same if we replace the four resistors with an equivalent R_{eq}; that is,

$$i_s = \frac{v_s}{R_{eq}} \quad \text{provided we define} \quad \frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_4}$$

This equivalence is illustrated in the figure below.



More generally, for 'k' resistors in parallel, we have

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots = \sum_1^k \frac{1}{R_k}$$

Two Resistors in Parallel

For the special case of **two resistors in parallel**, we have

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}.$$

$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

It is important to note that this algorithm does not hold for any more than 2 resistors. You ***cannot*** *use if for 3, 4, 5, …or more resistors.*

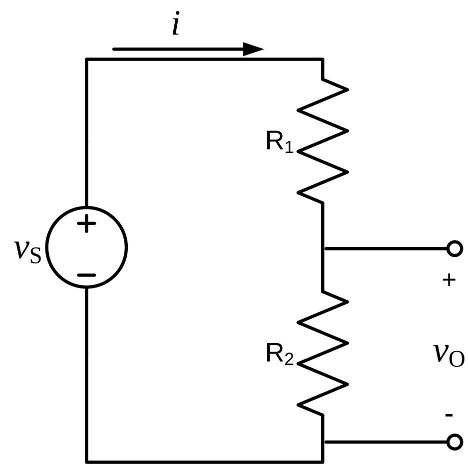
We also note the equivalent resistance R_{eq} for resistors in parallel is always smaller than any of the resistances in the original set. This fact can be used as a quick "check" that your equivalent is correct.

3.3 The Voltage-Divider and Current-Divider Circuits

Voltage Divider Rule (VDR)

The voltage divider is a configuration that occurs often. Sometimes it arises as a result of simplification of a more complicated circuit.

An analysis of the circuit on the right gives the following results for the "output" voltage v_o.

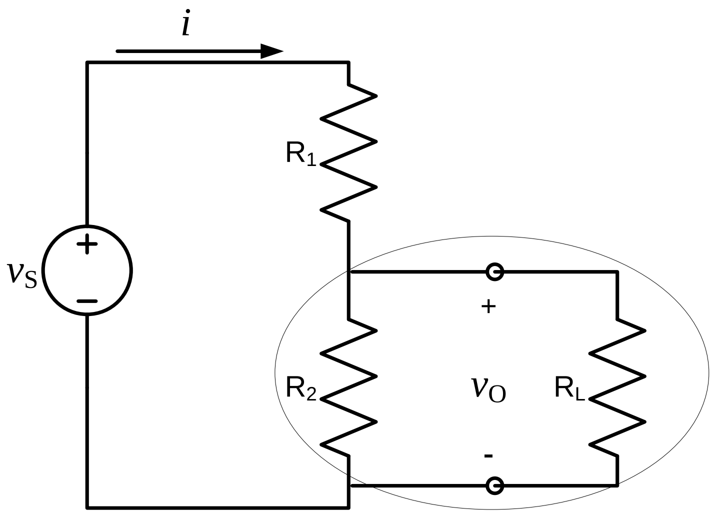


$$v_s = iR_1 + iR_2 \Rightarrow i = \frac{v_s}{R_1 + R_2}.$$

$$\boxed{v_o = iR_2 = v_s \frac{R_2}{R_1 + R_2}}.$$

So the source voltage has been "divided" by the two resistors; we have calculated the fraction of the source voltage that appears across R_2. We could have done a similar thing for R_1. This is the ***voltage divider rule*** (VDR).

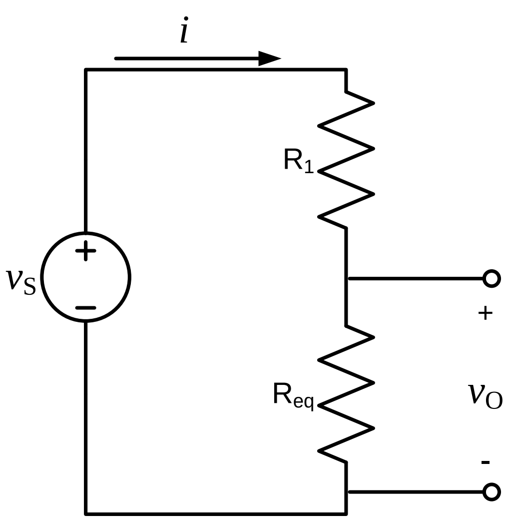
What if we connect a "load" resistance R_L to resistor R_2?



If we add a resistor R_L, the voltage v_O will change because the current through R_2 is now different; some of it is being "drawn off" by R_L. (The current *i* will change, too.) Let's do the calculation…

The figure to the left shows a load resistor connected across R_2. The equivalent resistance for R_2 and R_L in parallel is:

$$R_{eq} = \frac{R_L R_2}{R_L + R_2}$$



The voltage v_O is now

$$v_o = v_s \frac{R_1 R_{eq}}{R_1 + R_{eq}},$$

which we get by applying VDR.

Note carefully that we *cannot* apply the voltage divider rule in the circuit above using R_1 and R_2; this circuit is not the same as the one we used to derive VDR. However, if we combine R_2 and R_L into a parallel equivalent, the combination of R_1 and R_{eq} *is* the same as the VDR circuit.

The voltage v_O can also be written (after some algebra)

$$v_O = v_S \frac{R_1 R_{eq}}{R_1 + R_{eq}} = v_S \frac{R_2}{R_1\left(1+\frac{R_2}{R_L}\right)+R_2}.$$

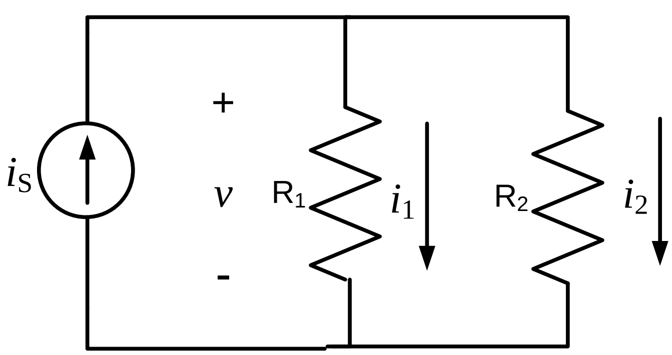
This makes it clear that the VDR formula in its original form does not apply here.

Note:

If R_L is very large (infinite), the second equation for v_S reduces to the simple voltage divider equation, as it should (the term $\left(1+\frac{R_2}{R_L}\right)$ approaches 1).

Current Divider Rule (CDR)

Just as voltage can be “divided” by two resistors in series, current can be “divided” by two resistors in parallel. The circuit below shows how the current in each of the resistors can be found.



The voltage v is $v = i_1 R = i_2 R$.

If we combine the resistors in parallel we get

$$v = i_S R_{eq} = i_S \frac{R_1 R_2}{R_1 + R_2}.$$

Equating the expressions for v from the first equations gives

$$i_1 = \frac{v}{R_1} = i_S \frac{R_2}{R_1 + R_2}$$

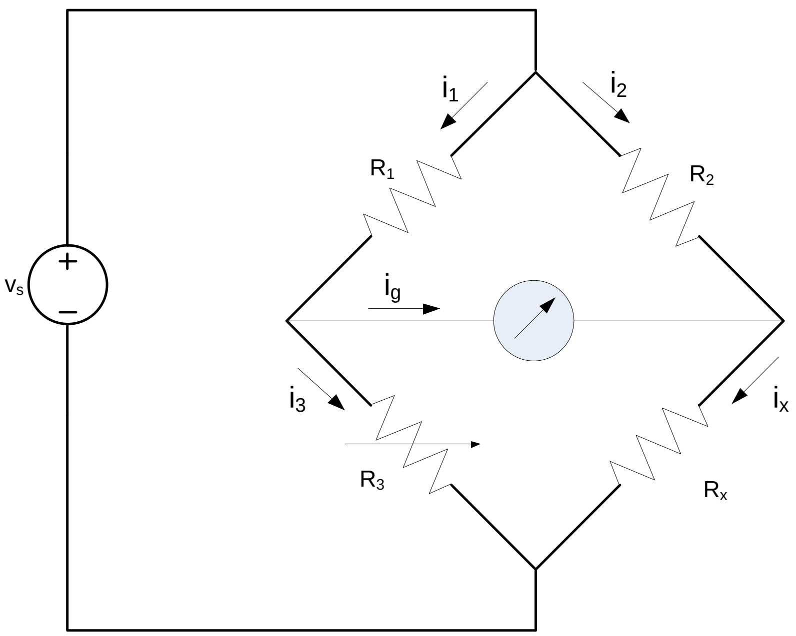
$$i_2 = \frac{v}{R_2} = i_S \frac{R_1}{R_1 + R_2}$$

Be careful not to use VDR and CDR where they don’t apply!! Students lose a lot of credit on quizzes and exams because they use VDR and CDR where they don’t apply. They do this because they are not keeping in mind the derivation of these rules. We will see examples of this in class…

3.4 Measuring Resistance – The Wheatstone Bridge

The Wheatstone Bridge (or just "bridge") configuration is useful in making a variety of electrical measurements. For example, we can use it in the circuit shown below to measure the unknown resistance R_x.

To measure R_x, we insert a sensitive d'Arsonval meter movement (which we will call a *galvanometer*) into the center branch, as shown. The arrow through R_3 means that it is adjustable. Then, we adjust R_3 until the current i_g is zero. In that case, we have $R_x = R_3 \frac{R_2}{R_1}$. Let's prove that result…



$i_g = 0 \implies i_1 = i_3$ and $i_2 = i_x$.

Also, by KVL through R_1, R_2, and the d'Arsonval, $i_3 R_3 = i_x R_x$ and $i_1 R_1 = i_2 R_2$. Solving for R_x…

$R_x = \frac{i_3}{i_x} R_3$. We also have

$\frac{i_3}{i_x} = \frac{i_1}{i_2} = \frac{R_2}{R_1}$. So finally we get

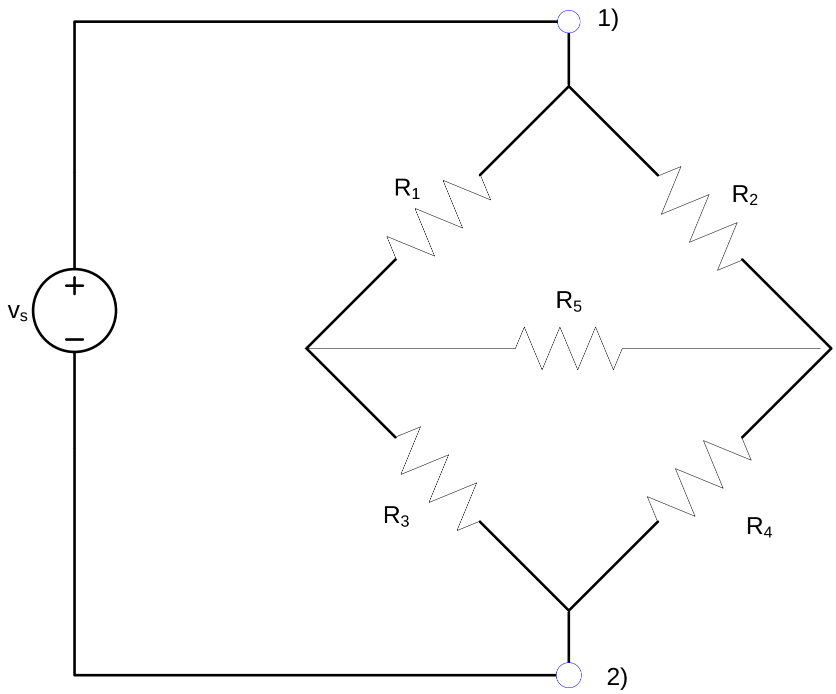
$$R_x = R_3 \frac{R_2}{R_1} \ . \ \ QED$$

Typically we make R_1 and R_2 adjustable in decades, i.e., R_1 and R_2 can be set to 1 [Ω], 10 [Ω], 100 [Ω], and 1000 [Ω]. That means $R_2/R_1 = 0.001, 0.01, \ldots 100, 1000$. R_3 is set to vary from, say, 1 [Ω] to 1000 [Ω] in increments of 1 [Ω]. That will give us a wide range of possible unknown resistances. For practical reasons we should expect to be able to measure

$$1\ [\Omega] < R_x < 1\ [\mathrm{M}\Omega] \ .$$

3.7 Delta-to-Wye (Pi-to-Tee) Equivalent Circuits

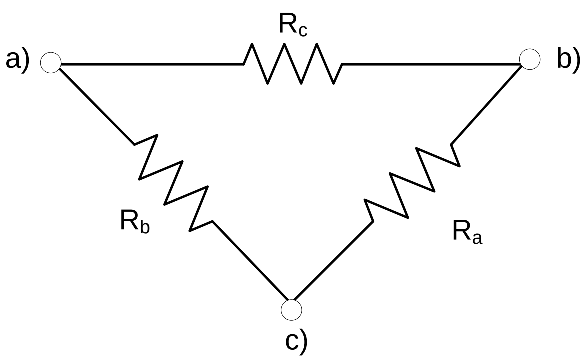
The Wheatstone bridge is an example of a circuit for which resistances cannot be reduced using parallel and series combinations. Suppose we model the galvanometer with a resistance…



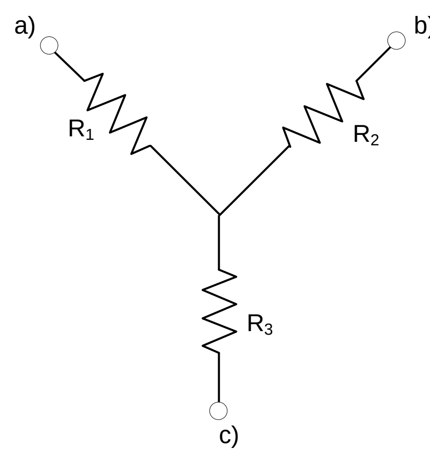
What is the equivalent resistance at the terminals a) and b) ?? We cannot use series/parallel combinations here because there are no resistors in series or parallel!!

$$R_{eq,12} = ??$$

We will not prove it here, but in this case we can use the following transformation…



$\Longleftrightarrow$



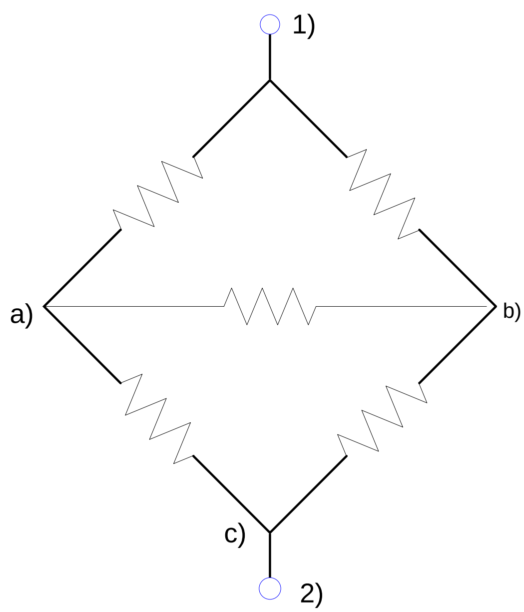
These configurations are interchangeable, but be careful to note the positions of the terminals (a, b, c) relative to the resistors. The formulae for the transformations are…

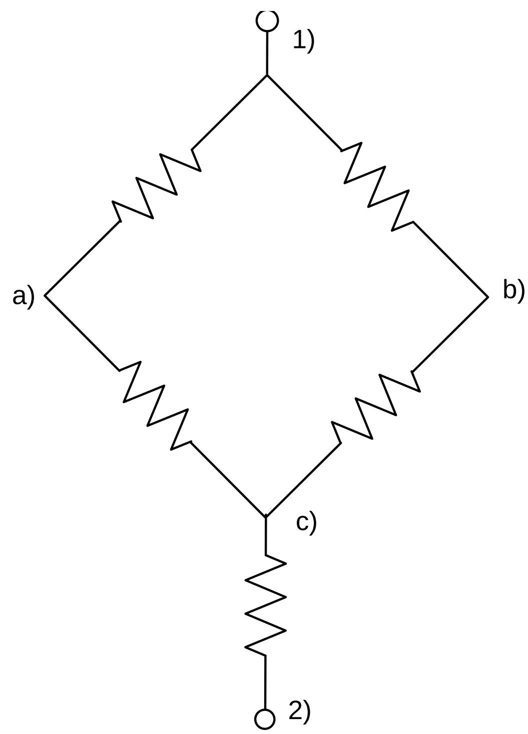
$$R_a = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_1} \qquad R_1 = \frac{R_bR_c}{R_a + R_b + R_c}$$

$$R_b = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_2} \qquad R_2 = \frac{R_aR_c}{R_a + R_b + R_c}$$

$$R_c = \frac{R_1R_2 + R_2R_3 + R_3R_1}{R_3} \qquad R_3 = \frac{R_aR_b}{R_a + R_b + R_c}$$

Now we can transform the bridge resistors like this:





$$\Longrightarrow$$

Watch where you put the terminals a, b, c!!