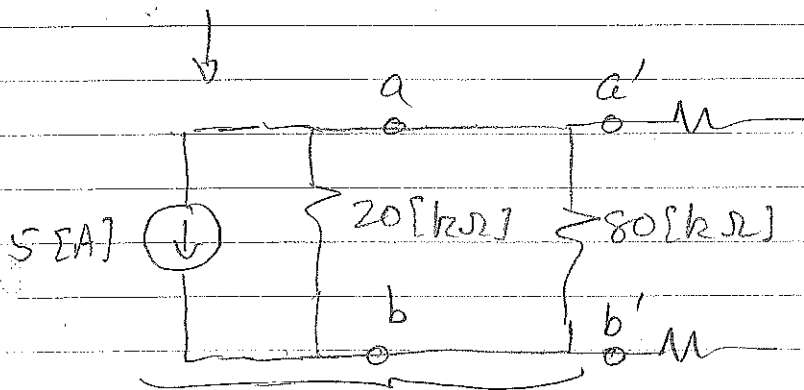
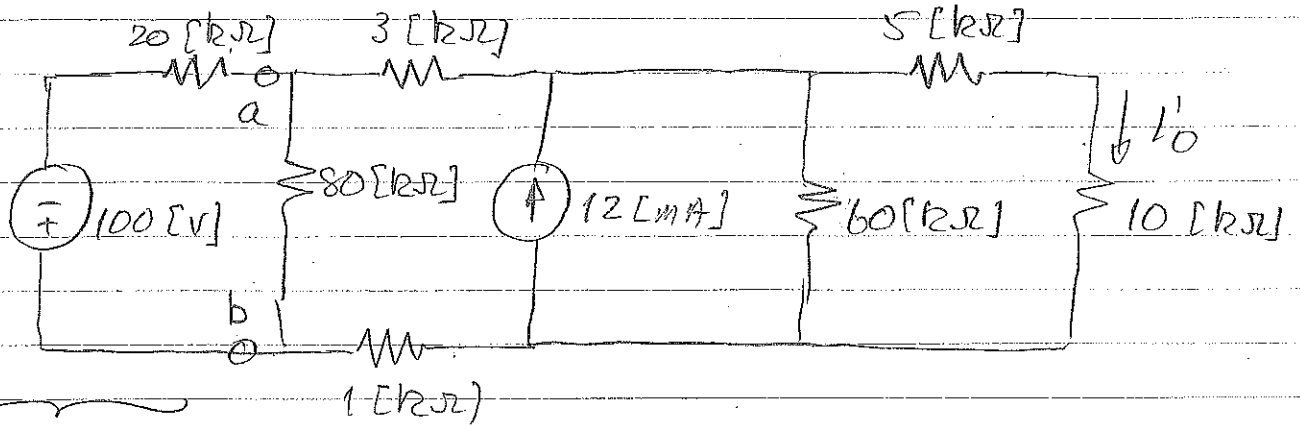


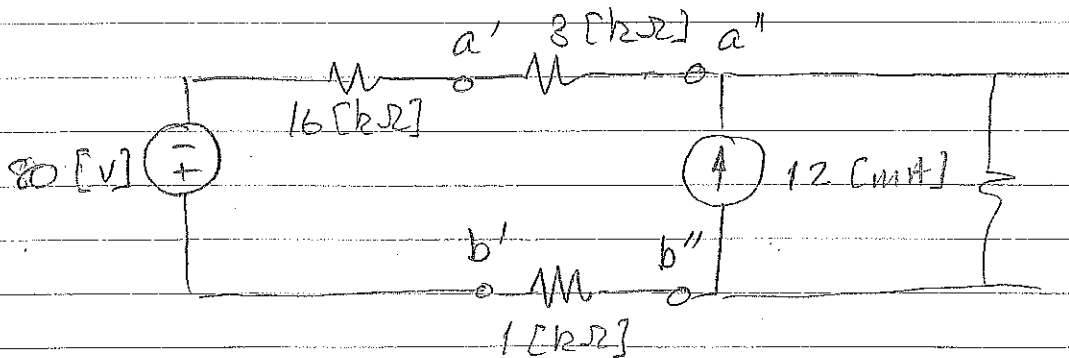
PROBLEM 4.05

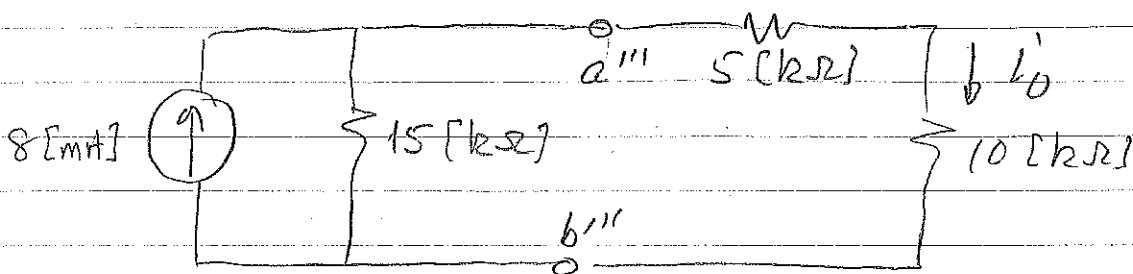
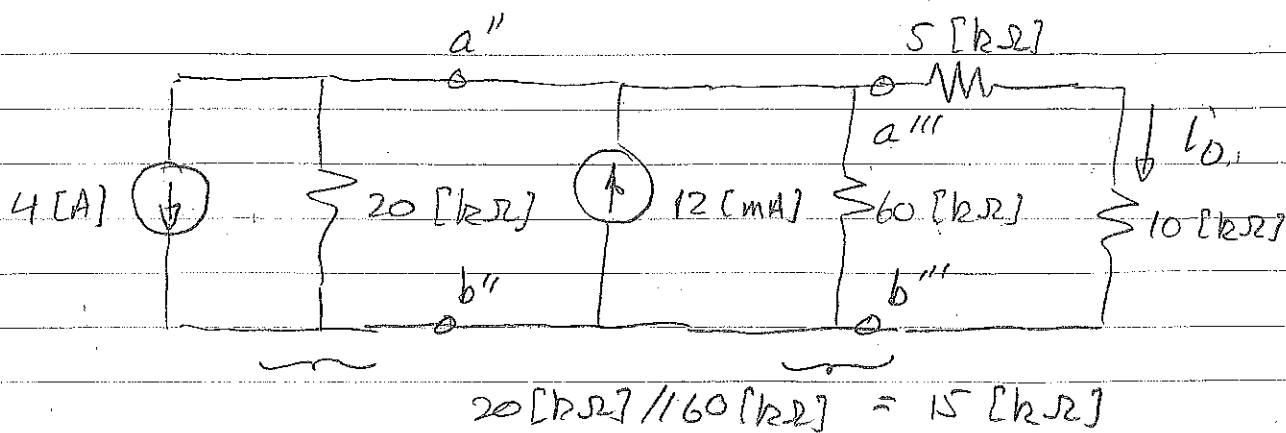
Find  $i_o$  using SOURCE TRANSFORMATIONS.



Watch polarities in this transformation!

$$20 \text{ [k}\Omega\text{]} \parallel 80 \text{ [k}\Omega\text{]} = 16 \text{ [k}\Omega\text{]}$$





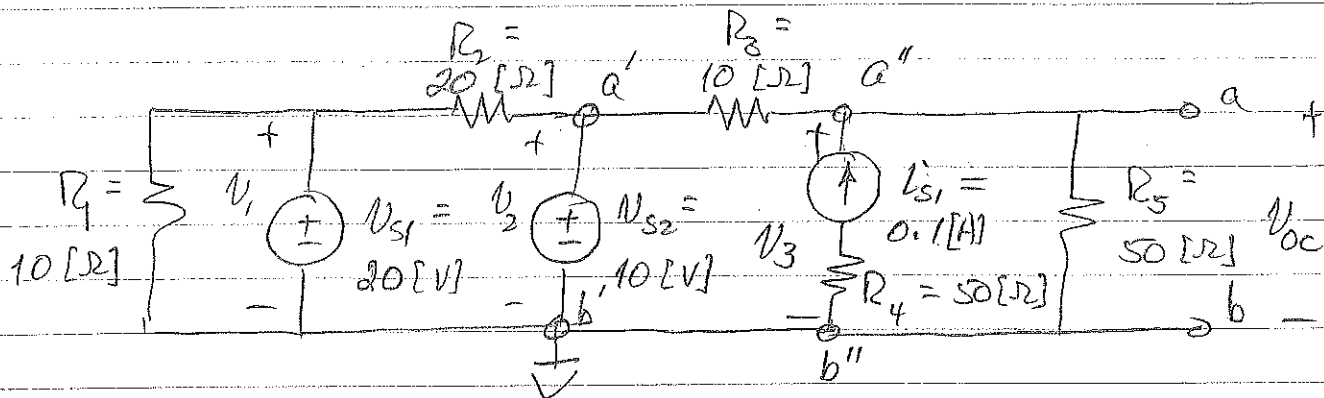
We can use KCL to show that 12 [mA] "up" and 4 [mA] "down" is equivalent to 8 [mA] "up".

Alternatively, we could have used three more source transformations (2 current-voltage and 1 voltage  $\rightarrow$  current) to arrive at the circuit above. In any case we can now use CDR:

$$I_o = 8 [mA] \cdot \frac{15 [k\Omega]}{15 [k\Omega] + 15 [k\Omega]} = 4 [mA]$$

It is important to note that source transformations are done with respect to specific terminals: components outside those terminals remain fixed. As we went through this example, we labelled the terminals we were working with; a, b; a'', b''; etc.

From chapter 4 on Source Transformations:  
 In the notes we consider this problem:



Consider solving for  $V_{oc}$  using node voltage, we have:

$$V_1 = V_{s1} = 20 \text{ [V]}$$

$$V_2 = V_{s2} = 10 \text{ [V]}$$

$V_3$  is the only unknown:

$$-I_{s1} + \frac{V_3 - V_2}{R_3} + \frac{V_3}{R_5} = 0$$

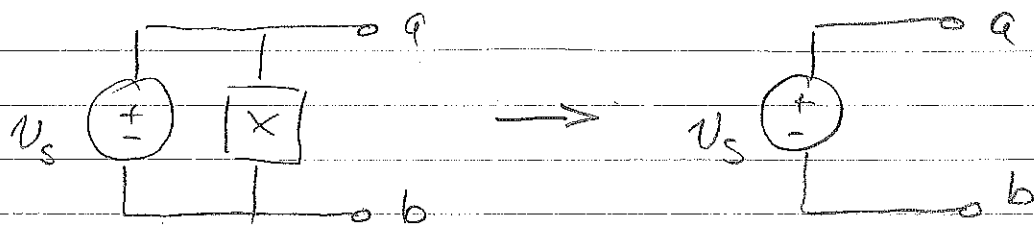
$$\Rightarrow V_3 = 9.167 \text{ [V]} = \underline{\underline{V_{oc}}}$$

1. The value of  $V_3$ , and hence  $V_{oc}$ , is completely unaffected by anything to the left of  $V_{s2}$ . This happens because the combination  $R_1$ ,  $V_{s1}$ , and  $R_2$  are in parallel with a voltage source. Thus anything to the left of  $a'$ ,  $b'$  is not "seen" by anything to the right of  $a'$ ,  $b'$ .

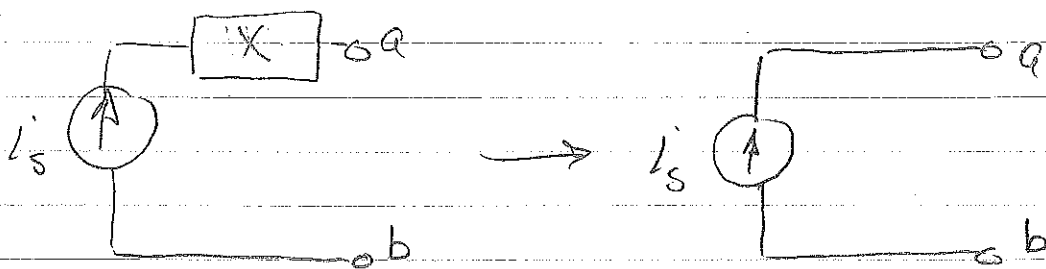
2. The value of  $V_3$ , and hence  $V_{oc}$ , is  $\rightarrow$

completely unaffected by  $R_4$ . That happens because  $R_4$  is in series with a current source. Thus anything outside of "a", "b" cannot "see"  $R_4$ .

So we have two special cases:



Something connected at a, b cannot "see" X.

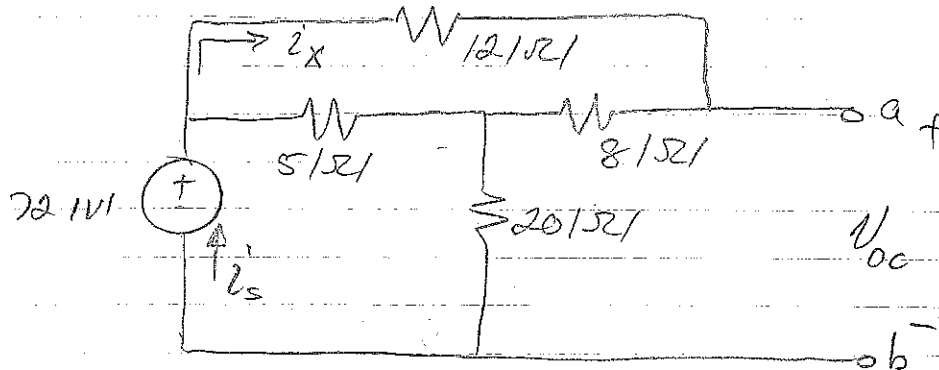


Something connected at a, b cannot "see" X.

BUT: if we need to know what's going on INSIDE X, for example the voltage across  $R_4$ , or the current through  $v_s$ , then we need to consider those components. But as long as we are interested in things outside X, we can ignore X.

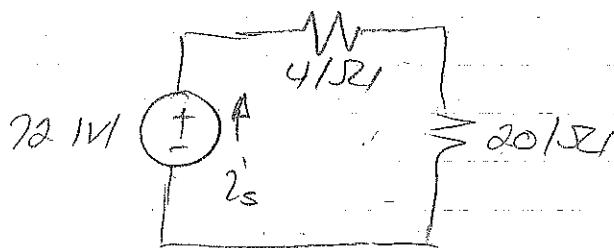
PROBLEM 4.1 (Nilsson & Riedel 8 ed)

- Find the Thevenin Equivalent at a, b.



Note that a, b is open circuit  $\Rightarrow$   
 $12\ \Omega$  is in series with  $8\ \Omega$ .

$$\text{So ... } (12 + 8) \parallel 5 = \frac{20 \cdot 5}{25} = 4\ \Omega$$



Of course now we have lost terminals a, b so we need to "unfold" this circuit:

$$i'_s = \frac{72}{24} = 3\text{A}$$

In original circuit we can use current divider to find  $i'_x$ :

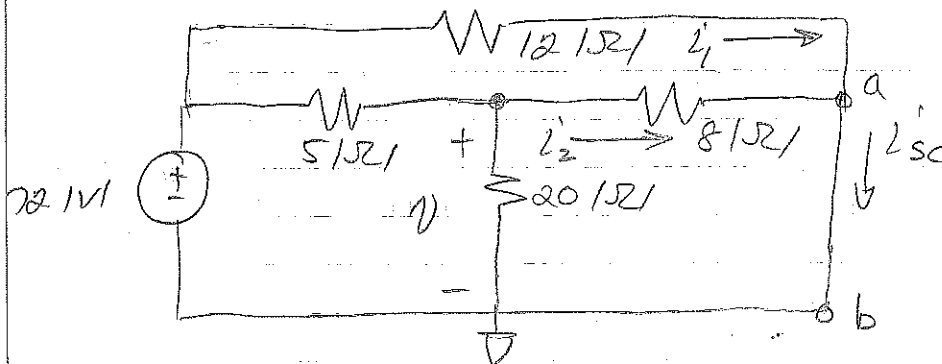
$$i'_x = i'_s \cdot \frac{5}{(12+8) + 5} = i'_s \cdot \frac{5}{25} = 0.6\text{A}$$

Now:  $V_{oc} - I'_s \cdot 20 - I'_x \cdot 8 = 0$

$$\underline{V_{oc}} = 20 I'_s + 8 I'_x = \underline{64.8 \text{ V}}$$

$$= V_{Th}$$

Short-circuit current:



Note that we must take  $I_{sc}$  in the direction  $a \rightarrow b$  if we take  $V_{oc}$  with positive polarity at  $a$ .

$$\frac{V}{20} + \frac{V}{8} + \frac{V - 72}{5} = 0$$

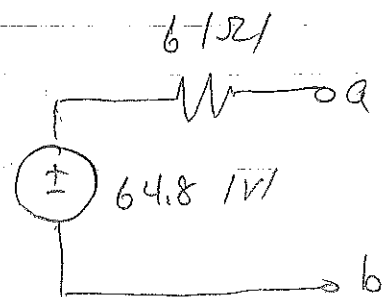
$$\Rightarrow V = 38.4 \text{ V}$$

$$\therefore I_2 = V/8 = 4.8 \text{ A}$$

$$I_1 = 72/12 = 6 \text{ A}$$

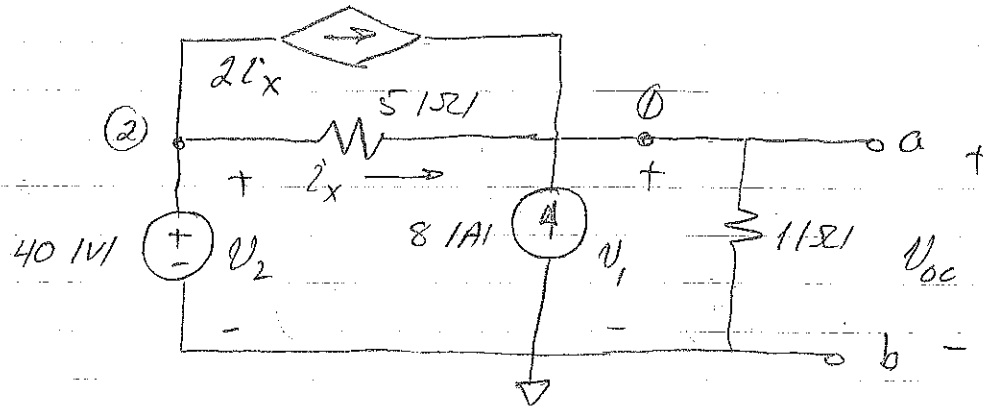
$$\Rightarrow I_{sc} = I_1 + I_2 = 10.8 \text{ A}$$

$$\therefore \underline{R_{Th}} = \underline{V_{oc}/I_{sc}} = \underline{6 \Omega}$$



PROBLEM 4.2 (Nilsson & Riedel 8ed)

Find the Thevenin Equivalent at a, b.



open-circuit voltage

$$\frac{V_1}{1} - 8 + \frac{V_1 - V_2}{5} - 2i'_x = 0$$

$$i'_x = \frac{V_2 - V_1}{5}$$

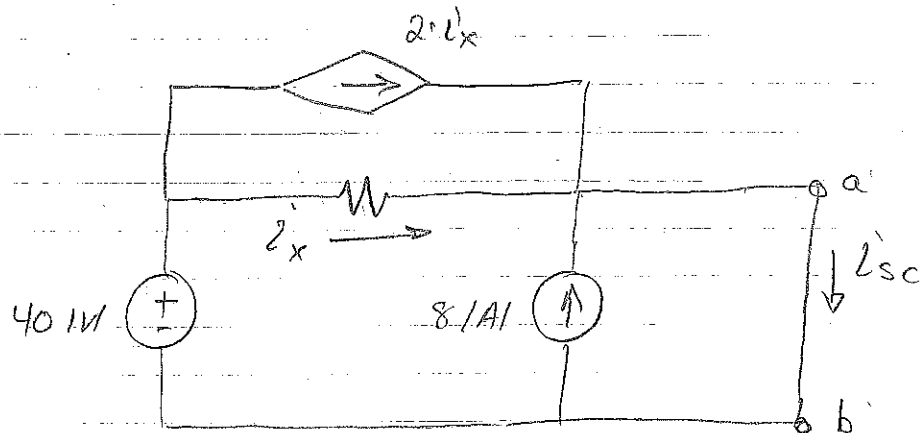
$$V_2 = 40 \text{ V}$$

Solution:  $V_1 = 20 \text{ V}$

$$i'_x = 4 \text{ A}$$

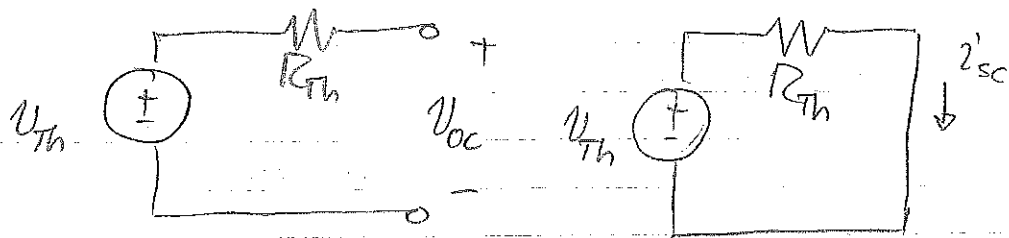
Then  $V_{oc} = \underline{\underline{V_{TH}}} = V_1 = \underline{\underline{20 \text{ V}}}$

## Short-circuit current



Note that  $I_{sc}$  must be taken in the direction  $a \rightarrow b$  if  $V_{oc}$  has positive polarity at  $a$ .

Why is that? Go back to the Thevenin equivalent:



For  $V_{Th} > 0$  and  $R_{Th} > 0$ ,  $V_{oc}$  and  $I_{sc}$  are positive as indicated.

Note also that we were able to remove the  $1/2$  resistor because the short circuit  $\Rightarrow$  no current through it.



(PROBLEM 4.2 cont.)

$$\text{KVL: } -40 + 5i_x = 0$$

$$\text{KCL: } i_{sc} - 8 - i_x - 2i_x = 0$$

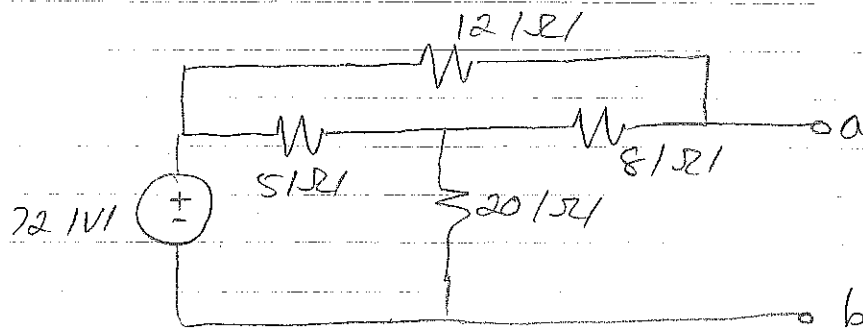
$$\text{Solution: } i_x = 8 \text{ A}$$

$$i_{sc} = 32 \text{ A}$$

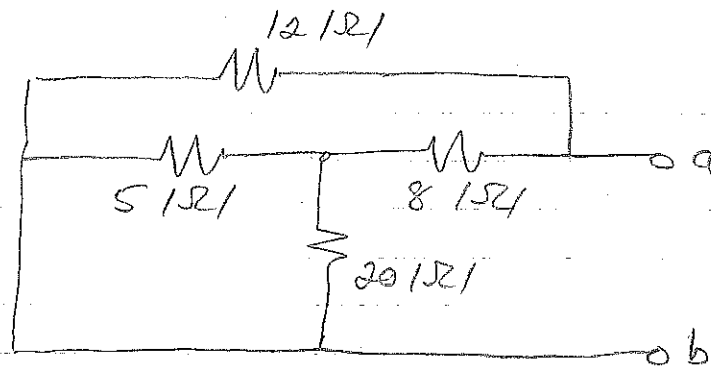
$$\therefore P_{Th} = V_{oc}/i_{sc} = 20/32 = 5/8 \text{ W}$$

# PROBLEM 4.1 REVISITED : $R_{TH}$ calculation

Use test-source idea — but here is a case where we don't need to apply the test source!

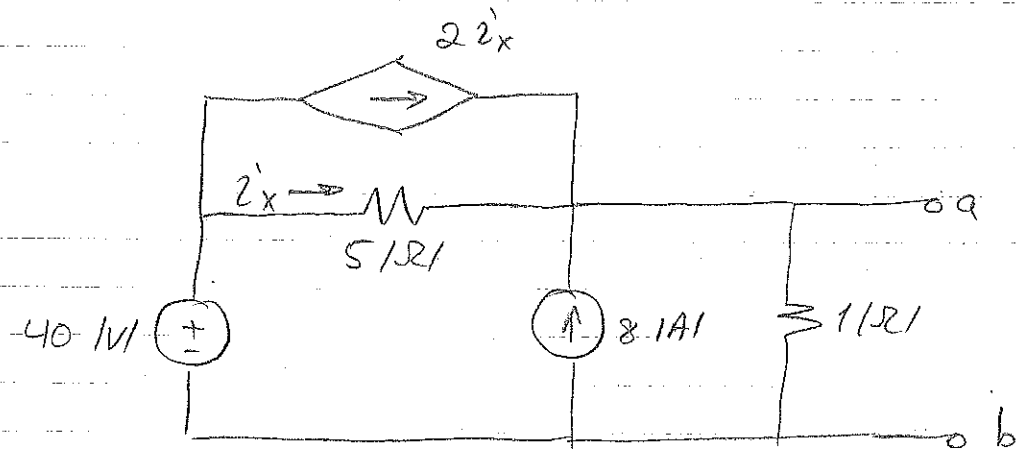


De-activate independent sources:

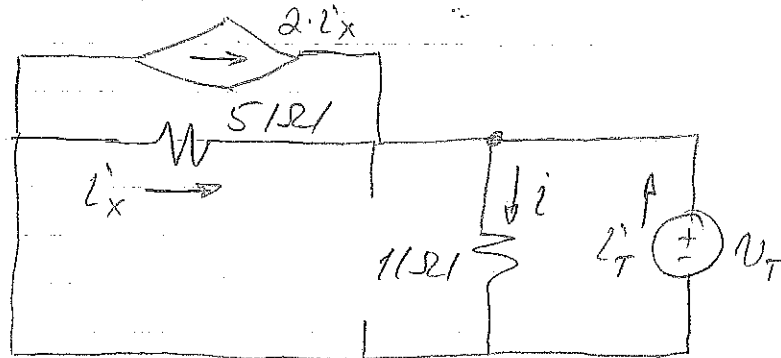


$$\text{Now } R_{TH} = (20 \parallel 5 + 8) \parallel 12 = 6 \Omega$$

PROBLEM 4.2 REVISITED:  $R_{Th}$  calculation



Apply test voltage and de-activate independent sources:



$$\text{KCL: } -i_T - i_x - 2i_x + i = 0$$

$$i = i_T/1$$

$$\text{KVL: } V_T + 5i_x = 0 \Rightarrow i_x = -V_T/5$$

$$\therefore i_T = -3i_x + \frac{V_T}{1}$$

$$= \frac{3}{5}V_T + V_T/1 = V_T \left( \frac{3}{5} + 1 \right)$$

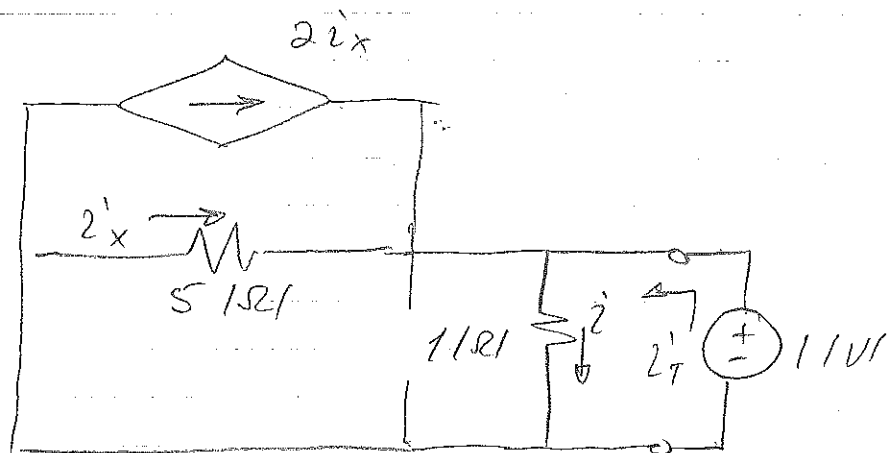
$$\Rightarrow \frac{V_T}{i_T} = \frac{1}{(8/5)} = \frac{5}{8} \text{ } 1\Omega = \underline{\underline{R_{Th}}}$$

(PROBLEM 4.2 REVISITED CON'T)

Note that we need to find  $V_T/i_T'$  so we manipulated our equations to arrive at this.

BUT: Since any test source will work, it is often useful to choose a specific value, say  $1 \text{ V}$  (or  $1 \text{ A}$ ).

Looking again at Problem 4.2, we apply a  $1 \text{ V}$  test source:



Now

$$i_T' = 1 \text{ A}$$

$$1 + 5i_x' = 0 \Rightarrow i_x' = -0.2 \text{ A}$$

$$\therefore i_T' = -3i_x' + i_T$$

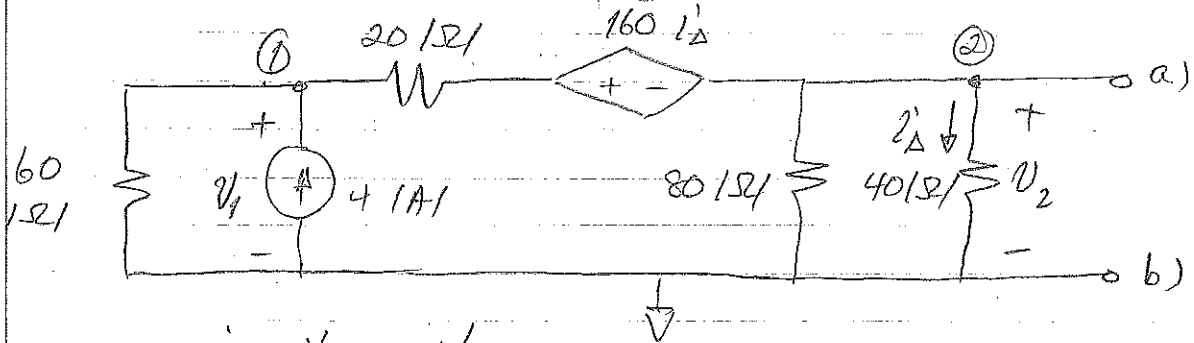
$$= +0.6 + 1 = 1.6 \text{ A}$$

$$\Rightarrow R_{Th} = V_T/i_T' = 1/1.6 = 0.625 \text{ } \Omega$$

( =  $5/8 \text{ } \Omega$  )

PROBLEM 4.3 (Nilsson & Riedel, 8ed)

Find the Thevenin Equivalent at a), b).



open-circuit voltage:

$$\frac{v_2}{40} + \frac{v_2}{80} + \frac{v_2 - v_1 + 160 i'_{\Delta}}{20} = 0$$

$$\frac{v_1}{60} - 4 + \frac{v_1 - v_2 - 160 i'_{\Delta}}{20} = 0$$

$$i'_{\Delta} = v_2/40$$

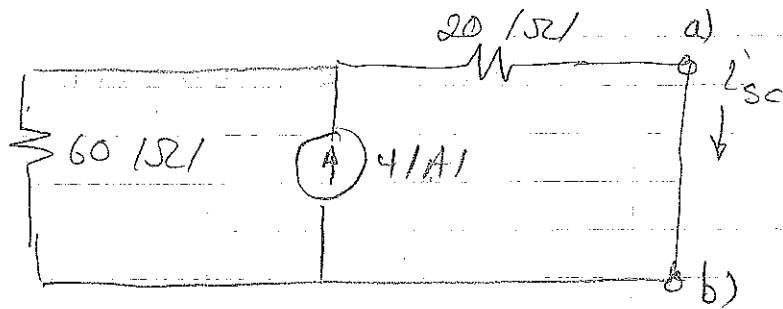
Solution:  $v_2 = v_{oc} = \underline{v_{Th} = 30 \text{ V}}$

$$i'_{\Delta} = 0.75 \text{ A}$$

$$v_1 = 172.5 \text{ V}$$

Short-circuit current

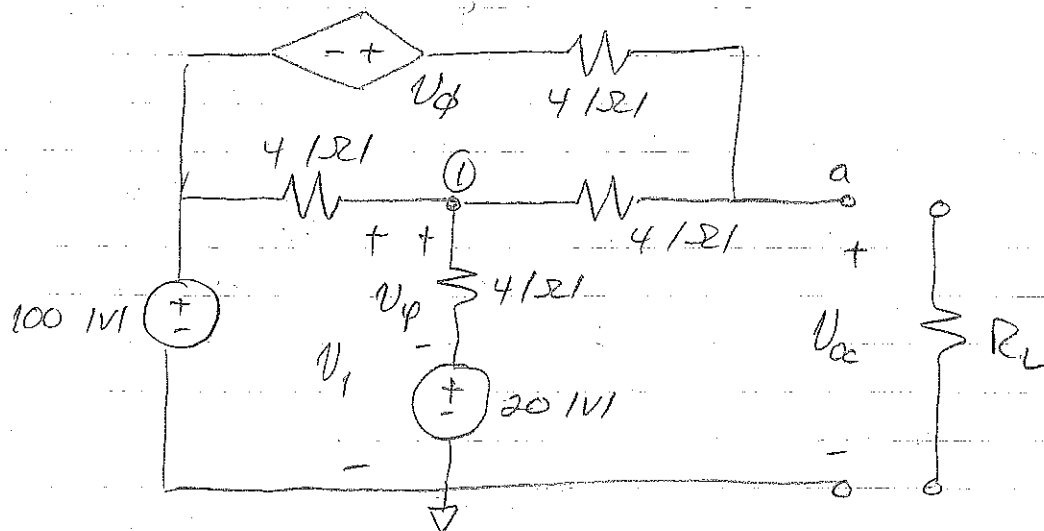
with a), b) short-circuited, there is no current in the 80 ohm or 40 ohm resistors. Also,  $i'_{\Delta} = 0$ , so the circuit simplifies considerably:



Then  $i_{sc} = 4 \cdot \frac{60}{60+20} = 3A$

So  $R_{Th} = V_{oc}/i_{sc} = 10 \Omega$

PROBLEM 4.4 (Nilsson & Riedel 8ed)



- a) Find  $R_L$  for maximum power transfer  
 b) Find maximum power transferred.

Since we need both the value of  $R_L$  for max power and the max power, we need both  $R_{Th}$  and  $V_{Th}$ . For this we need two of the following: open circuit voltage; short circuit current; equivalent resistance.

Although only two are required, we will find all three for illustration purposes.

open-circuit voltage

$$\frac{V_1 - 100}{4} + \frac{V_1 - 20}{4} + \frac{V_1 - 100 - V_\phi}{8} = 0$$

(No current flows out through terminal a) under open-circuit conditions, so the two  $4\Omega$  resistors are in series)

$$V_1 - 20 - V_\varphi = 0$$

Solving these equations:

$$V_1 = 80 \text{ V}$$

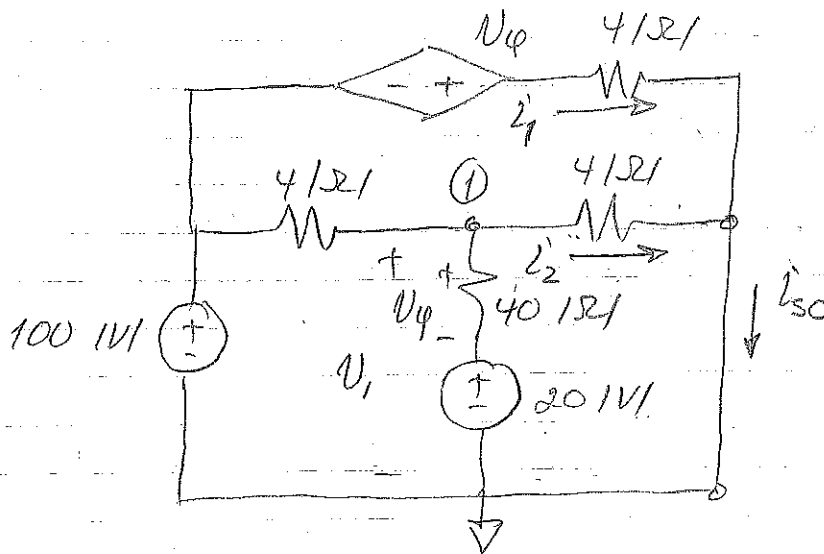
$$V_\varphi = 60 \text{ V}$$

Then  $V_{oc} = 2 \cdot 4 + V_1$

$$I' = -\frac{V_1 - 100 - 2V_\varphi}{8} = 10 \text{ A}$$

$$\therefore V_{oc} = 120 \text{ V}$$

Short-circuit current





(PROBLEM 4.4 cont)

$$\frac{V_1 - 20}{4} + \frac{V_1 - 100}{4} + \frac{V_1}{4} = 0$$

$$\Rightarrow V_1 = 40 \text{ V}$$

Now

$$Z_{sc} = Z'_1 + Z'_2$$

$$\text{KVL: } -100 - V_\varphi + 4Z'_1 = 0$$

$$V_\varphi = V_1 - 20 = 20 \text{ V}$$

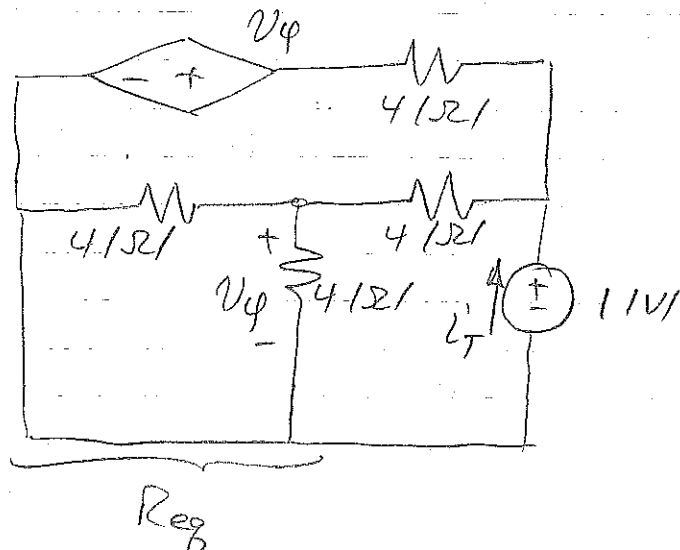
$$\therefore Z'_1 = \frac{100 + V_\varphi}{4} = 30 \text{ } \Omega$$

$$Z'_2 = \frac{V_1}{4} = 10 \text{ } \Omega$$

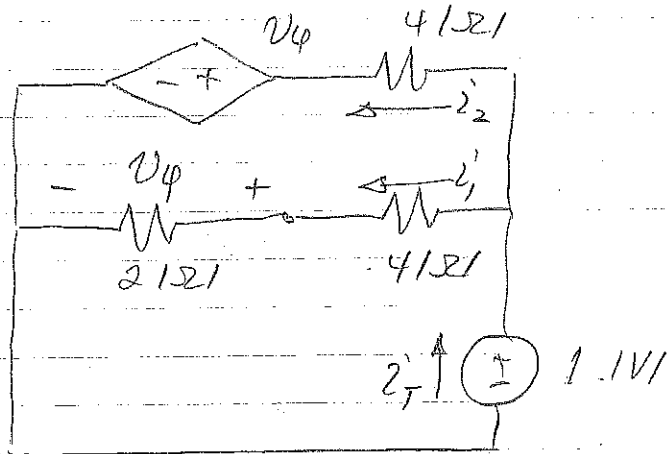
$$\Rightarrow Z_{sc} = 40 \text{ } \Omega$$

$$\therefore R_L = R_{Th} = V_{oc} / i_{sc} = 120 / 40 = \underline{\underline{3 \text{ } \Omega}}$$

Try test source for fun:



$$R_{eq} = 4 \parallel 4 = 2 \text{ } \Omega$$



voltage divider:  $V_\phi = 1 \cdot \frac{2}{4+2} = \frac{1}{3} \text{ V}$

$$i_1 = \frac{1}{6} \text{ A}$$

$$i_2 = \frac{1 - V_\phi}{4} = \frac{1 - \frac{1}{3}}{4} = \frac{\frac{2}{3}}{4} = \frac{1}{6} \text{ A}$$

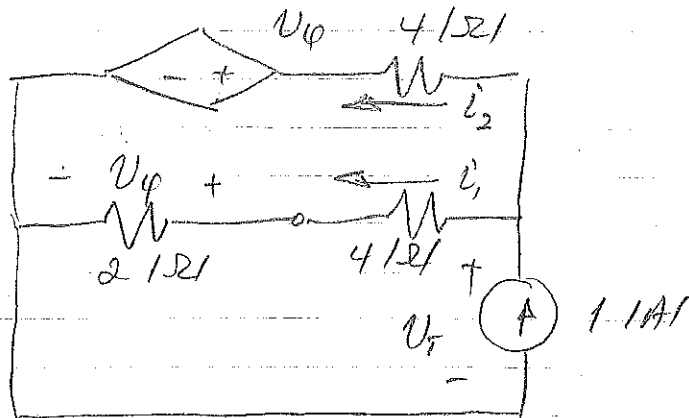
$$\therefore i_T = i_1 + i_2 = \frac{1}{3} \text{ A}$$

Finally  $R_{Th} = \frac{1 \text{ V}}{i_T} = 3 \text{ } \Omega$

... as we expect.

Let's use a test current source too...

(PROBLEM 4.4 cont.)



$$i_1 = V_T / 6$$

$$i_2 = \frac{V_T - V_\phi}{4}$$

$$V_\phi = 2 \cdot i_1 = V_T / 3$$

Then

$$i_1 + i_2 = 1 = \frac{V_T}{6} + \frac{V_T - V_T/3}{4}$$

$$= \frac{V_T}{6} + \frac{V_T}{6} = \frac{V_T}{3}$$

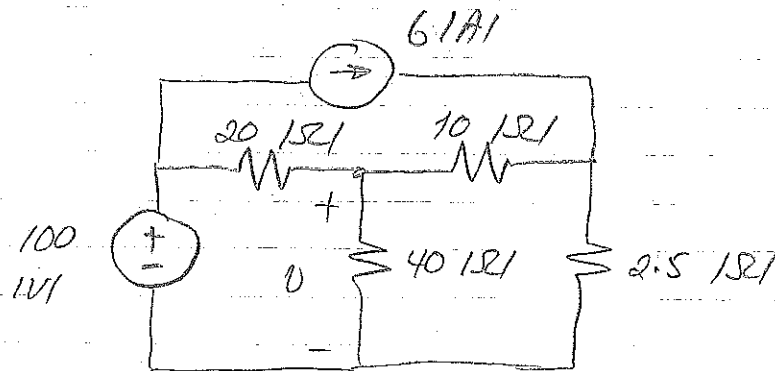
$$\therefore V_T = 3 \text{ V}$$

Finally  $R_{Th} = \frac{V_T}{1} = \underline{\underline{3 \Omega}}$

b)

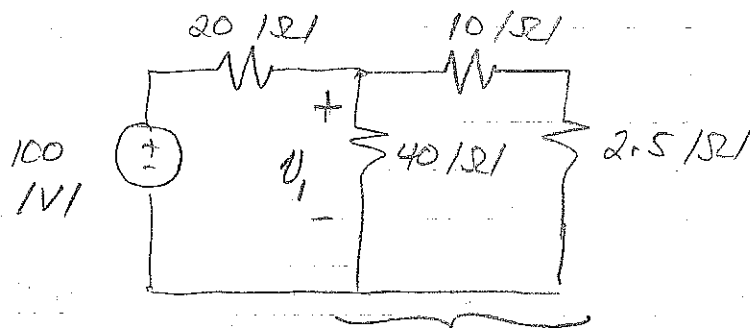
$$\underline{\underline{P_{max}}} = \frac{V_{Th}^2}{4 \cdot R_{Th}} = \frac{(120)^2}{4 \cdot 3} = \underline{\underline{1200 \text{ W}}}$$

PROBLEM 4.6



- a) Find  $V$  using SUPERPOSITION.  
 b) Find the power dissipated in  $40\ \Omega$  resistor.

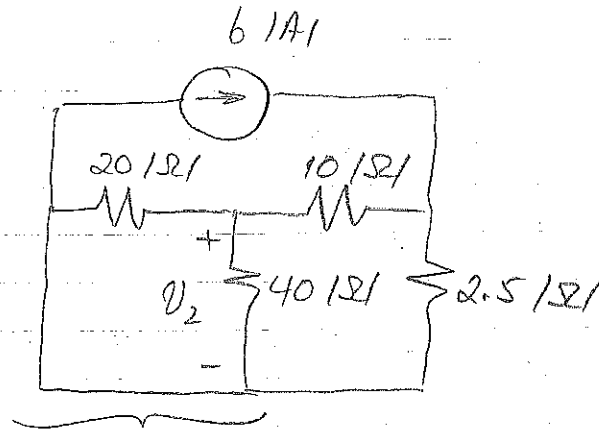
De-activate current source:



$$R_{eq} = (10 + 2.5) \parallel 40 = 9.52\ \Omega$$

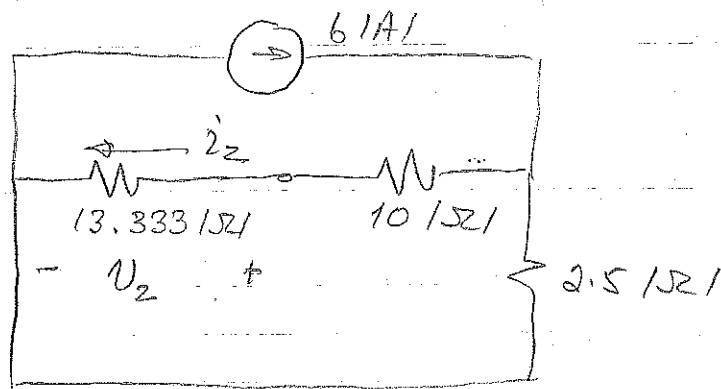
$$V_1 = 100 \cdot \frac{R_{eq}}{R_{eq} + 20} = 32.25\ \text{V}$$

De-activate voltage source:



$$R_{eq} = 40 \parallel 20 = 13.333 \Omega$$

Now:



current divider rule:

$$I_2' = 6 \cdot \frac{2.5}{(23.333 + 2.5)} = 0.58065 \text{ A}$$

$$V_2 = 13.333 \cdot I_2' = 7.742 \text{ V}$$

Now

$$V = V_1 + V_2 = 39.992 \text{ V}$$

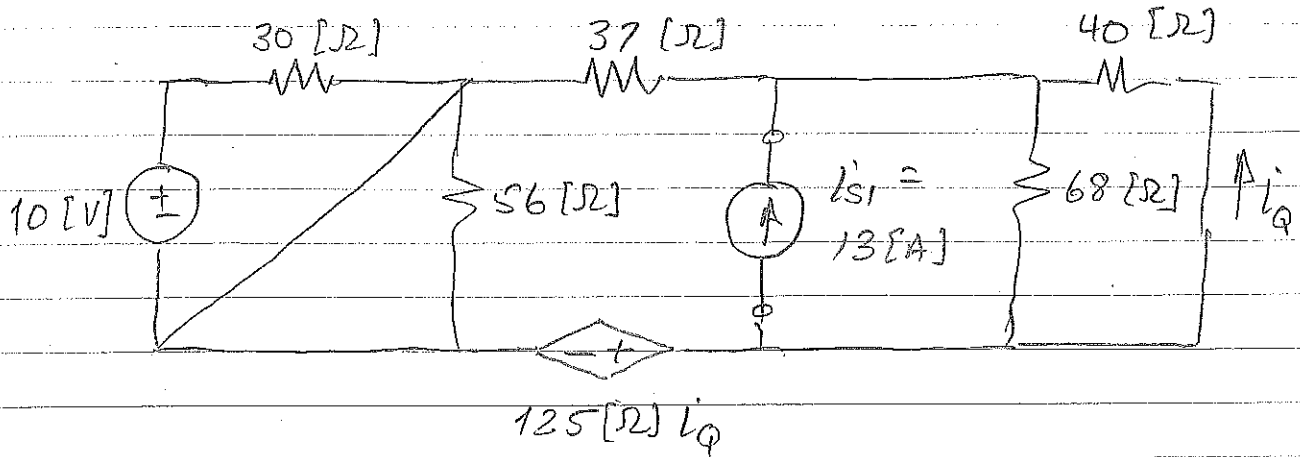
$$b) P_{abs} = V^2/R = 39.984 \text{ W}$$

IMPORTANT:  $P_{abs} \neq \frac{V_1^2}{R} + \frac{V_2^2}{R} = 29.5 \text{ W}$

Reason:  $p = V^2/R \neq V_1^2/R + V_2^2/R$

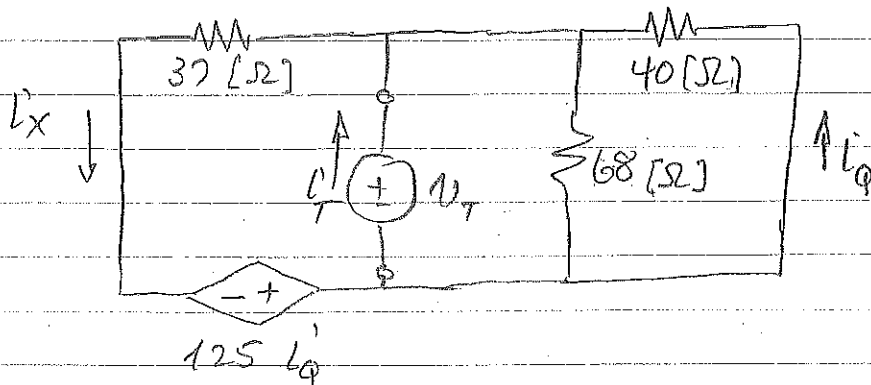
PROBLEM 4.5

Find the Thevenin Equivalent seen by the current source.



Note: "seen by the current source" means exactly that: what equivalent circuit does the current source see? For that, we need to disconnect the current source because the current source is not part of the equivalent circuit.

Because we have no independent sources, we must find  $R_{TH}$  with a test source.



We have removed several components that were in parallel with a short.

$$\text{KVL: } -V_T - 40i_q = 0$$

$$\text{Set } V_T = 1 \text{ [V] (arbitrary)} \Rightarrow i_q = -25 \text{ [mA]}$$

$$\text{KVL: } -V_T + 37i_x - 125i_q = 0$$

$$\Rightarrow i_x = \frac{1 + 125(-0.025)}{37} = -57.43 \text{ [mA]}$$

$$\text{KCL: } i_T = i_x - i_q + \frac{1}{68} = -17.72 \text{ [mA]}$$

$$\therefore \underline{R_{TH} = -56.42 \text{ } [\Omega]}$$

Note that the terminals of the Thevenin Equivalent do not "see" any independent sources, which means...

$$V_{TH} = 0$$

So the Thevenin Equivalent is just a resistance:

$$R_{TH} = -56.42 \text{ } [\Omega]$$

