Chapter 6: Inductance and Capacitance

We introduce here the two remaining basic circuit elements: the inductor and the capacitor.

The **behavior of the inductor** is based on the properties of the magnetic field generated in a coil of wire. In fact, the inductor is basically a coil of wire.

Ampere's Law: current in a coil → magnetic field

Faraday's Law: Time-varying magnetic field → induced voltage (emf)

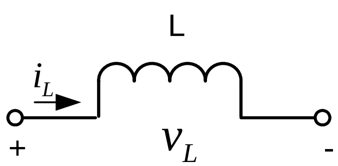
In circuits that we will study, the time-varying magnetic field is produced by a changing current.

The **behavior of the capacitor** is based on the properties of the electric field created in a dielectric (non-conductor) placed between two conductors. The capacitor is basically a non-conductor sandwiched between two conductors.

Energy can be stored in, but not generated by, an inductor or a capacitor, so these are passive devices. The inductor stores energy in its magnetic field; the capacitor stores energy in its electric field.

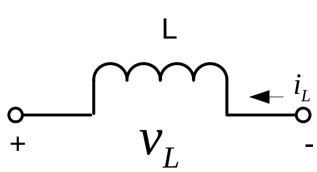
6.1 The Inductor

Circuit symbol



There is a relationship between current and voltage for an inductor, just as there is for a resistor. However, for the inductor, the voltage is related to the change in the current, as follows.

$$v_L = L\frac{di_L}{dt}$$



This relationship holds when the voltage and current are drawn in the passive sign convention. When they are in the active sign convention, we need a '-' sign:

$$v_L = -L\frac{di_L}{dt}$$

The relationship between current and voltage involves the time derivative of the current. This is because a changing current produces a changing magnetic field, which induces a voltage.

Units:

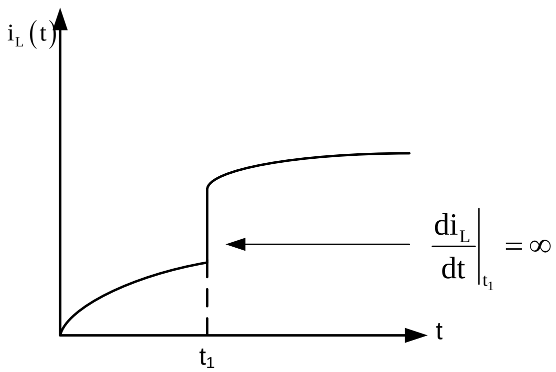
$$[L] = \frac{voltage}{current/time} = \frac{volt - \sec}{Amp} = Henry[H]$$

Very Important Points:

1. $v_L = L\dfrac{di_L}{dt} \Rightarrow$ if current i_L is constant (but NOT necessarily 0), there is no voltage. In other words, **under constant current conditions, the inductor is a short**.

2. An instantaneous change in current would generate an infinite voltage! Therefore we assume (and in reality this is always the case) that **in an inductor, there cannot be an instantaneous change in current.**

Here is what we mean by "instantaneous change in current":



Here, the current changes from one value to another over a time span of 0 [s] at t_1, i.e., instantaneously. This produces a derivate, and hence a voltage, that is infinity large. We can't have this!!

Current in terms of voltage

The current-voltage relationship we discussed above gives the inductor voltage if we know the inductor current. But sometimes we have the inductor voltage and need to find the current So we need to integrate…

$$v_L(t) = L\frac{di_L}{dt}$$

Integrate both sides:

$$\int_{t_0}^{t} v_L(t)dt = \int_{i_L(t_0)}^{i_L(t)} L di_L$$

$$\therefore i_L(t) - i_L(t_0) = \frac{1}{L}\int_{t_0}^{t} v_L(t)dt$$

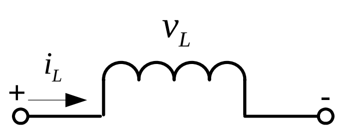
Do a little algebra, assume that $t_0 = 0$, and voila!

$$i_L(t) = \frac{1}{L}\int_0^t v_L(t)\,dt + i_L(0)$$

We have integrated the voltage from an "initial" time t_0 to the "final" time t (which is arbitrary). If we know the value of the current at the initial time t_0, we can find the current as a function of time. In the last equation above, we assume for simplicity that the initial time is 0.

The current-voltage relationship is a **first-order differential equation**. To solve it (meaning that we find a complete numerical expression for the current as a function of time) we need to know the initial condition $i_L(t_0)$. This will either be given, or we have to have a way to find it.

Power and Energy



Not surprisingly, we will want to know about energy stored in the inductor and about power delivered to/from the inductor. In what follows we assume passive sign convention.

$$p_{abs} = v_L \cdot i_L$$

$$= i_L \cdot L\frac{di_L}{dt}$$

$$= Li_L\frac{di_L}{dt} = \frac{dw}{dt}$$

Now we can integrate…

$$\int_0^w dw = \int_0^{i_L} Li_L\,di_L$$

So the energy stored in an inductor that is carrying a current i_L is..

$$W = \frac{1}{2}Li_L^2$$

Units

$$units\,of\;w = [L]\cdot[i_L]^2$$

$$= H\cdot Amp^2 = Volt.\text{sec}.Amp = Volt\cdot Coul$$

But we know that

$$1Volt = 1 \, {}^{Joul}\!/\!_{Coul}$$

$$units\,of\;w = [Joul]$$

So w is an energy, as we expected.

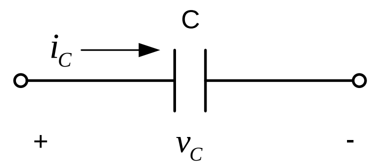
Construction: We can make an inductor by wrapping a coil of wire around a core of magnetic material.

Modeling: Any physical device that involves a coil of wire can be modeled using inductance. An obvious example is a motor, whose windings have an inductance. More generally, a device with a current-induced magnetic field that interacts like an inductor will have inductance.

6.2 The Capacitor

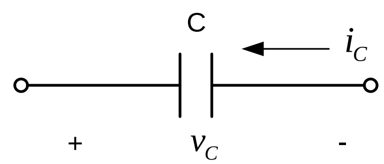
Circuit symbol

There is a relationship between current and voltage for a capacitor, just as there is for a resistor. However, for the capacitor, the current is related to the change in the voltage, as follows.



$$i_C = C\frac{dv_C}{dt}$$

This relationship holds when the voltage and current are drawn in the passive sign convention. When they are in the active sign convention, we need a '-' sign:



$$i_C = -C\frac{dv_C}{dt}$$

The relationship between current and voltage involves the time derivative of the voltage. This is because a changing voltage produces a changing electric field, which induces a current.

Very Important Points:

1. $i_C = C\frac{dv_C}{dt} \Rightarrow$ if voltage v_C is constant (but NOT necessarily 0), there is no current. In other words, **under constant voltage conditions, the capacitor is an open circuit**.

2. An instantaneous change in voltage would generate an infinite current! Therefore we assume (and in reality this is always the case) that **in a capacitor, there cannot be an instantaneous**

change in voltage. What is meant by "instantaneous change in voltage" can be seen by looking at the graph above for instantaneous change in inductor current – just substitute v_C for i_L.

Units

$$\text{units of C} = \frac{\text{Amp}\cdot\text{sec}}{\text{Volt}} = \frac{\text{Coul}}{\text{Volt}} = \text{Farad [F]}$$

Voltage in terms of Current

The current-voltage relationship we discussed above gives the capacitor current if we know the capacitor voltage. But sometimes we have the capacitor current and need to find the voltage. So we need to integrate…

$$i_C = C\frac{dv_C}{dt}$$

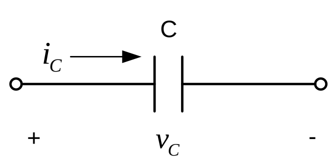
$$\Rightarrow \int_{t_0}^{t} i_C dt = \int_{v_C(t_0)}^{v_C(t)} C dv_C$$

$$\therefore v_C(t) = v_C(t_0) = \frac{1}{C}\int_{t_0}^{t} i_C dt$$

$$v_C(t) = \frac{1}{C}\int_{0}^{t} i_C dt + v_C(0)$$

Power and Energy

We will want to know about energy stored in the capacitor and about power delivered to/from the capacitor. In what follows we assume passive sign convention.



$$p_{abs} = v_C \cdot i_C$$

$$p_{abs} = v_C \cdot C\frac{dv_C}{dt}$$

$$p_{abs} = Cv_C\frac{dv_C}{dt} = \frac{dw}{dt}$$

Now we integrate…

$$\int_0^w dw = \int_0^t C v_C dv_C$$

So the energy stored in a capacitor that has a voltage v_C across it is

$$w = \frac{1}{2} C v_C^2$$

Units

$$units\,of\; w = [C] \cdot [v_C]^2$$

$$= F \cdot Volt^2 = Coul \cdot Volt$$

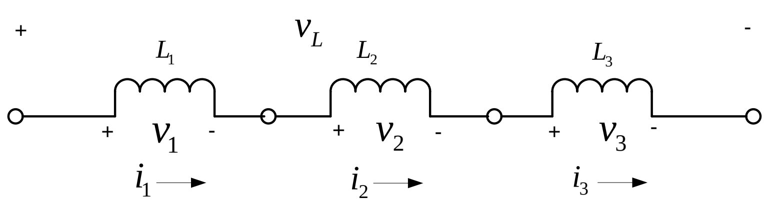
$$= [Joul]$$

Construction: We can make a capacitor by sandwiching an insulator between two conductors.

Modeling: Any physical device that involves conducting plates or wires with insulation between them can be modeled using capacitance. Two wires stranded together that connect two devices will have capacitance. Semiconductor devices are often made from some combination of metal and semiconductor layers that have capacitance. More generally, a device with an electric field that interacts like a capacitor will have capacitance.

6.3 Series – Parallel Combinations of Inductance and Capacitance

Inductors in Series



Since $i_1 = i_2 = i_3$, we have

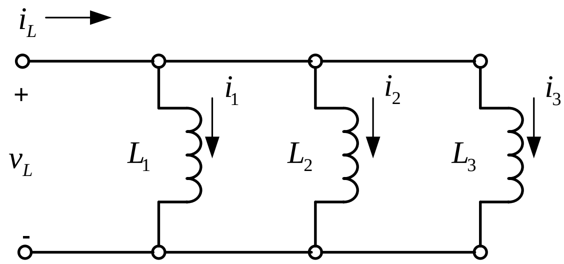
$$v_L = v_1 + v_2 + v_3 = (L_1 + L_2 + L_3)\frac{di_L}{dt}$$

$$L_{eq} = L_1 + L_2 + L_3$$

In general

$$L_{eq} = \sum_n L_n$$

Inductors in Parallel



Since

$$i_L = \frac{1}{L}\int_{t_0}^{t} v_L(t)\,dt + i_L(t_0)$$

we have

$$i_L = i_1 + i_2 + i_3 = \left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}\right)\int_{t_0}^{t} v_L(t)\,dt + i_1(t_0) + i_2(t_0) + i_3(t_0)$$

$$i_L = \frac{1}{L_{eq}}\int_{t_0}^{t} v_L(t)\,dt + i_L(t_0)$$

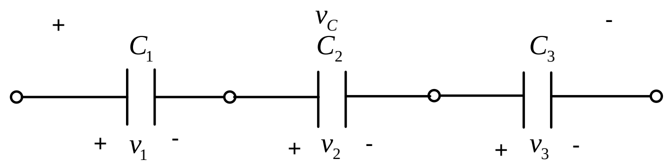
$$L_{eq} = \left(\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}\right)$$

$$i_L(t_0) = i_1(t_0) + i_2(t_0) + i_3(t_0)$$

In general

$$\frac{1}{L_{eq}} = \sum_n \frac{1}{L_n}$$

Capacitors in Series



$$v_C = \frac{1}{C}\int_{t_0}^{t} i_C dt + v_C\left(t_0\right)$$

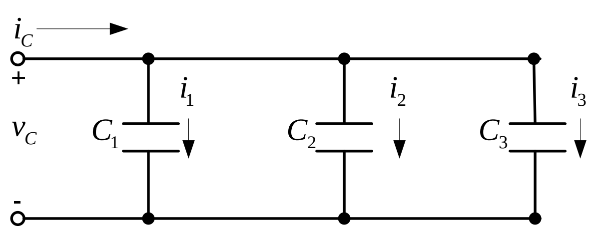
$$v_C = \left(\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}\right)\int_{t_0}^{t} i_C dt + v_1\left(t_0\right)+v_2\left(t_0\right)+v_3\left(t_0\right)$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}$$

In general

$$\frac{1}{C_{eq}} = \sum_n \frac{1}{C_n}$$

Capacitors in Parallel



$$i_C = C\frac{dv_C}{dt}$$

$$i_C = i_1 + i_2 + i_3 = \left(C_1 + C_2 + C_3\right)\frac{dv_C}{dt}$$

$$i_C = C_{eq}\frac{dv_C}{dt}$$

In general

$$C_{eq} = \sum_n C_n$$