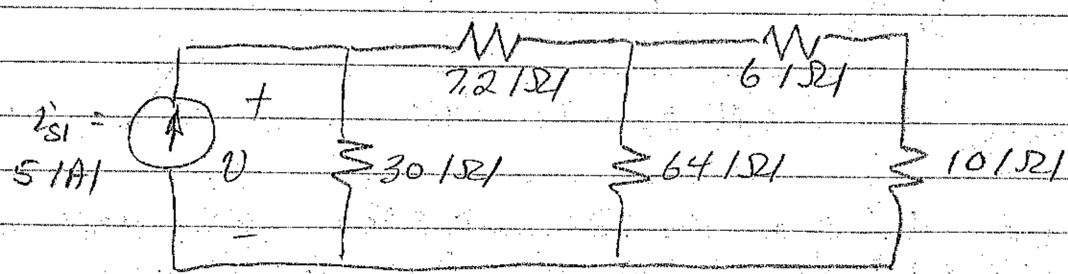
Example 3.1

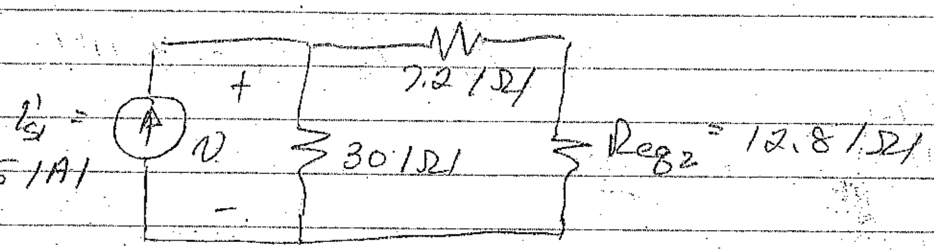
For the circuit shown we want to find a) v, b) power delivered by i_{s1}, c) power dissipated by the $10\,\Omega$ resistor.



$$R_{eq1} = 6 + 10 = 16\,\Omega$$

$$R_{eq2} = 16 \parallel 64 = \frac{16 \cdot 64}{16 + 64} = 12.8\,\Omega$$

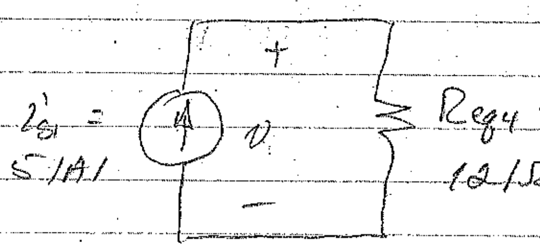
We re-draw the circuit now, for clarity.



$$R_{eq3} = 7.2 + 12.8 = 20\,\Omega.$$

$$R_{eq4} = 30 \parallel 20 = \frac{30 \cdot 20}{30 + 20} = 12\,\Omega$$

A final re-draw gives

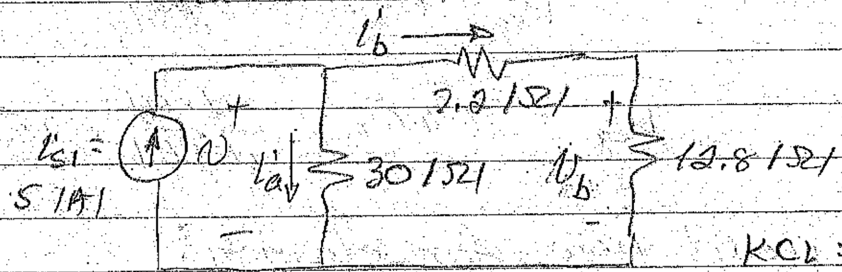


Note that because our series and parallel combinations are equivalent to the original network of resistors, the voltage v does not change.

a) $$v = i_{s_1}' R_{eq_4} = 60\ [V]$$

b) $$P_{del, i_{s1}} = i_{s_1}' \cdot v = 300\ [W]$$

c) To get to the $10\ [\Omega]$ resistor, we need to "unfold" the circuit. But let's use what we know:

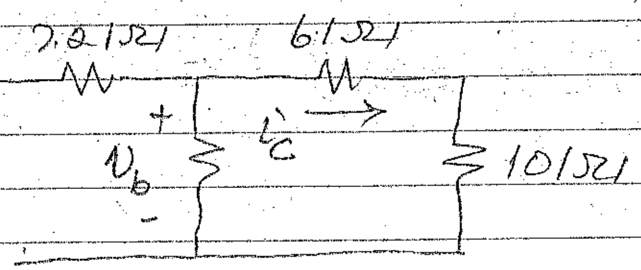


$$i_a' = \frac{v}{30} = 2\ [A]$$

$$KCL: \ -i_{s_1}' + i_a' + i_b' = 0$$

$$\Rightarrow \ i_b' = i_{s_1}' - i_a' = 3\ [A]$$

$$\therefore \ v_b' = (12.8)(3) = 38.4\ [V]$$

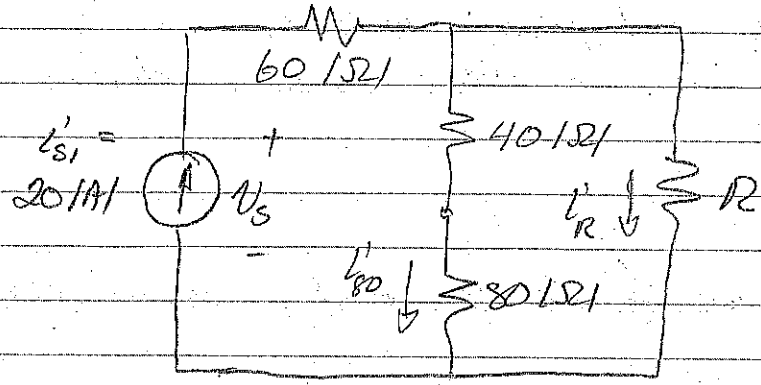


$$-v_b + i_c'(10 + 6) = 0$$

$$i_c' = v_b/16 = 2.4\ [A]$$

c) Finally $$P_{abs,10} = i_c'^2 \cdot 10 = 57.6\ [W]$$

Example 3.2



a) Find R so that $i'_{80} = 4\ |A|$.

CDR ⇒

$$i'_{80} = 20 \cdot \frac{R}{(40+80)+R} = 4$$

We need to solve for R:

$$\frac{R}{120+R} = 0.2 \Rightarrow R = 24 + 0.2R$$

$$0.8R = 24 \Rightarrow \underline{\underline{R = 30\ |\Omega|}}$$

b) Find the power absorbed by R.

CDR:

$$i'_R = 20 \cdot \frac{(40+80)}{(40+80)+30} = 16\ |A|$$

(Alternatively, $i'_R = 20 - i_{80} = 16\ |A|$)

Then

$$P_{abs,R} = i_R^2 \cdot (30) = 7680\ |W|$$

(This is a large value compared with power dissipation in your typical lab kit resistors. Those are rated for a max power dissipation of 1/4 [W]!)

c) Find v_s.

To do this, we first simplify using resistor combinations.

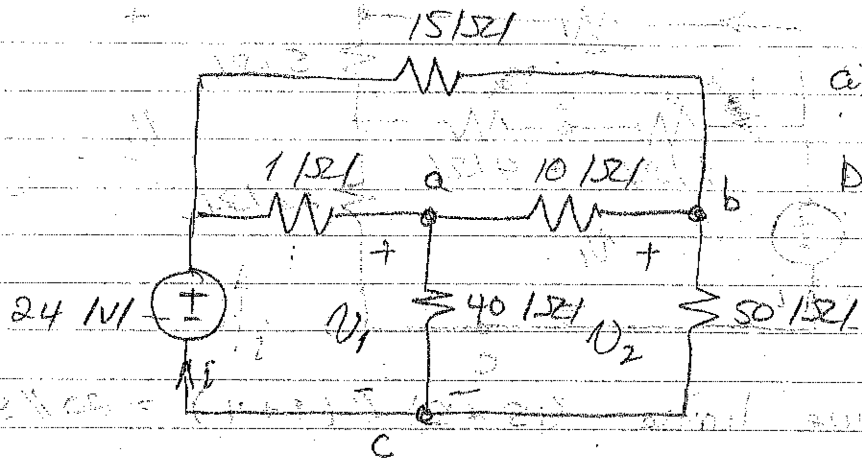
$$R_{eq} = 60 + [(40+80) \parallel 30]$$

$$= 60 + 24 = 84 \ [\Omega]$$

So $v_s = 20 \cdot 84 = 1680 \ [V]$

$$P_{del, i_{s1}} = i_{s1} \cdot v_s = 33{,}600 \ [W]$$

Example 3.3

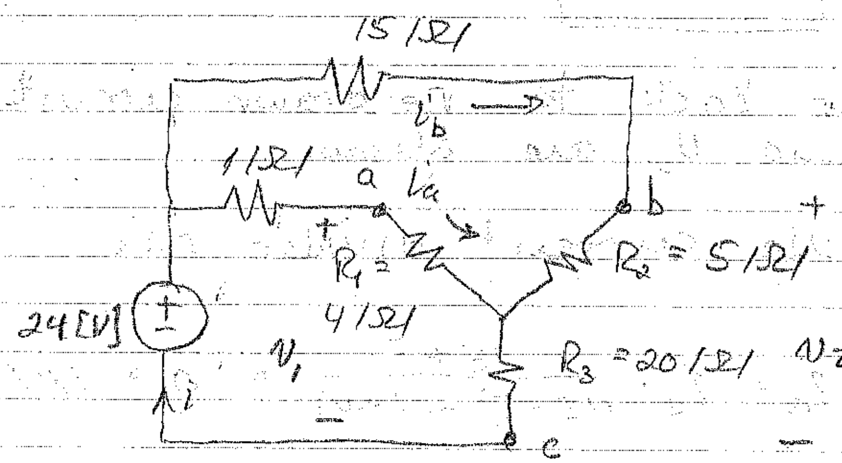


a) Find i.

b) Find v_1, v_2

a) We have: $R_a = 50\,[\Omega]$, $R_b = 40\,[\Omega]$, $R_c = 10\,[\Omega]$

Transformation:

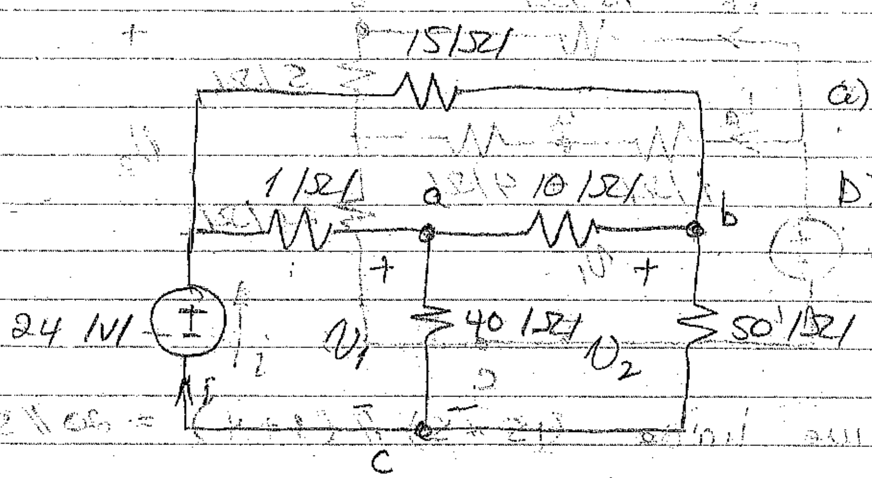


$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} = \frac{40(10)}{100} = 4\,[\Omega]$$

$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c} = \frac{10(50)}{100} = 5\,[\Omega]$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} = \frac{40(50)}{100} = 20\,[\Omega]$$

Example 3.3

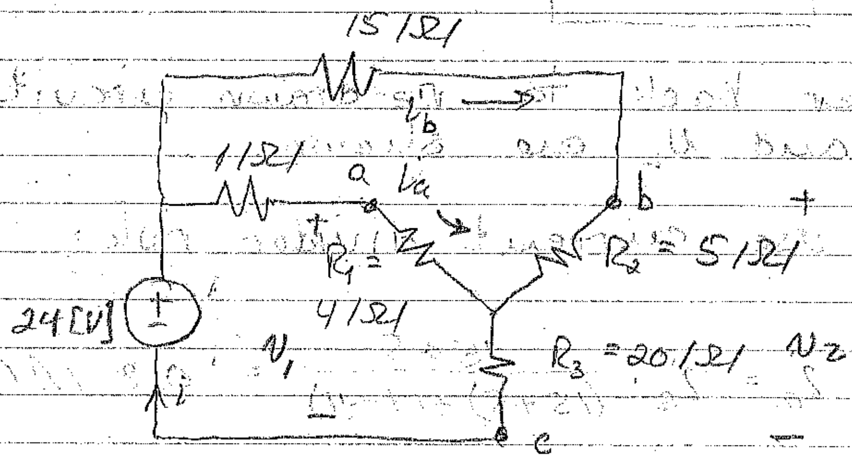


a) Find i.

b) Find v_1, v_2

a) We have: $R_a = 50\,\Omega$, $R_b = 40\,\Omega$, $R_c = 10\,\Omega$

Transformation:

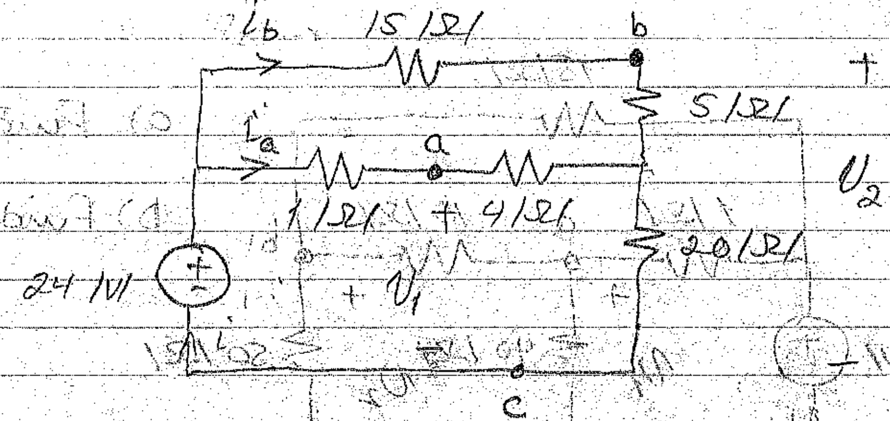


$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c} = \frac{40(10)}{100} = 4\,\Omega$$

$$R_2 = \frac{R_a R_c}{R_a + R_b + R_c} = \frac{10(50)}{100} = 5\,\Omega$$

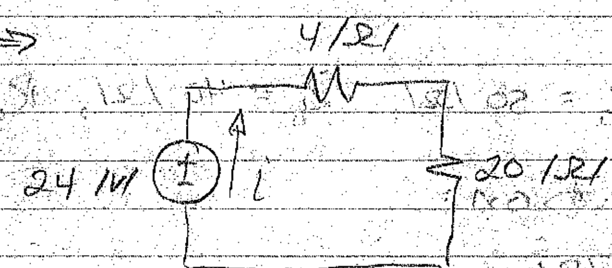
$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c} = \frac{40(50)}{100} = 20\,\Omega$$

Re-draw



Now we have $(15+5) \parallel (1+4) = 20 \parallel 5 = 4\ \Omega$

$\Rightarrow$



$\boxed{i_s' = 24/24 = 1\ A}$

b) Refer back to re-drawn circuit where v_1 and v_2 are shown.

Then use current divider rule:

$$i_a' = i_s' \cdot \frac{15+5}{(15+5)+(1+4)} = 0.8\ A$$

$$i_b' = i_s' \cdot \frac{1+4}{(15+5)+(1+4)} = 0.2\ A$$

KVL: $-24 + 1 \cdot i_a' + v_1 = 0 \Rightarrow \boxed{v_1 = 23.2\ V}$

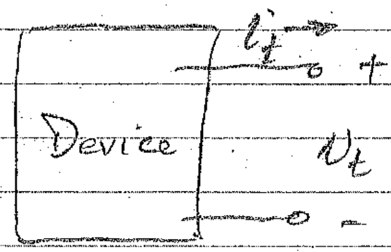
KVL: $-24 + 15 i_b' + v_2 = 0 \Rightarrow \boxed{v_2 = 21\ V}$

Example 3.4 Device Modeling

This problem is taken from Nilsson & Reidel, 8 ed. p. 42.

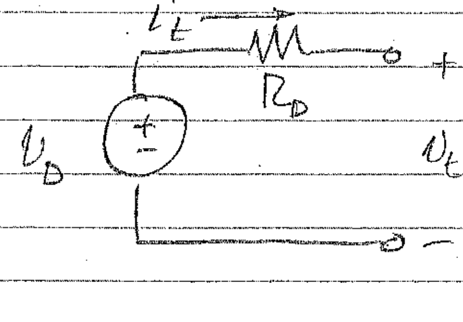
The device shown below can be modeled by a voltage source in series with a resistor. The table shows the measured terminal voltage and current for several cases.

Find the model parameters, that is, the values of the voltage source and resistance.



v_t [V]	i_t [A]
25	0
15	0.1
5	0.2
0	0.25

We begin by drawing the proposed model:



We need to find v_D and R_D.

For the data in this particular case, the problem is easy, because they give us values for the case where v_t and i_t are 0.

(Ex 3.4 p.2)

- For $i_t = 0$, v_t is the same as v_D.
- For $v_t = 0$, $i_t = v_D/R_D$.

From these two cases, it is clear that

$\boxed{v_D = 25\ [V]}$ and $R_D = v_D/i_t = 25/0.25 =$

$$\boxed{R_D = 100\ [\Omega]}$$

We could also do this problem even if we had only been given the 2nd and 3rd table entries. But for that we need a general formula:

Looking at our model, we have

$$v_t - v_D + i_t R_D = 0$$

Now we have v_t and i_t in terms of v_D and R_D.

$v_t = 15\ [V] \Rightarrow i_t = 0.1\ [A]$

$$\Rightarrow 15 - v_D + 0.1 R_D = 0 \quad (1)$$

$v_t = 5\ [V] \Rightarrow i_t = 0.2\ [A]$

$$\Rightarrow 5 - v_D + 0.2 R_D = 0 \quad (2)$$

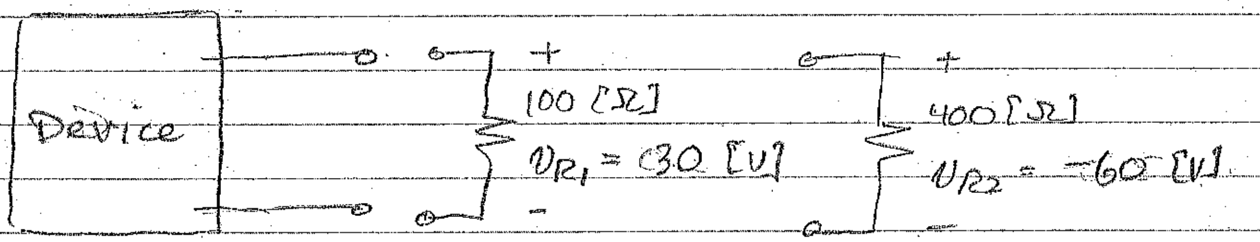
Solving gives $\boxed{v_D = 25 [V]}$ and $\boxed{R_D = 100\ [\Omega]}$

Example 3.5 Device Modeling

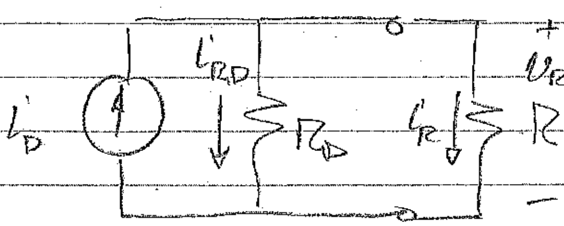
The device shown below can be modeled by a current source in parallel with a resistor.

When a 100 [Ω] resistor is connected to the device terminals, the voltage across it is −30 [V]. When a 400 [Ω] resistor is connected, the voltage across it is −60 [V]

Find the model parameters, that is, the values of the current source and resistance.



We start by drawing the proposed model:



We need to find R_D and i_D.

As for Example 2.4, we need a formula to describe this circuit. We have by KCL

$$i_D = i_{RD} + i_R$$

(Ex 3.8 p.2)

But the information we are given is in terms of $v_R = i_R \cdot R$. So we re-write this as

$$i_D' = \frac{v_R}{R_D} + \frac{v_R}{R}$$

Now we have i_D' and R_D, which we are looking for, in terms of v_R and R, which we have.

$R = 100\ [\Omega] \Rightarrow v_R = 30\ [V]$

$$\Rightarrow i_D' = \frac{30}{R_D} + \frac{30}{100} \qquad (1)$$

$R = 400\ [\Omega] \Rightarrow v_R = -60\ [V]$

$$\Rightarrow i_D' = \frac{-60}{R_D} + \frac{-60}{400} \qquad (2)$$

Solving gives $\boxed{i_D' = 150\ [mA]}$ and $\boxed{R_D = -200\ [\Omega]}$

WAIT! WHAT?? How can we have a negative R_D?? Remember that i_D' in parallel with R_D is a model only. We are not saying that inside the box there is a current source and a resistor. We are saying that whatever is inside, which may be quite complicated, can be modeled with i_D' and R_D. So R_D in that case can be negative.

But just to be clear: actual resistors have a positive resistance.