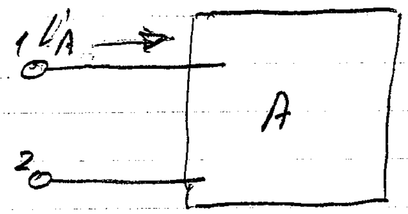
Example 1.1 Calculate Charge from Current



GIVEN:

$i_A = 0 \quad t < 0$

$i_A = 20 e^{-5000t}$ [A] $\quad t \geq 0$

FIND: Total charge in μCoul entering terminal 1.

We have $\quad dq = i_A \, dt$

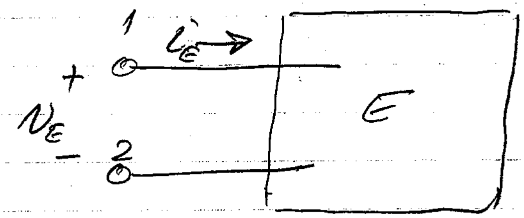
$$\Rightarrow \quad q = \int i_A \, dt = \int_0^\infty 20 e^{-5000t} \, dt$$

How do we know the integration limits? We are asked to find the "total charge" which we interpret as the charge over all time, i.e., from 0 to ∞. This is OK (it will not diverge) since for very large t, $i \rightarrow 0$. So...

$$q = \int_0^\infty 20 e^{-5000t} \, dt = -\frac{20}{5000} e^{-5000t} \Big|_0^\infty$$

$$q = 20/5000 \text{ [C]} = 4000 \text{ [}\mu\text{C]}$$

Example 1.2: Total Energy



GIVEN:

From Example 1.1 ...

$i_E = 0 \quad t < 0$

$i_E = 20e^{-5000t}$ [A] $\quad t \geq 0$

Also,

$v_E = 0 \quad t < 0 \qquad v_E = 10e^{-5000t}$ [kV] $\quad t \geq 0$

FIND: Total energy in [Joules] delivered to the circuit element, E

We have that $p_{abs,E} = v_E \cdot i_E$. Now, energy and power absorbed by the circuit element is the same thing as energy and power delivered to the circuit element. Thus, what we want here is in fact $p_{abs,E}$. So...

$$p_{abs,E} = \frac{d}{dt}(w_{abs,E}) \Rightarrow w_{abs,E} = \int p_{abs,E}\,dt$$

$$w_{abs,E} = \int_0^{\infty} v_E \cdot i_E \, dt$$

We interpret "total energy" as energy delivered over all time.

(Ex 1.2, p.2)

So...

[kV]

$$w_{abs,E} = \int_0^\infty (20e^{-5000t})(10e^{-5000t} \times 10^3)\,dt$$

$$= 2\times10^5 \int_0^\infty e^{-10\,000t}\,dt$$

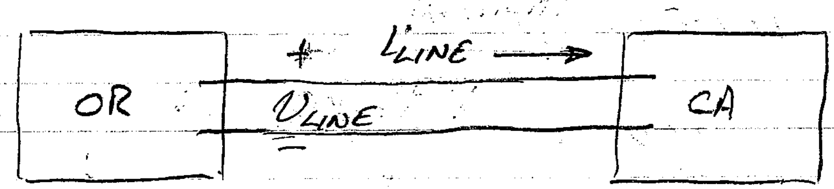
$$= \frac{2\times10^5}{-10^4} e^{-10\,000t}\Big|_0^\infty = 20\ [J]$$

In summary,

$$w_{\text{delivered to box}} = w_{\text{absorbed by box}} = 20\ [J]$$

Example 1.3 Power Transfer

$v_{LINE} = 800$ [kV]
$i_{LINE} = 1.8$ [kA]



Celilo, Oregon   Sylmar, California

FIND: Power in [MW] at the OR end of the line and state direction of flow.

At Oregon, the diagram indicates that current is flowing from the negative terminal to the positive terminal. Thus

$$P_{abs,OR} = -v_{LINE} \cdot i_{LINE}$$

$$= -(1800)(800 \times 10^3) = -1.44 \times 10^9 \text{ [W]}$$

$$P_{abs,OR} = -1440 \text{ [MW]}$$

Since $P_{abs,OR}$ is negative, power is being delivered by the Oregon end. So power is flowing from Celilo to Sylmar.

We should get the same answer if we look at the CA end...

(Ex 1.3 p.2)

At CA, current flows in the direction of the voltage drop, so

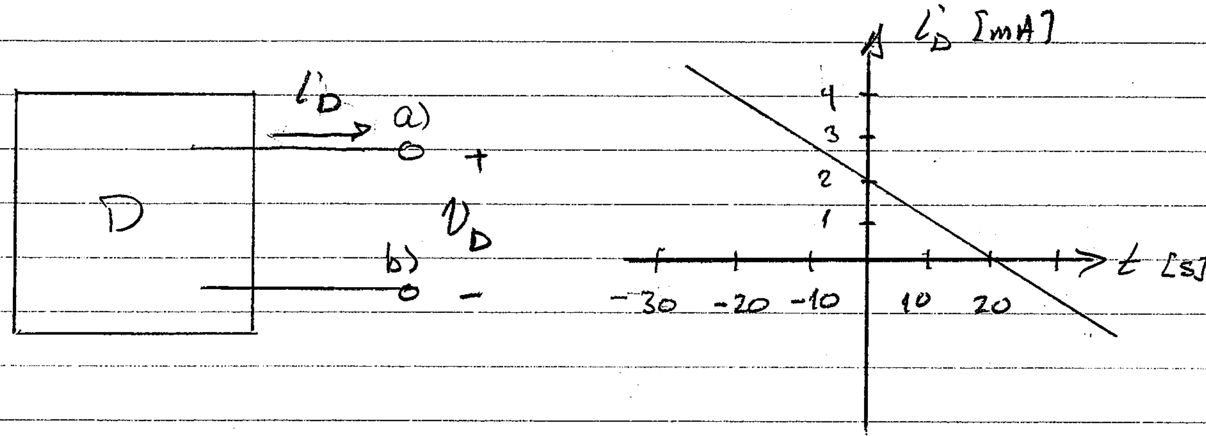
$$P_{abs,\,CA} = v_{LINE} \cdot i_{LINE} = +1440\ [MW]$$

So power flow is again Celilo to Sylmar.

Example 1.4 Graphical Analysis

A device D is measured and found to have a terminal voltage $v_D = -32$ [V]. The current i_D is varying in time, as shown in the graph.

Find the energy delivered to the device from $t = -2$ [s] to $t = +2$ [s].



We have that

$$p_{abs,D} = -v_D i_D$$

The energy delivered to D is the same as the energy absorbed by it, so we calculate $p_{abs,D}$.

At D, current is going in the direction of the voltage rise (active sign convention), so $p_{abs,D} = -v_D i_D$.

(Ex 1.4 p.2 -)

We need an equation for $i_D(t)$.

$$i_D(t) = mt + b$$

From the graph, $t = 0 \Rightarrow i_D = 2\,[mA] \Rightarrow b = 2\,[mA]$.

Also, $i_D = 0 \Rightarrow t = 20\,[s] \Rightarrow m = \dfrac{-2\,[mA]}{20\,[s]} = -0.1\left[\dfrac{mA}{s}\right]$.

So

$$i_D(t) = -0.1\left[\frac{mA}{s}\right]t + 2\,[mA]$$

$$\therefore\ p_{abs,D} = -(-32)(-0.1t + 2) \quad [mW]$$

Volts x milliamps

$$w_{abs,D} = \int_{-2[s]}^{2[s]} -(-32)(-0.1t + 2) \times 10^{-3}\,dt$$

$$= -\frac{1}{2}(3.2)t^2 \times 10^{-3}\Big|_{-2}^{+2} + 64 \times 10^{-3}\,t\Big|_{-2}^{+2}$$

$$= 0 + 64 \times 10^{-3}(4) = 0.256\ [J]$$

$$\boxed{w_{abs,D} = 256\ [mJ]}$$