



ECE 2300

Circuit Analysis

CIRCUIT ANALYSIS MADE EASY

PART III: KVL, KCL, AND OHM'S LAW

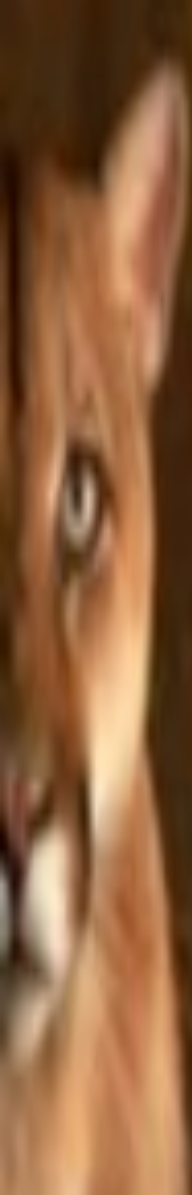
University of Houston

Len Trombetta

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WHERE WE'RE GOING...

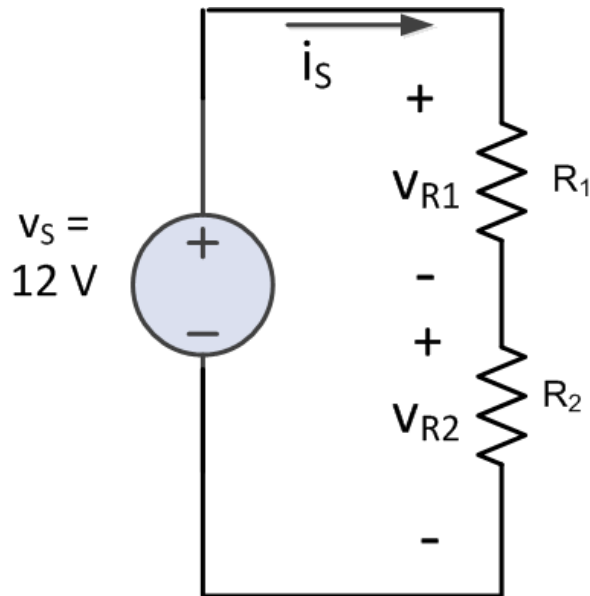
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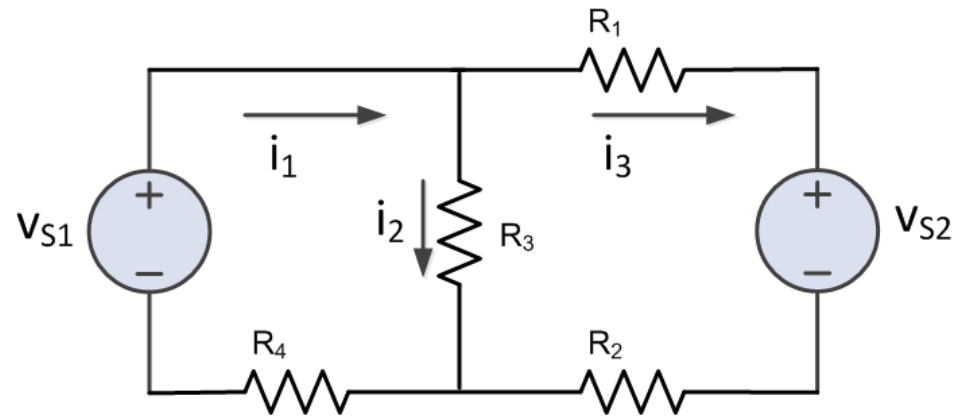
Where Are We Going?

Where we've been:



This is a simple circuit with one loop, and one current. We analyzed it using KVL and Ohm's Law.

Where we're going:



We will analyze circuits like this one, which has three currents. This will require Kirchhoff's Current Law (KCL).



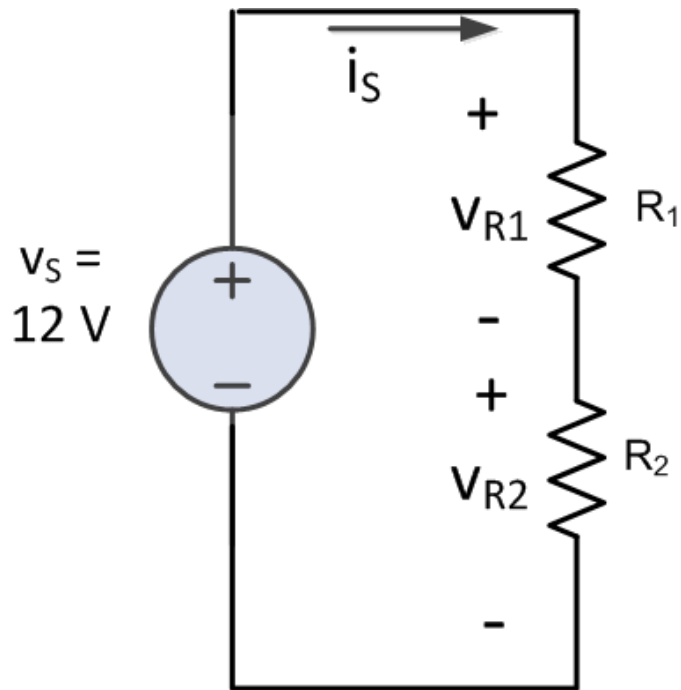
VOLTAGE DIVIDER RULE

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Voltage Divider Rule



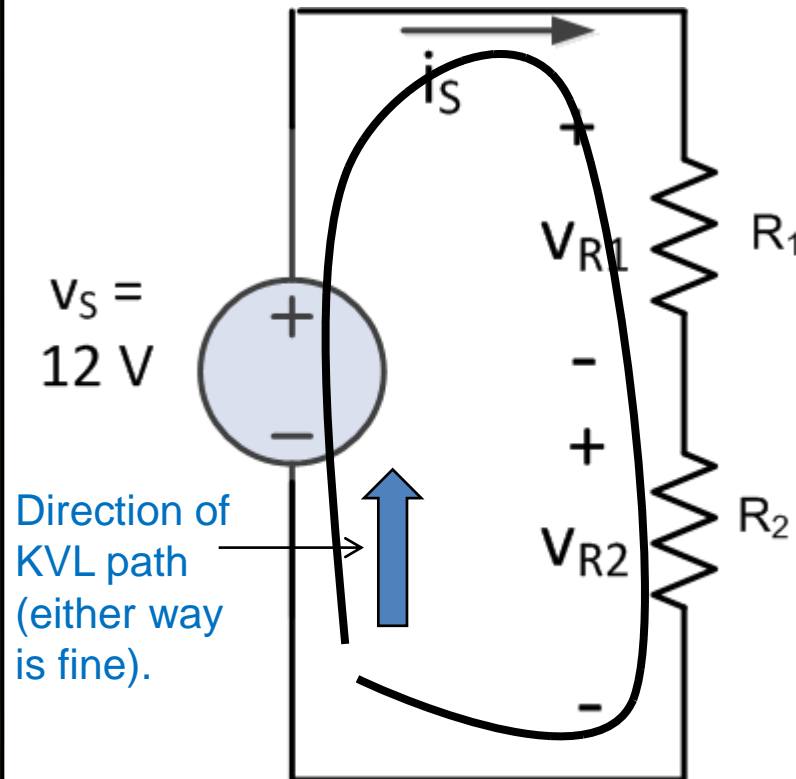
- Ohm's Law and KVL give us the following equations...

$$v_{R1} = v_S \left(\frac{R_2}{R_1 + R_2} \right)$$

$$v_{R2} = v_S \left(\frac{R_1}{R_1 + R_2} \right)$$

How can we derive those results?

Voltage Divider Rule



Direction of KVL path (either way is fine).

KVL

$$v_S = v_{R1} + v_{R2}$$

Ohm's Law

$$v_{R1} = i_S R_1 \quad v_{R2} = i_S R_2$$

Algebra

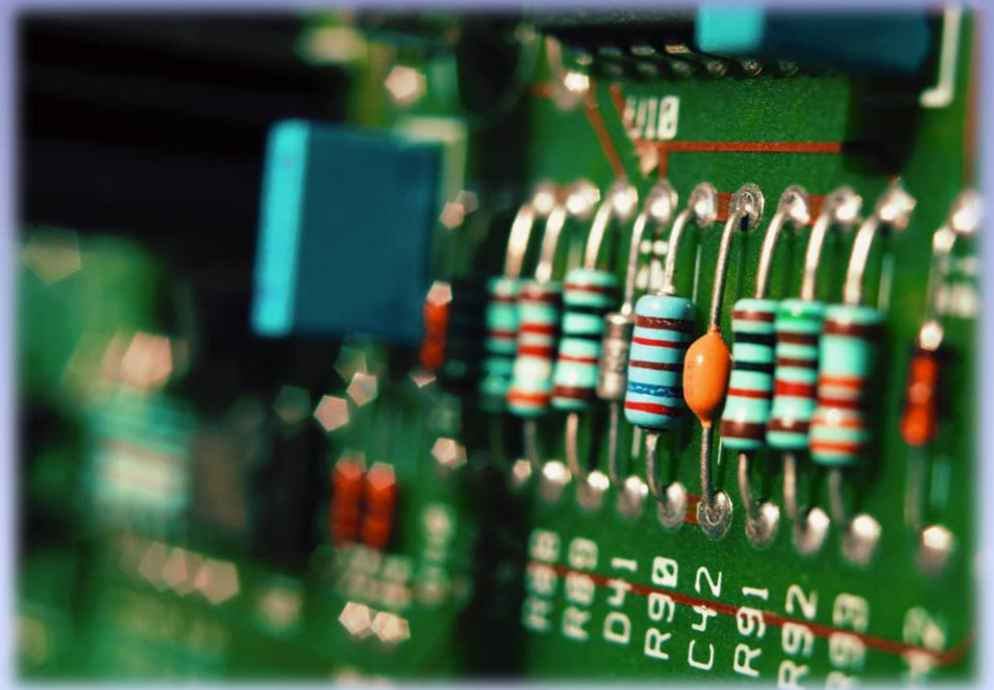
$$i_S = \frac{v_S}{R_1 + R_2}$$

Then...

$$v_{R1} = i_S R_1$$

Finally...

$$v_{R1} = v_S \left(\frac{R_1}{R_1 + R_2} \right)$$



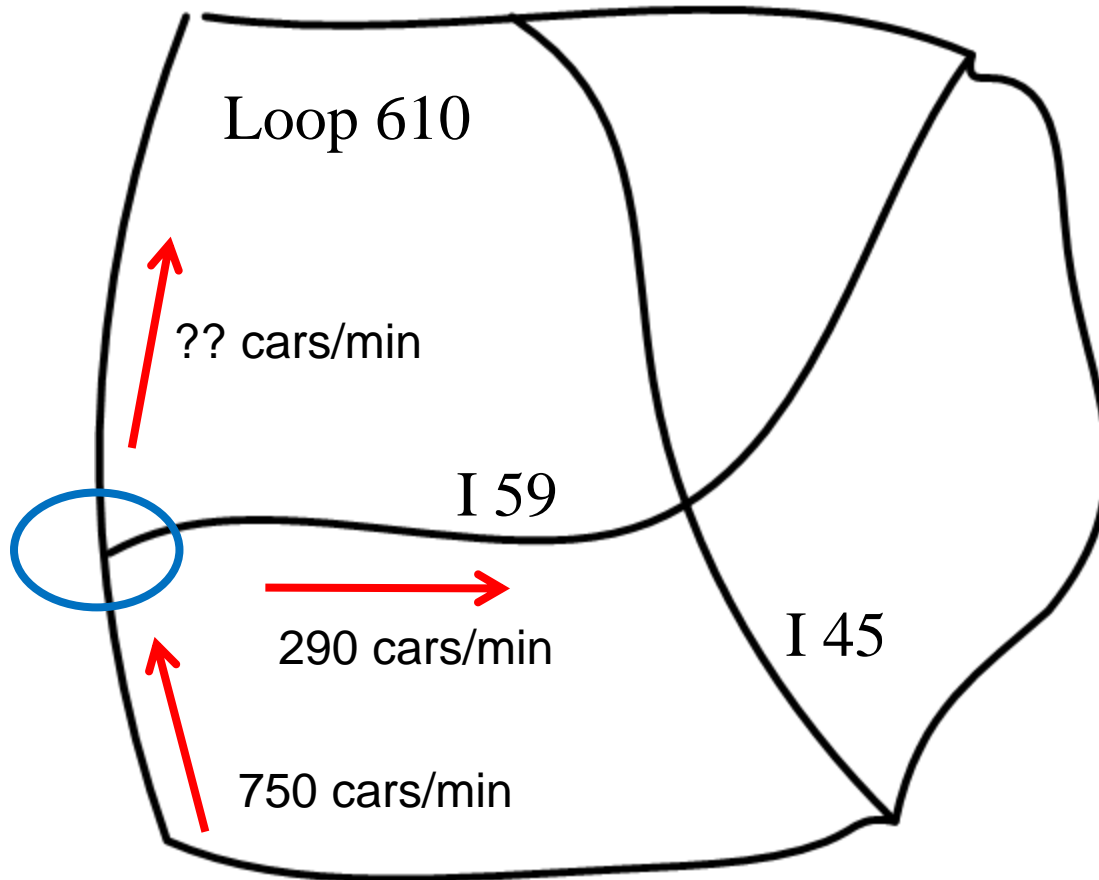
KIRCHHOFF'S CURRENT LAW

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Houston Traffic

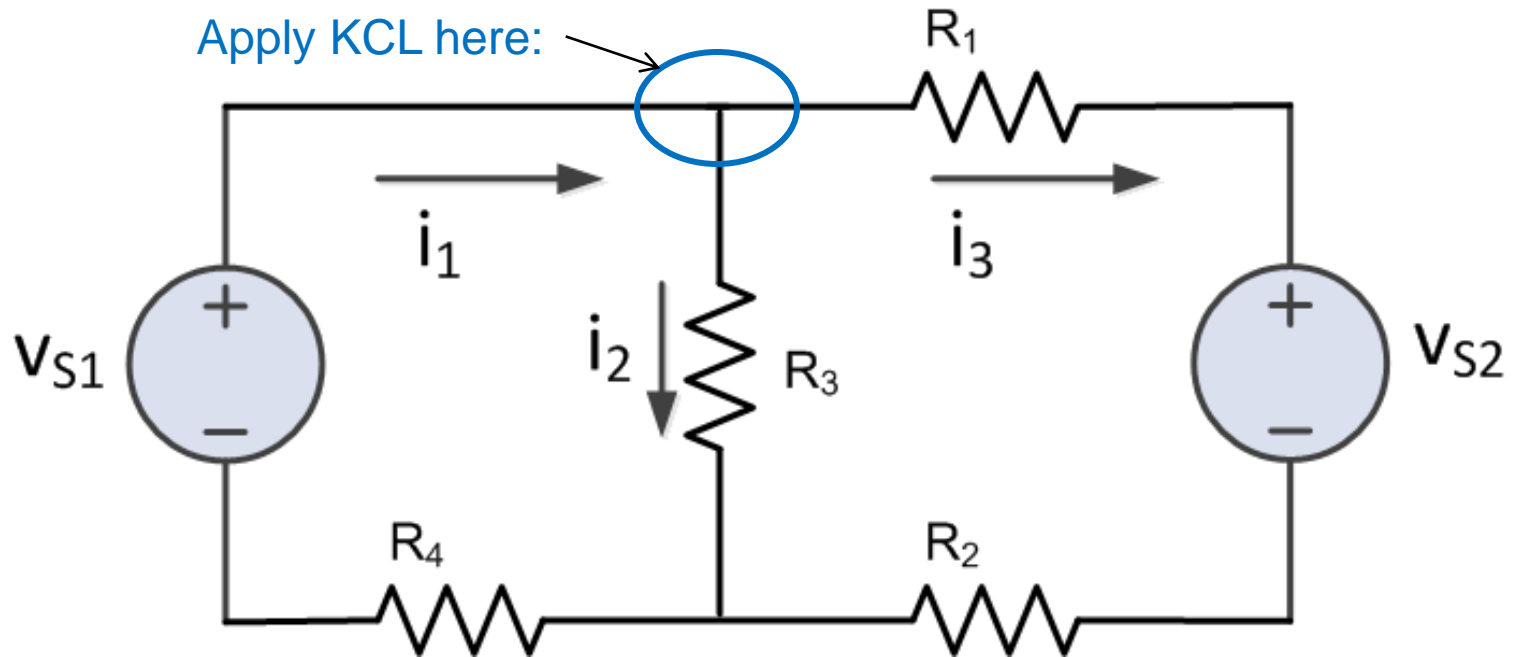


Kirchhoff's Current Law:

$$\text{in} = \text{out}$$

All other roads are closed for repairs!

Kirchhoff's Current Law



What current is flowing in R_4 ? ... R_2 ?... V_{S2} ... V_{S1} ?

What is the relationship between i_1 , i_2 , and i_3 ?

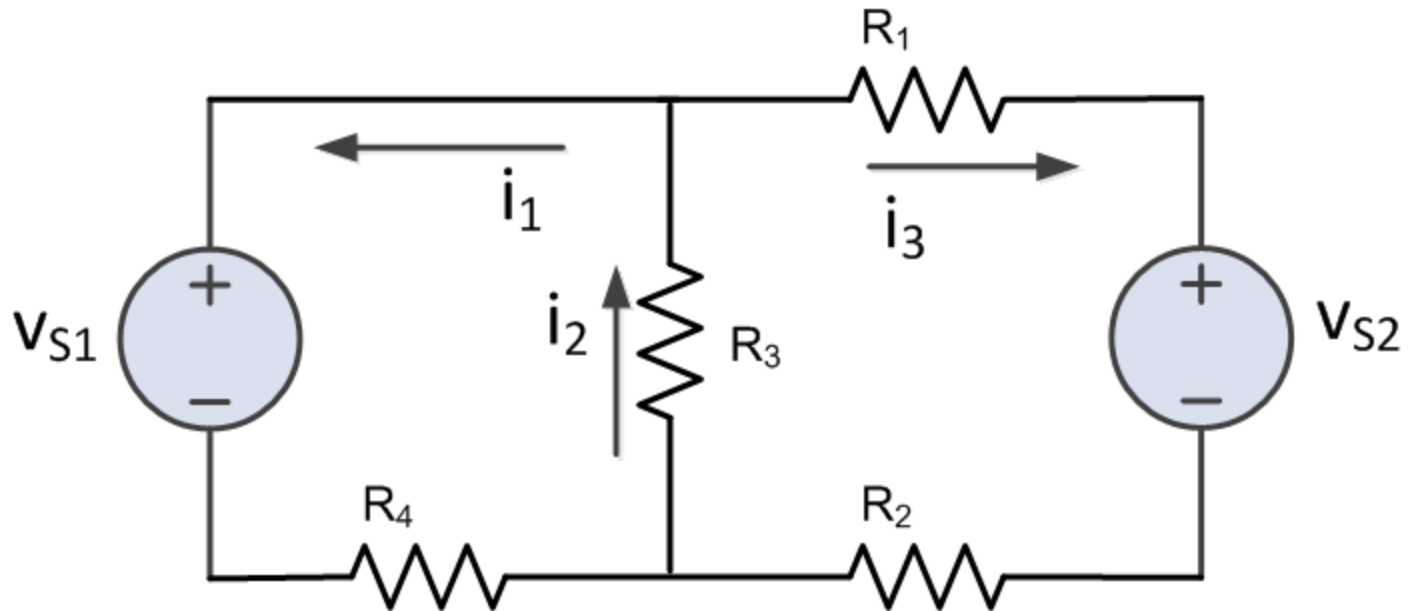
KCL:

$$i_1 = i_2 + i_3$$

In = out

Kirchhoff's Current Law

What if we had drawn it this way? Now what is KCL?



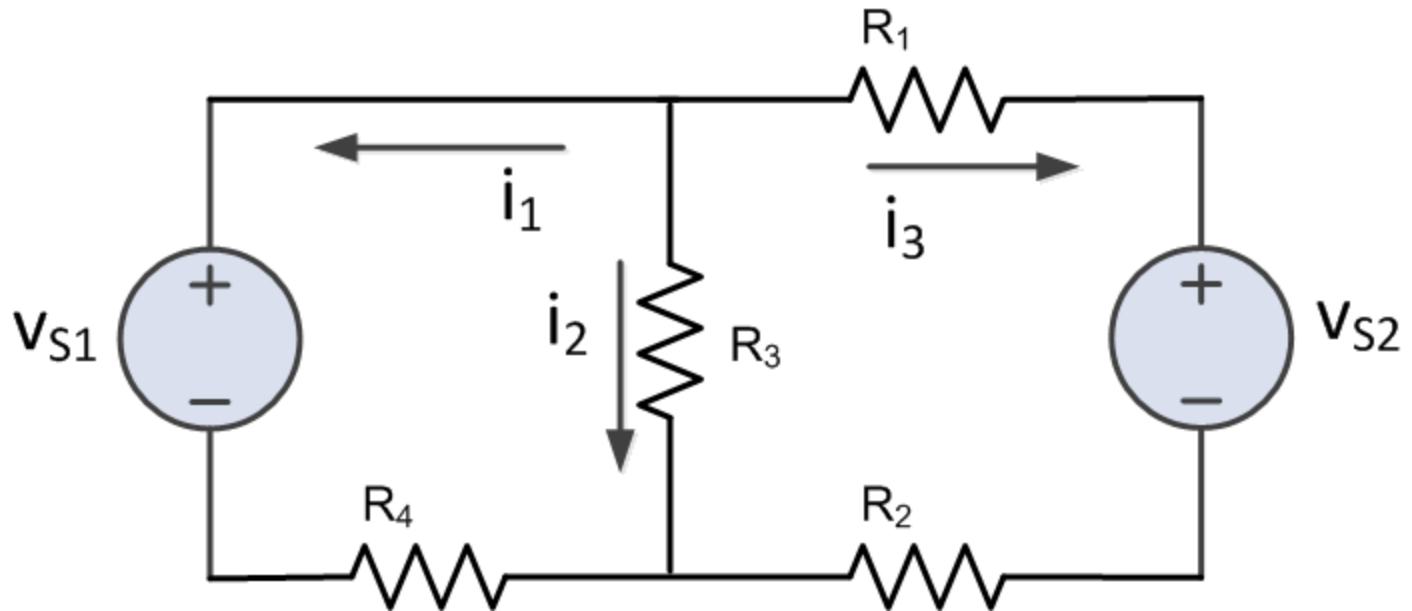
KCL:

$$i_2 = i_1 + i_3$$

in = out

Kirchhoff's Current Law

Can I do THIS????



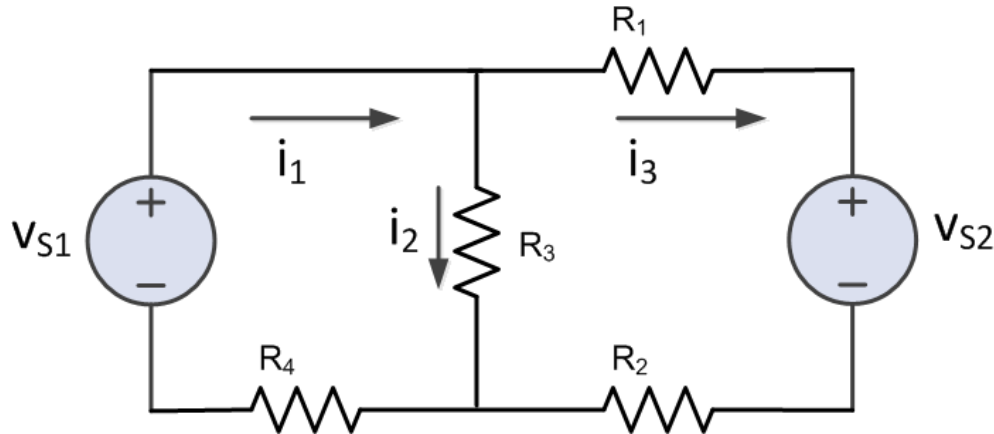
Sure!!

in = out

KCL:

$$0 = i_2 + i_1 + i_3$$

Kirchhoff's Current Law



Find the indicated currents...

$$i_1 = 70 \text{ mA}$$

$$i_2 = 15 \text{ mA}$$

$$i_3 = ??$$

$$i_1 = -24 \text{ mA}$$

$$i_2 = ??$$

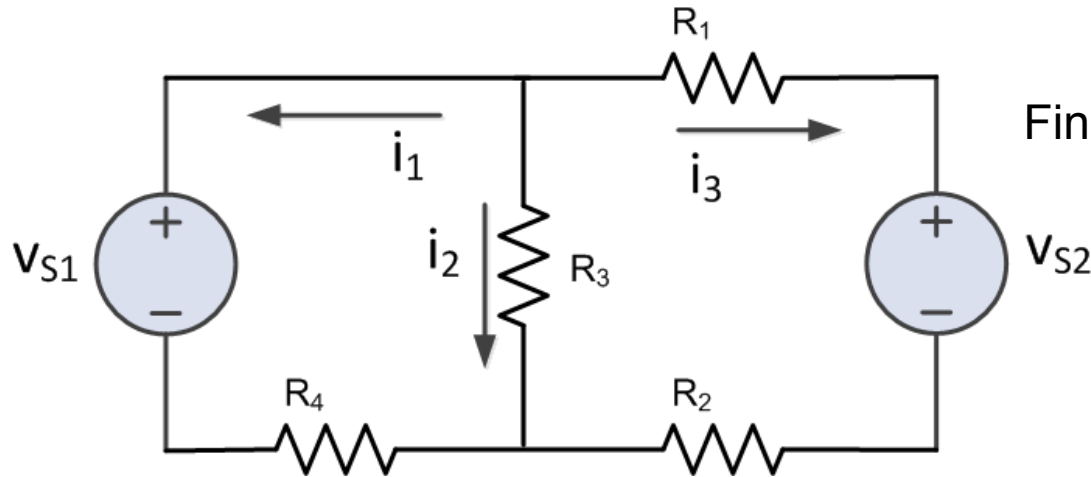
$$i_3 = 10 \text{ mA}$$

$$i_1 = ??$$

$$i_2 = 0.25 \text{ mA}$$

$$i_3 = 0.20 \text{ mA}$$

Kirchhoff's Current Law



Find the indicated currents...

$$i_1 = 240 \text{ mA}$$

$$i_2 = -150 \text{ mA}$$

$$i_3 = ??$$

$$i_1 = -240 \text{ mA}$$

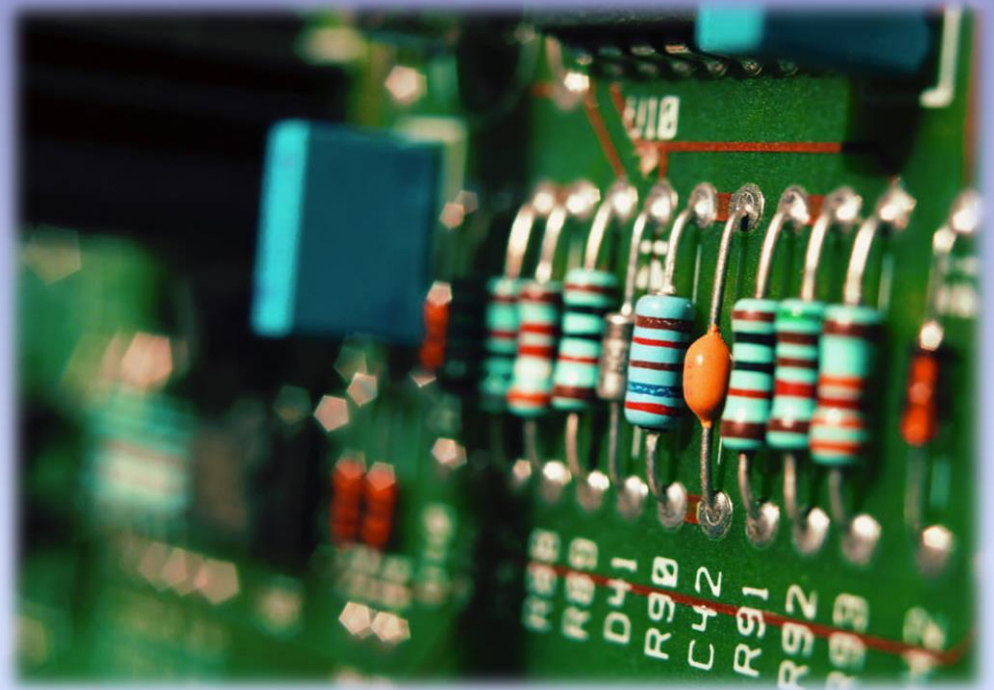
$$i_2 = ??$$

$$i_3 = 150 \text{ mA}$$

$$i_1 = ??$$

$$i_2 = 0.75 \text{ mA}$$

$$i_3 = 0.20 \text{ mA}$$



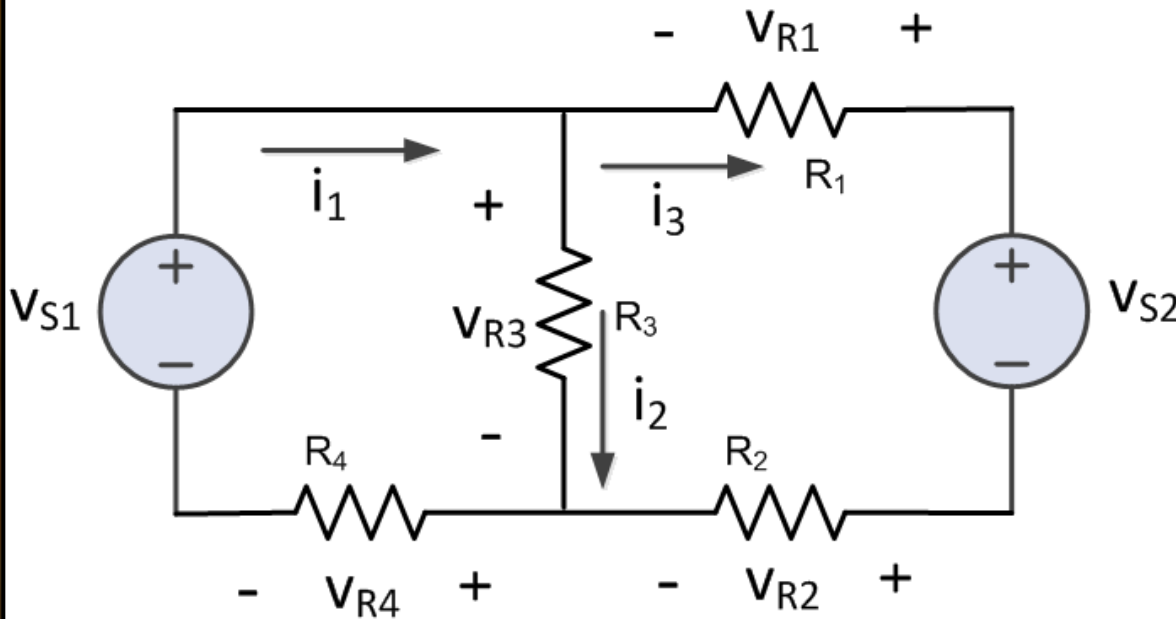
CIRCUIT ANALYSIS USING KVL, KCL, AND OHM'S LAW

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Analyze This!



Usually we know the voltage source values and the resistor values. We will assume that here.

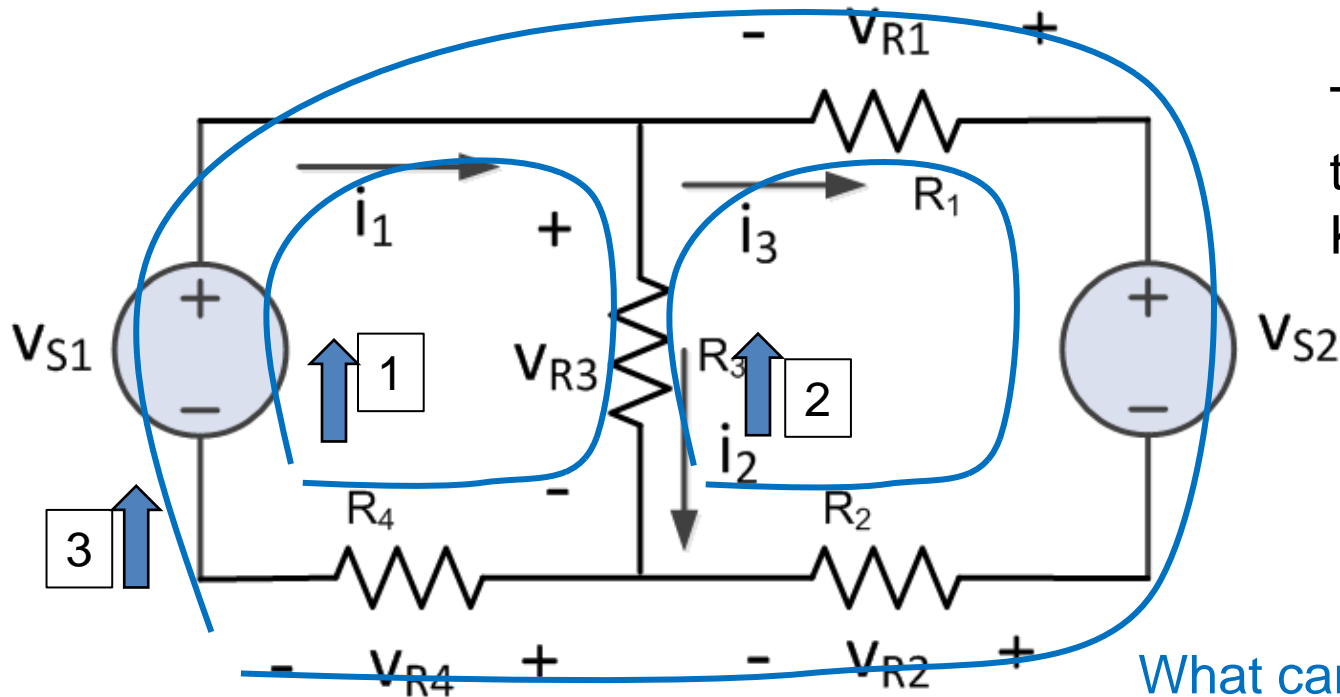
How many KVL's can we write for this circuit?

of KVL's: 3

How many KCL's can we write for this circuit?

of KCL's: 2

Let's Do It: KVL



There are three possible KVL paths.

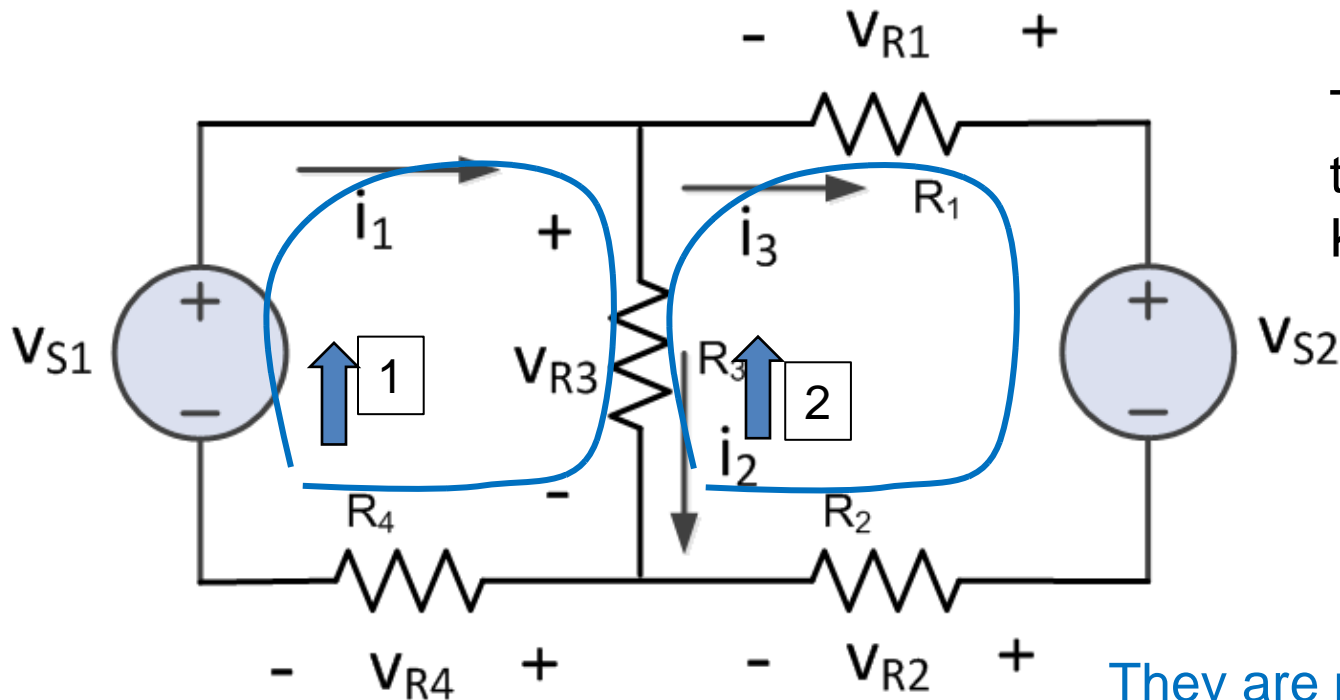
$$1 \quad -v_{S1} + v_{R3} + v_{R4} = 0$$

$$2 \quad -v_{R3} - v_{R1} + v_{S2} + v_{R2} = 0$$

$$3 \quad -v_{S1} - v_{R1} + v_{S2} + v_{R2} + v_{R4} = 0$$

What can we say about the relationship among these three equations?

Let's Do It: KVL



There are three possible KVL paths.

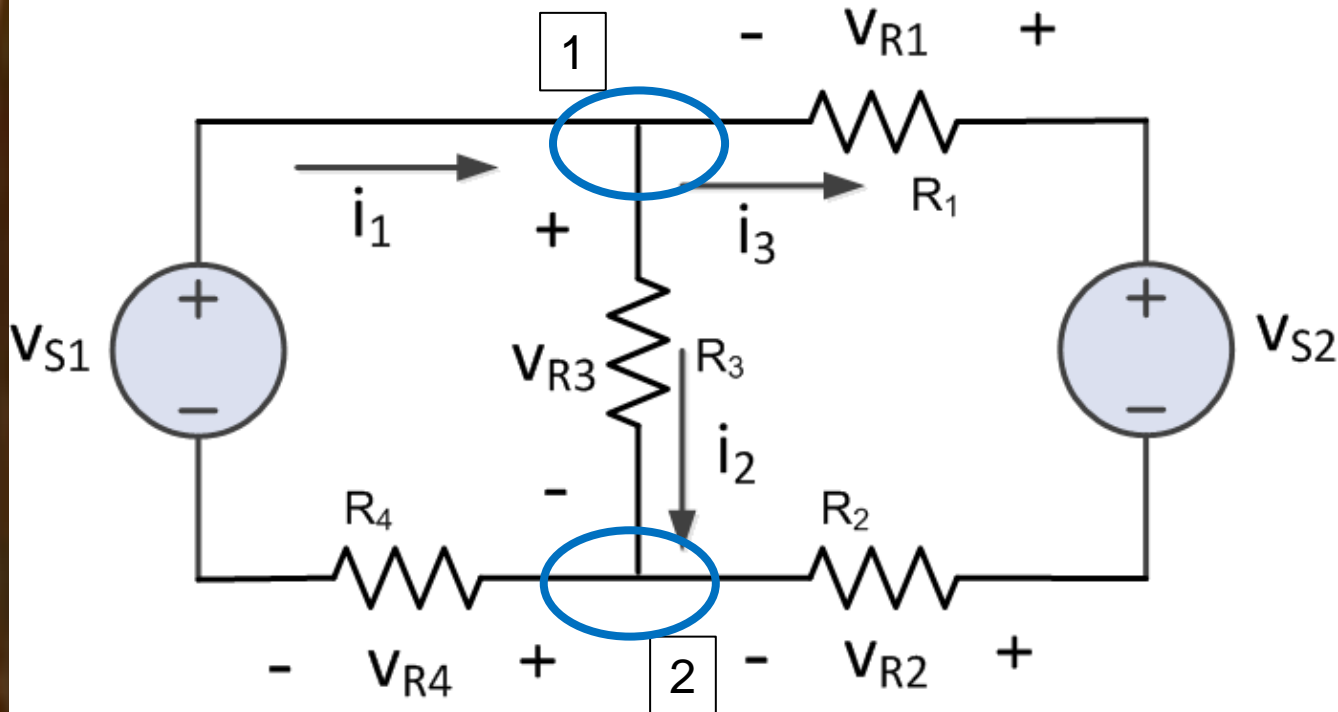
$$\boxed{1} \quad -v_{S1} + v_{R3} + v_{R4} = 0$$

$$\boxed{2} \quad -v_{R3} - v_{R1} + v_{S2} + v_{R2} = 0$$

~~$$\boxed{3} \quad -v_{S1} - v_{R1} + v_{S2} + v_{R2} + v_{R2} = 0$$~~

They are not algebraically independent – we can only use two of them (any two will do).

Let's Do It: KCL



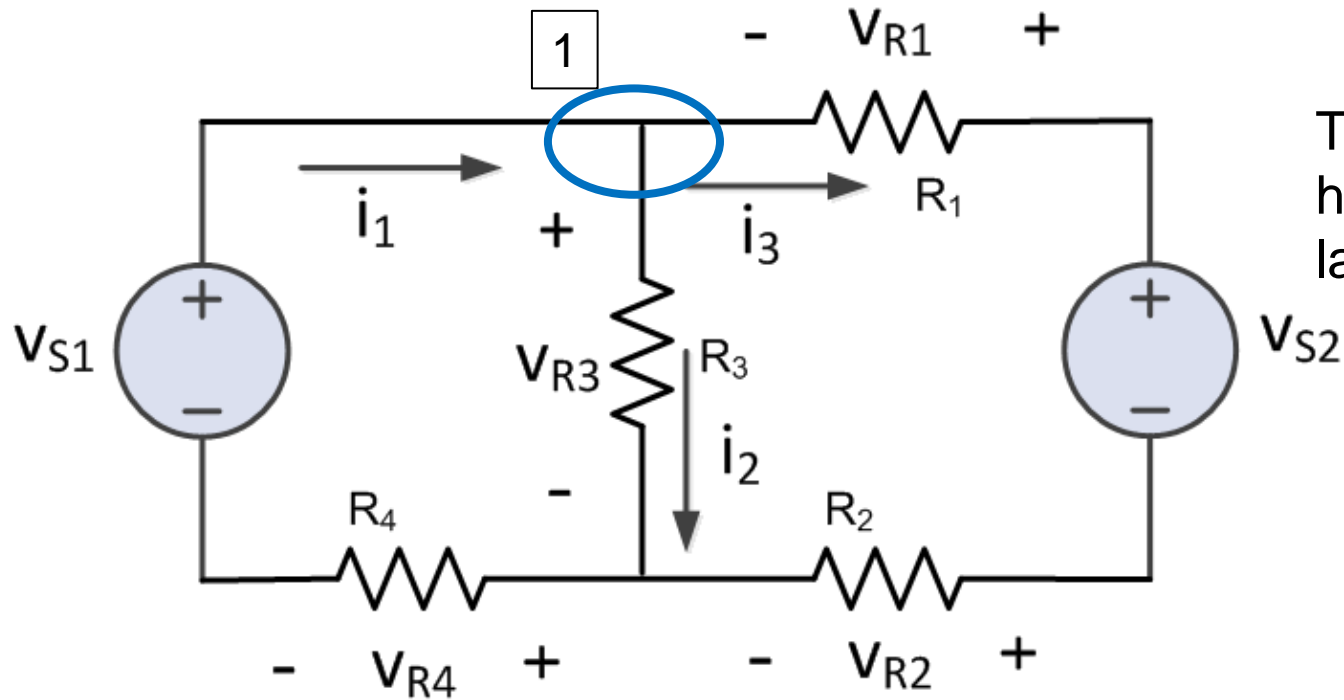
There are two nodes for KCL.

$$\boxed{1} \quad i_1 = i_2 + i_3$$

$$\boxed{2} \quad i_2 + i_3 = i_1$$

What can we say about these two KCL equations?

Let's Do It: KCL



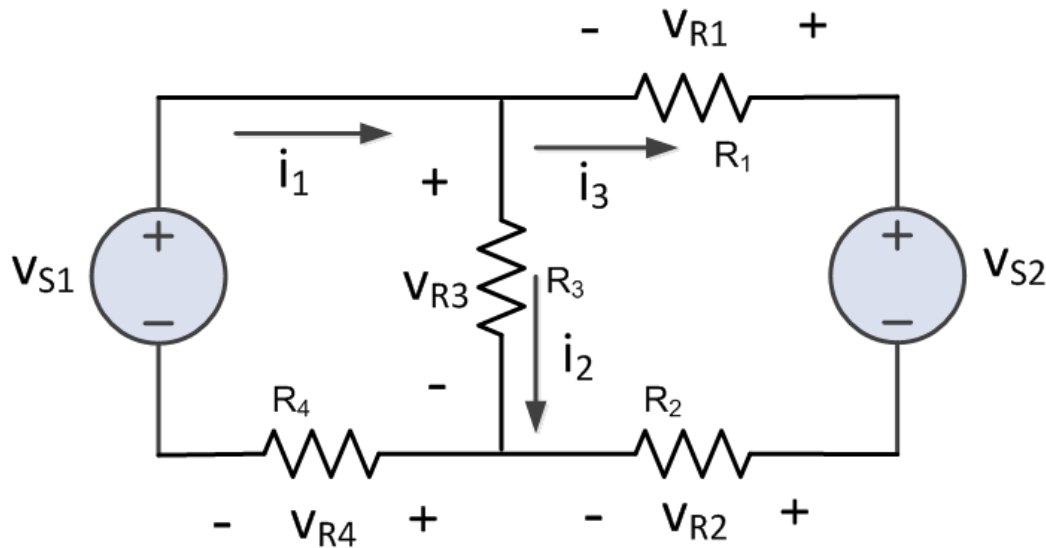
Two nodes have been labeled.

1 $i_1 = i_2 + i_3$

~~2 $i_2 + i_3 = i_1$~~

They are algebraically identical. We can only use one of them.

Let's Do It: KCL and KVL



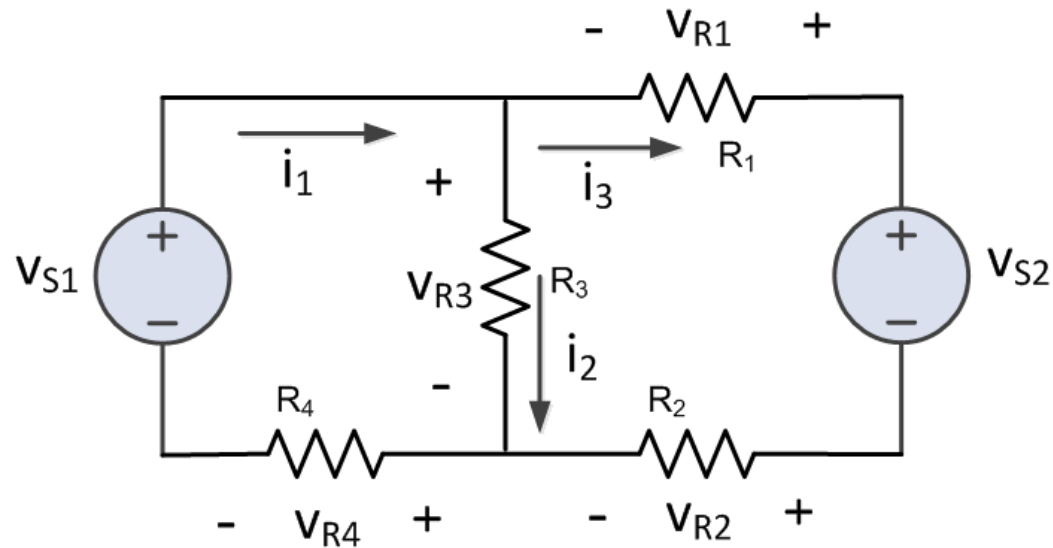
We have three equations: 2 KVL's and 1 KCL.

Remember: we assume we know the voltage sources and resistor values.

- 1 $i_1 = i_2 + i_3$
- 2 $-v_{S1} + v_{R3} + v_{R4} = 0$
- 3 $-v_{R3} - v_{R1} + v_{S2} + v_{R2} = 0$

But now what? We have too many **unknowns** to go further – the resistor voltages and the three currents are all unknown.

Ohm's Law!!



$$v_{R1} = -i_3 R_1$$

$$v_{R2} = i_3 R_2$$

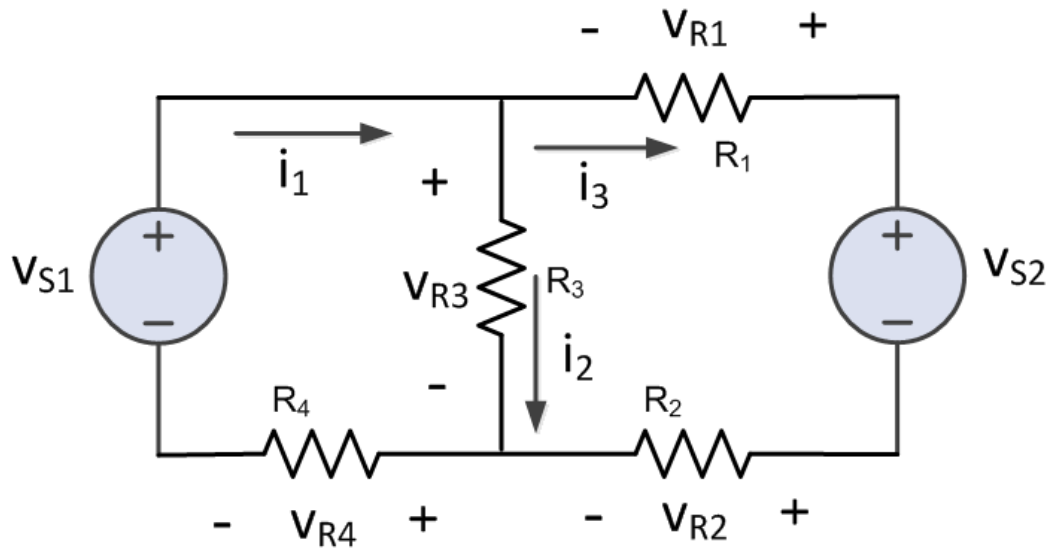
$$v_{R3} = i_2 R_3$$

$$v_{R4} = i_1 R_4$$

We can use Ohm's Law to eliminate the resistor voltages...watch the signs!

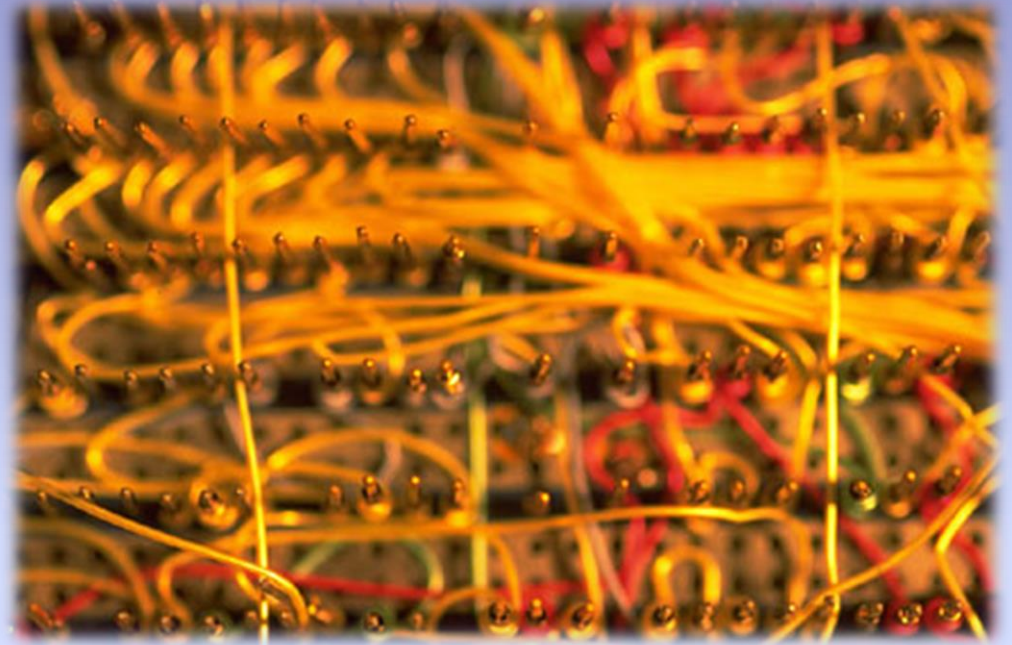
Substitute these equations into KVL and get...

Let's Do It: KCL and KVL



Now we have three equations in three **unknown** currents! We can solve for the currents and then use Ohm's Law to get the resistor voltages.

- 1 $i_1 = i_2 + i_3$
- 2 $-v_{S1} + i_2 R_3 + i_1 R_4 = 0$
- 3 $-i_2 R_3 + i_3 R_1 + v_{S2} + i_3 R_2 = 0$



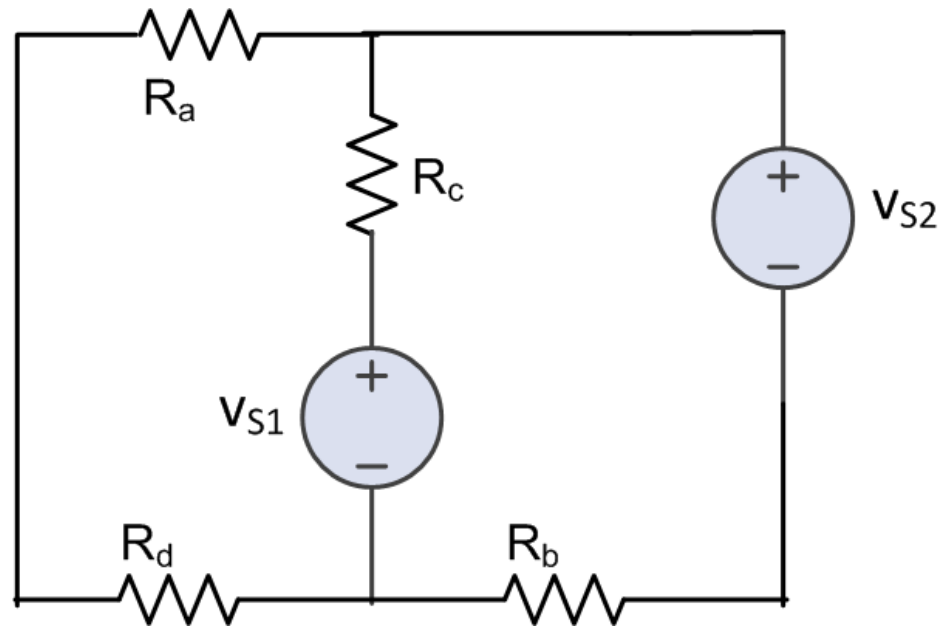
YOU TRY!

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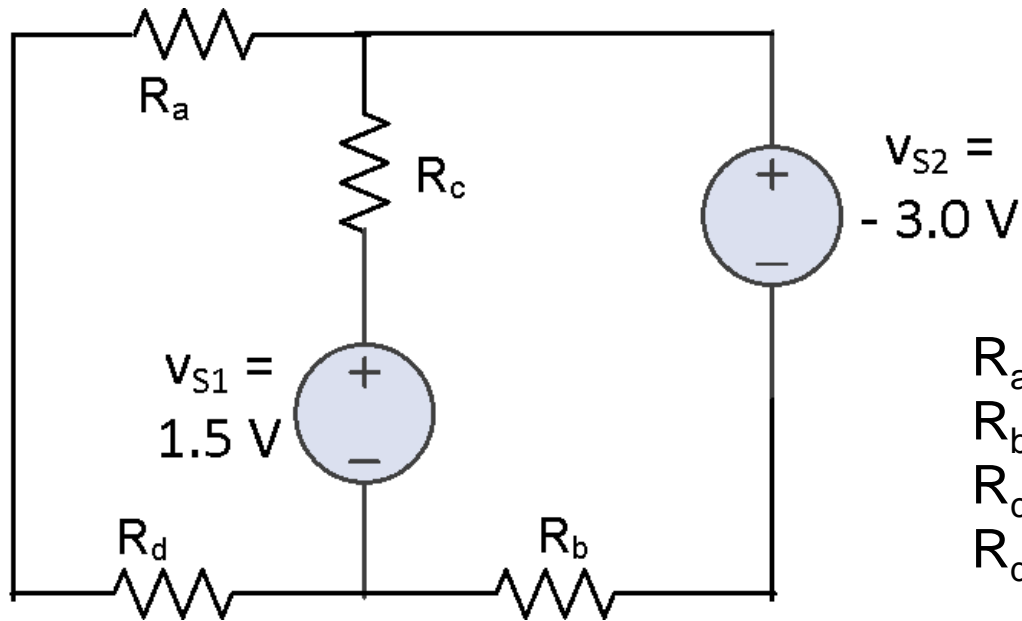
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You Try!



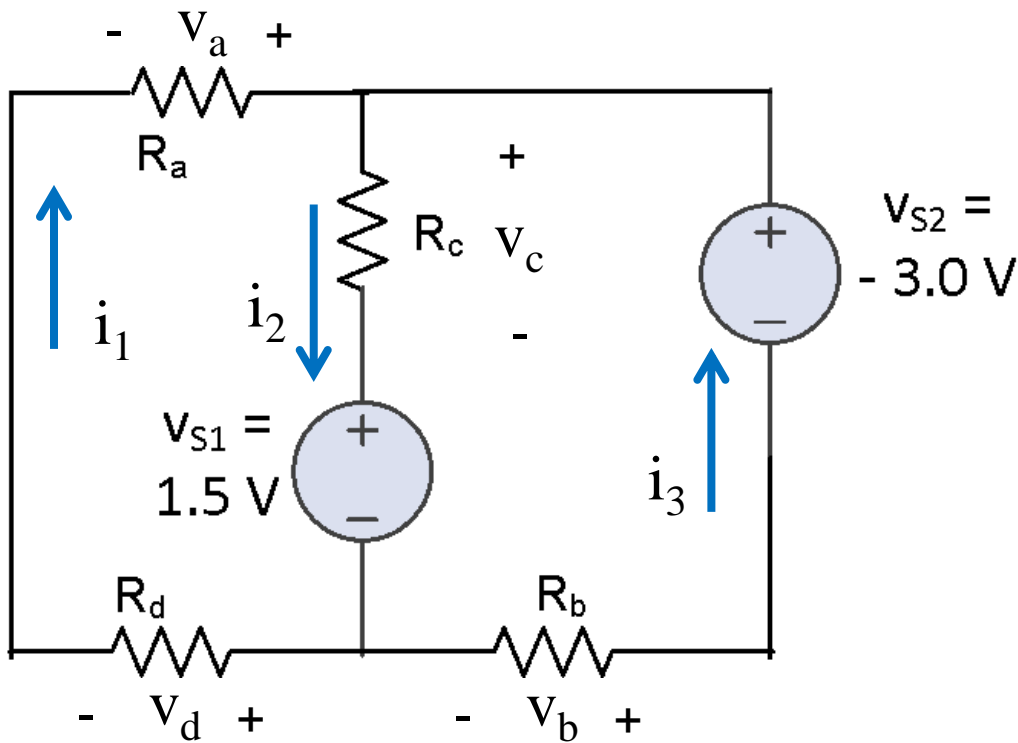
1. Label 3 currents and all resistor voltages. Watch the label rules!
2. Write 2 KVL equations and 1 KCL equation.
3. Write Ohm's Law for each of the resistor voltage variables.
4. Re-write your equations in terms of three unknown currents.

Numerical Values



$$\begin{aligned}R_a &= 1.1 \text{ [k}\Omega\text{]} \\R_b &= 4.7 \text{ [k}\Omega\text{]} \\R_c &= 150 \text{ [}\Omega\text{]} \\R_d &= 2.2 \text{ [k}\Omega\text{]}\end{aligned}$$

My Answers



$$\begin{aligned}R_a &= 1.1 \text{ [k}\Omega\text{]} \\R_b &= 4.7 \text{ [k}\Omega\text{]} \\R_c &= 150 \text{ [}\Omega\text{]} \\R_d &= 2.2 \text{ [k}\Omega\text{]}\end{aligned}$$

If your currents and voltages are labeled the way I have them, then the circuit variables are...

$$\begin{aligned}i_1 &= -0.39497 \text{ [mA]} \\i_2 &= -1.31070 \text{ [mA]} \\i_3 &= -0.91562 \text{ [mA]} \\v_a &= 0.4345 \text{ [V]} \\v_b &= 4.3034 \text{ [V]} \\v_c &= -0.1966 \text{ [V]} \\v_d &= -0.8689 \text{ [V]}\end{aligned}$$

If you have labeled things differently, your results will be different.

The End

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