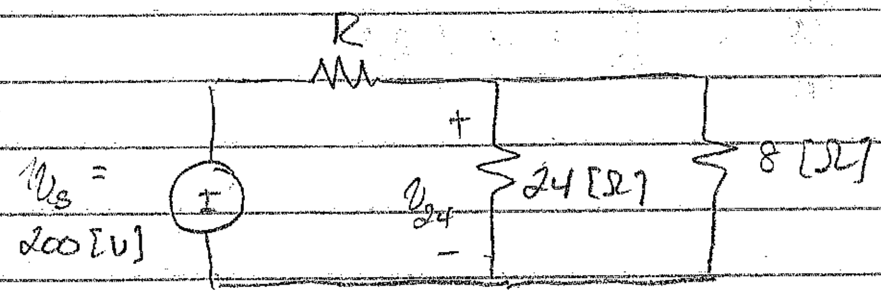
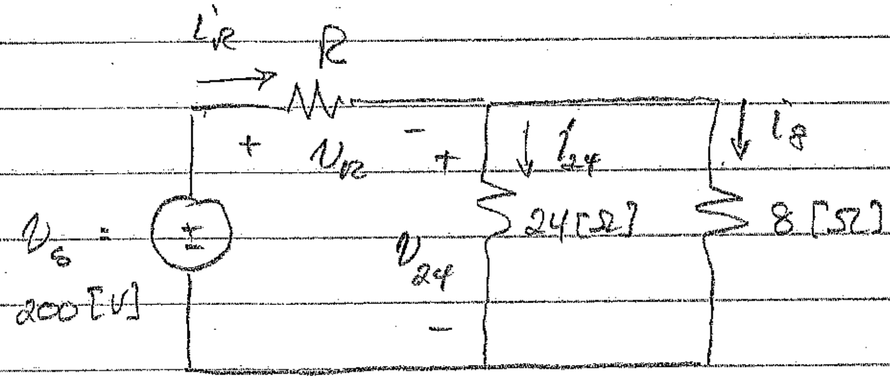
Example 2.1 KVL, KCL, Ohm's Law

- From Nilsson & Reidel 8 ed. pg. 42



Find: R, Given: $v_{24} = 120$ [V]

Approach: If we know the ratio of voltage to current in R, we can find R. We will need to label these variables, as well as some others...



$$R = \frac{v_R}{i_R}$$

ohm: $i_{24} = \frac{v_{24}}{24} = 5$ [A]

KVL: $24\,i_{24} - 8\,i_8 = 0 \Rightarrow i_8 = \frac{24\,i_{24}}{8} = \frac{v_{24}}{8} = 15$ [A]

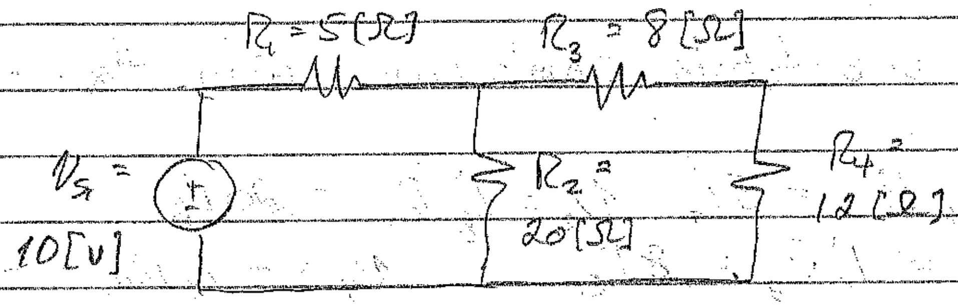
KCL: $i_R = i_{24} + i_8 = 20$ [A]

KVL: $V_R + V_{dep} - 200 = 0 \Rightarrow V_R = 80$ [V]

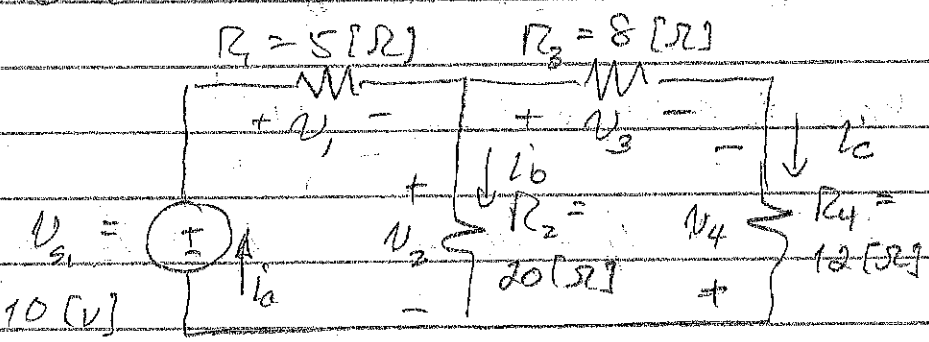
$\therefore R = \frac{V_R}{i_R} = \frac{80}{20} = 4$ [Ω]

Example 2.2

Solve the circuit shown by finding values for all unknown currents.



Idea: There are 3 unknown currents, which have been labeled on the diagram below. Resistor voltages have also been labeled, although as we will see, we don't need to do this - there's a shortcut!



We can write 1 independent KCL and 2 independent KVLs:

KCL: $i_a = i_b + i_c$

KVL $v_1 + v_2 - v_{s} = 0$

KVL $v_3 - v_4 - v_2 = 0$ *

We now substitute Ohm's Law into the KVL's:

$$v_1 = i_a R_1 = 5 i_a \qquad v_2 = i_b R_2 = 20 i_b$$
$$v_3 = i_c R_3 = 8 i_c \qquad v_4 = - i_c R_4 = -12 i_c$$

Then the KVLs become: (and using $v_s = 10$ [V])

$$5 i_a + 20 i_b - 10 = 0$$

$$8 i_c - (-12 i_c) - 20 i_b = 0$$

Solving KCL & 2 KVLs:

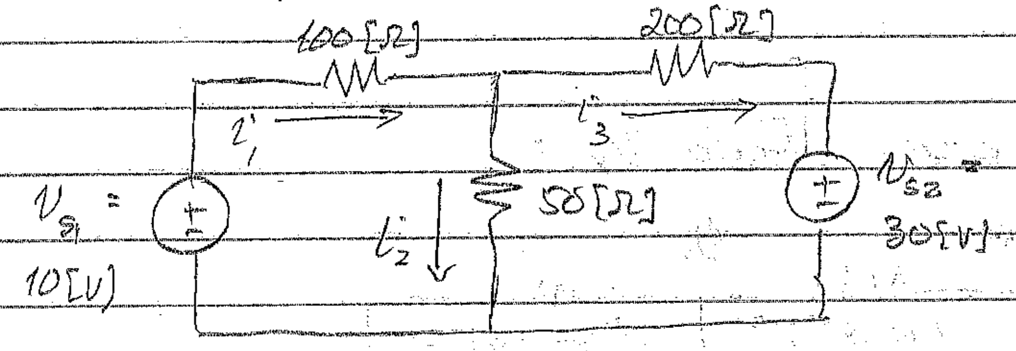
$i_a = 2/3$ [A] $\qquad i_b = 1/3$ [A] $\qquad i_c = 1/3$ [A]

We could have written the last two KVLs directly, without having to label resistor voltages. See Trombetta's notes, Chapter 2.

* We have written v_4 differently from the others to illustrate the use of Ohm's Law with the active sign relationship.

Example 2.3

Find the power delivered by each of the voltage sources in the circuit below.



Approach: We'll find the unknown currents, which have been labeled on the diagram. We will need to solve simultaneous equations to find them, but once we have them, we can find anything else, including source power, with a single equation.

KCL $\quad i_1 = i_2 + i_3$

KVL $\quad 200\, i_3 + 30 - 50\, i_2 = 0$

KVL $\quad 100\, i_1 + 50\, i_2 - 10 = 0$

Solving gives $\quad i_1 = 28.57$ [mA]

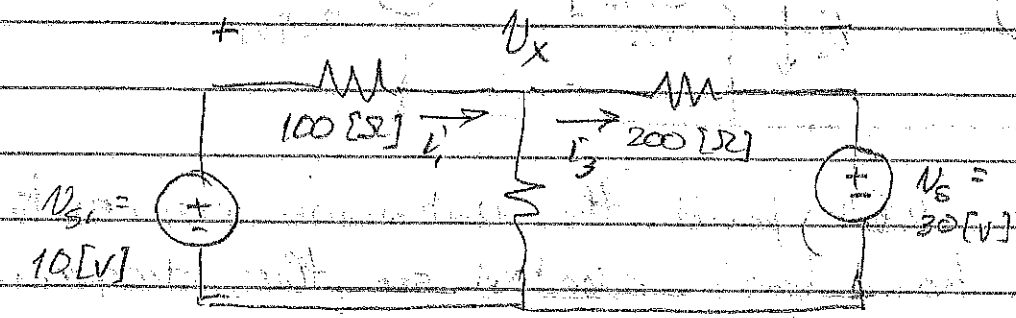
$i_2 = 142.86$ [mA]

$i_3 = -114.29$ [mA]

$P_{del\ by\ v_{s1}} = v_{s1} \cdot i_1' = 285.7\ [mW]$

$P_{del\ by\ v_{s2}} = -v_{s2} \cdot i_3' = 3.429\ [W]$

Something else: Find v_x:



We can use v_x in a KVL:

$$v_x - 200 i_3' - 100 i_1' = 0$$

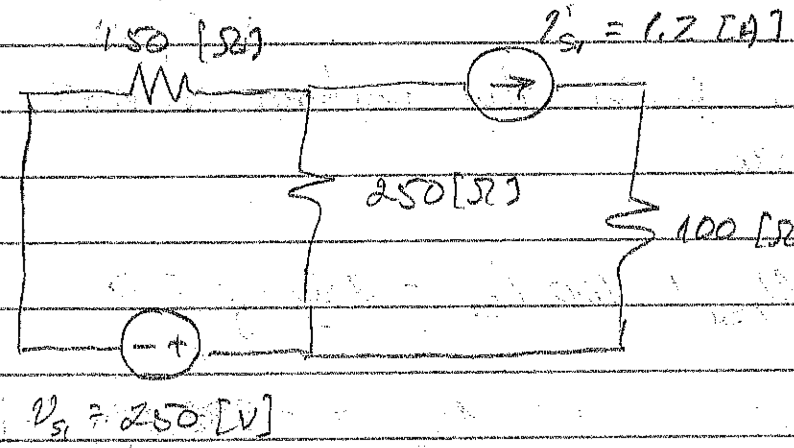
$$\Rightarrow v_x = -20\ [V]$$

Also: $v_x + v_{s2} - v_{s1} = 0$

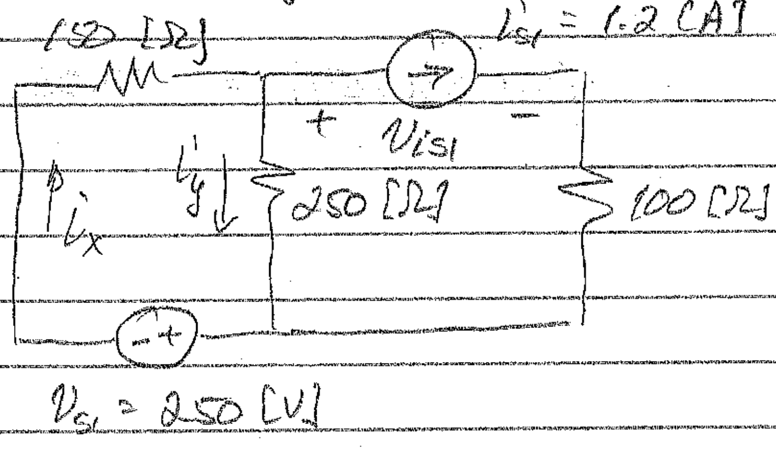
$$\Rightarrow v_x = -20\ [V]$$

Example 2.4

In the circuit below, find the power delivered by the sources i_{s_1} and v_{s_1}.



We can find the two unknown currents, which we have labeled below, with two simultaneous equations. Once we have all the branch currents (the third one, i_{s_1}, is known), we can find anything else we need.



branch currents i_x, i_y, i_{s_1}

$$\left.\begin{array}{ll} \text{KCL:} & i'_x = i'_y + 1.2 \\ \text{KVL:} & 150\, i'_x + 250\, i'_y + 250 = 0 \end{array}\right\} \quad \begin{array}{l} i'_x = 0.125 \text{ [A]} \\ i'_y = -1.075 \text{ [A]} \end{array}$$

Now we can find the voltage across the current source:

KVL: $v_{i_{s1}} + 100\, i'_{s1} - 250\, i'_y = 0$

$$\Rightarrow v_{i_{s1}} = -388.75 \text{ [V]}$$

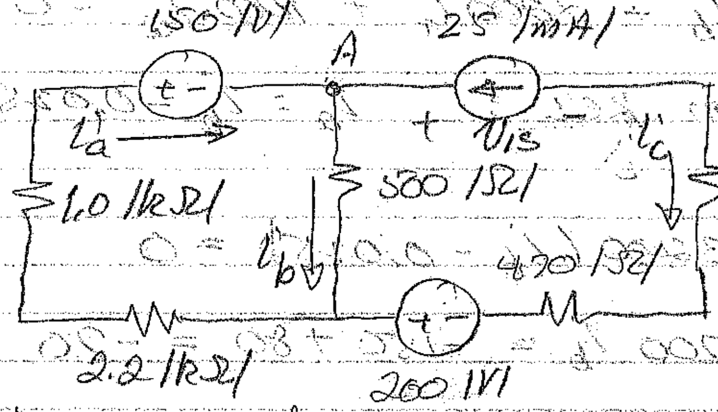
$\therefore$ $P_{del\ by\ i_{s1}} = -i'_{s1} \cdot v_{i_{s1}}$

$$= -(1.2)(-388.75) = 466.5 \text{ [W]}$$

$P_{del\ by\ v_{s1}} = -i'_x \cdot v_{s1}$

$$= -(0.125)(250) = -31.25 \text{ [W]}$$

Example 2.5:



FIND the power delivered by the current source.

We have 3 branches and therefore 3 unknown currents: i_a, i_b, i_c. However, our work is simplified because one of the currents is obvious from the diagram:

$$i_c = -25 \text{ mA}$$

So we need two more equations.

KCL at A:

$$-i_a + i_b + i_c = 0$$

KVL?

Important note: I do not know the voltage across the current source v_{is}. Therefore I will not use a KVL in the right-hand branch, because it will introduce another unknown (v_{is}).

So I will do my KVL around the left-hand loop: going clockwise from the top-left corner I have ...

(Ex 2.5 p.2)

$$150 + 500 i_b' + 2200 i_a' + 1000 i_a' = 0$$

Substitute from KCL: $i_a' = i_b' \pm 0.025$

$$150 + 500 i_b' + 3200 (i_b' - 0.025) = 0$$

$$\Rightarrow \quad 3700 i_b' = -150 + 80 = -70$$

$$i_b' = -18.92 \text{ [mA]}$$

$$i_a' = i_b' + i_c' = -43.92 \text{ [mA]}$$

Now the circuit is "solved" and I can do the right-hand KVL to find v_s.

$$v_s + 1470 i_c' = 200 - 500 i_b' \quad \text{so}$$

$$v_s = 200 + 500(-0.01892) - 1470(-0.025)$$

$$\therefore \quad v_s = 222.3 \text{ [V]}$$

The absorbed power is $P_{abs} = + v_s i_c'$

$$P_{abs} = (222.3)(-0.025) = -5.682 \text{ [W]}$$

So the power delivered is

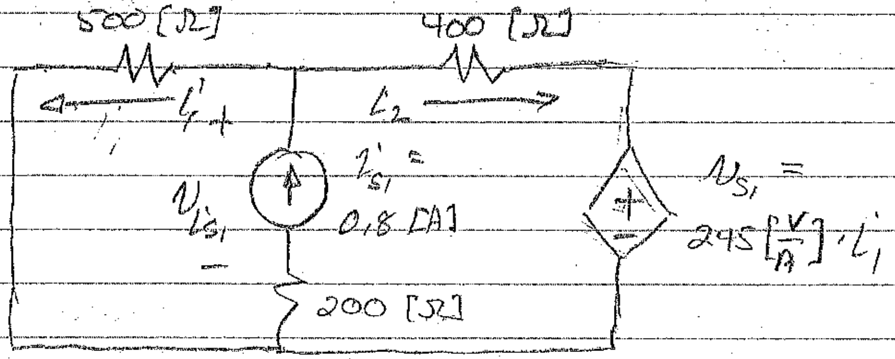
$$P_{del}(25 \text{ [mA]}) = +5.682 \text{ [W]}$$

Example 2.6 KVL, KCL, Ohm's Law, Dependent Source

For the circuit below ...

a) ... write a KCL and one KVL to solve for i_1 & i_2;

b) ... find the power delivered by the current source.



a) KCL: $i_{S_1} = i_1 + i_2$

KVL: $400\, i_2 + 245\, i_1 - 500\, i_1 = 0$

Note that a KVL through the current source means we have to include $v_{i_{S_1}}$, which we don't know. So I avoid the current source by going around the outside loop.

Now I have two equations in two unknowns. Solving gives:

$i_1 = 0.4885$ [A] $\quad i_2 = 0.3115$ [A]

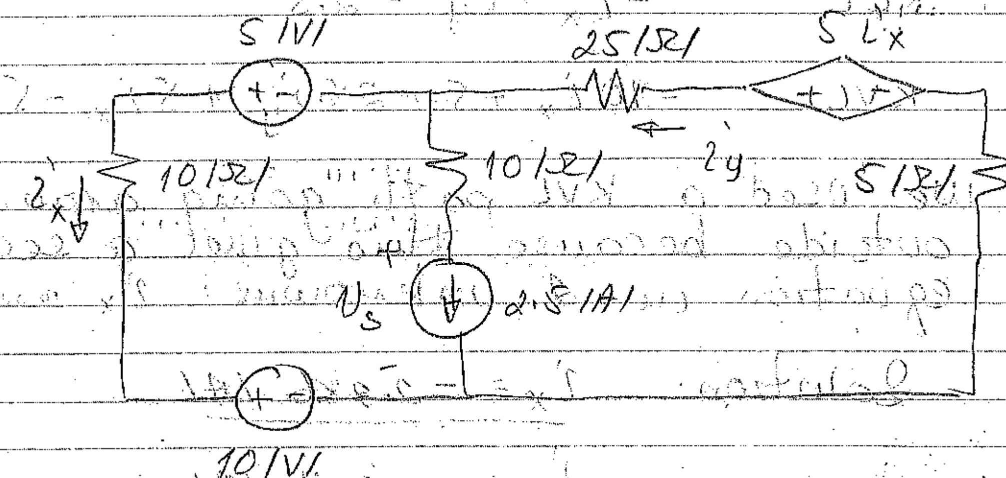
Now that I have i_1' and i_2, I can do a single KVL to find $v_{i_{S1}}$:

$$400 i_2' + 245 i_1' + 200 i_{S1} - v_{i_{S1}} = 0$$

$$v_{i_{S1}} = 404.28 \text{ [V]}$$

$$\therefore \quad \boxed{P_{del\ by\ i_{S1}} = i_{S1}' \cdot v_{i_{S1}} = 323.4 \text{ [W]}}$$

Example 2.7: KVL, KCL w/ Dependent Voltage Source



Note that although there are three branches, we have only two unknown currents since the current in the center branch is known to be 2.5 |A|.

We need only label current in the right-hand branch (i_y) since current in the left-hand branch is already labelled (i_x).

Be careful not to confuse the current-controlled dependent voltage source ($5 i_x$) — it is a voltage source, not a current source.

FIND: a) all branch currents

b) power delivered by 2.5 |A| source.

(Ex 2.7 p. 2)

KCL: $-i'_x + i'_y = 2.5$

KVL: $-10 i'_x + 5 - 25 i'_y + 5 i'_x - 5 i'_y - 10 = 0$

We used a KVL path going around the outside because this gives a second equation in 2 unknowns: i'_x and i'_y.

Solution: $i'_x = -2.286\ [A]$

$i'_y = 0.214\ [A]$

We need a KVL to find V_s.

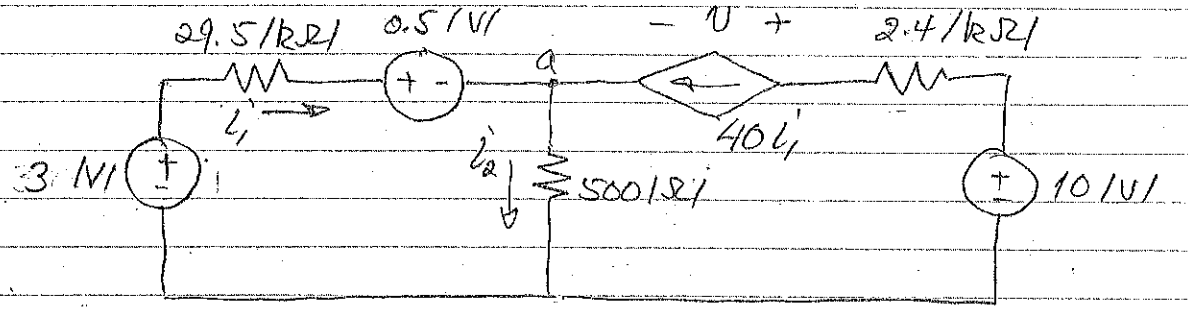
$10 - 10 i'_x + 5 + 2.5(10) + V_s = 0$

$V_s = -42.86\ [V]$

$P = 2.5\, V_s = -107.15\ [W]$

power delivered = 107.15 [W].

Example 2.8



Find a) i_1 in microamps and b) v in volts

KCL at a): $i_2 - i_1 - 40 i_1 = 0$

$$i_2 = 41 i_1 \qquad (1)$$

KVL: $-3 + 29500\, i_1 + 0.5 + 500\, i_2 = 0 \qquad (2)$

(1) ⇒ (2) ⇒

$$-3 + 29500\, i_1 + 0.5 + 500(41 i_1) = 0$$

$$50{,}000\, i_1 = 2.5$$

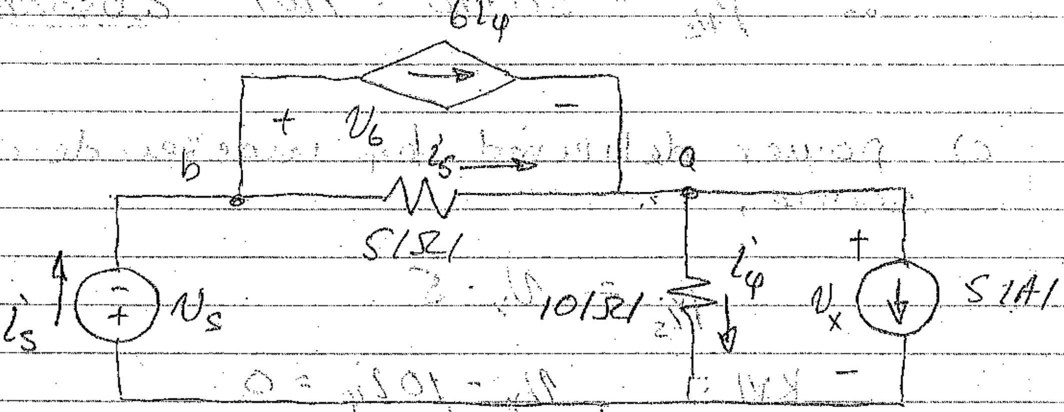
$$\underline{\underline{i_1}} = 5 \times 10^{-5}\ [A] = \underline{\underline{50\ [\mu A]}}$$

KVL:

$$-3 + 29500\, i_1 + 0.5 - v - 2400 \cdot (40 i_1) + 10 = 0$$

$$v = 7.5 - 66500\, i_1 = 4.175\ [V]$$

Example 2.9: KVL, KCL with a Twist



GIVEN: $i_\varphi = 5\,[A]$

CALCULATE:

a) v_s;

KVL: $v_s + 5i_s' + 10i_\varphi = 0$

We need i_s':

KCL at a): $-i_s' - 6i_\varphi + i_\varphi + 5 = 0$

$i_s' = 5 - 5i_\varphi = -20\,[A]$

$\therefore \underline{\underline{v_s}} = -5i_s' - 10i_\varphi = 100 - 50 = \underline{\underline{50\,[V]}}$

b) power absorbed by independent voltage source;

$P_{v_s} = v_s \cdot i_s$

KCL at b): $-i_s + 6i_\varphi + i_s' = 0$

$i_s = 6i_\varphi + i_s' = 30 - 20 = 10\,[A]$

(Ex 2-11 . p.2)

$\therefore \quad P_{v_s} = 50 \cdot 10 = [W]$ absorbed

c) power delivered by independent current source;

$P_{i'_s} = v_x \cdot 5$

KVL: $\quad v_x - 10 i'_\varphi = 0$

$v_x = 10 i'_\varphi = 50 \, [V]$

$\therefore \quad P_{i'_s} = 50(5) = 250 \, [W]$

This result is positive, so power is being absorbed. Therefore the power delivered is

$P_{i'_s} \text{ (delivered) } = -250 \, [W]$

d) power delivered by the controlled current source;

$P_{6i'_\varphi} = v_6 \cdot 6 i'_\varphi$

KVL: $\quad v_6 - 5 i'_s = 0$

$v_6 = 5 i'_s = -100 \, [V]$

$\therefore \quad P_{6i'_\varphi} = -100(6)(5) = -3000 \, [W]$ absorbed

$\Rightarrow \quad P_{6i'_\varphi} \text{ (delivered) } = +3000 \, [W]$

e) total power dissipated in two resistors

$P = i_s'^2 \cdot 5 + i_\varphi'^2 \cdot 10 = 400(5) + 25(10) = 2250 \, [W]$