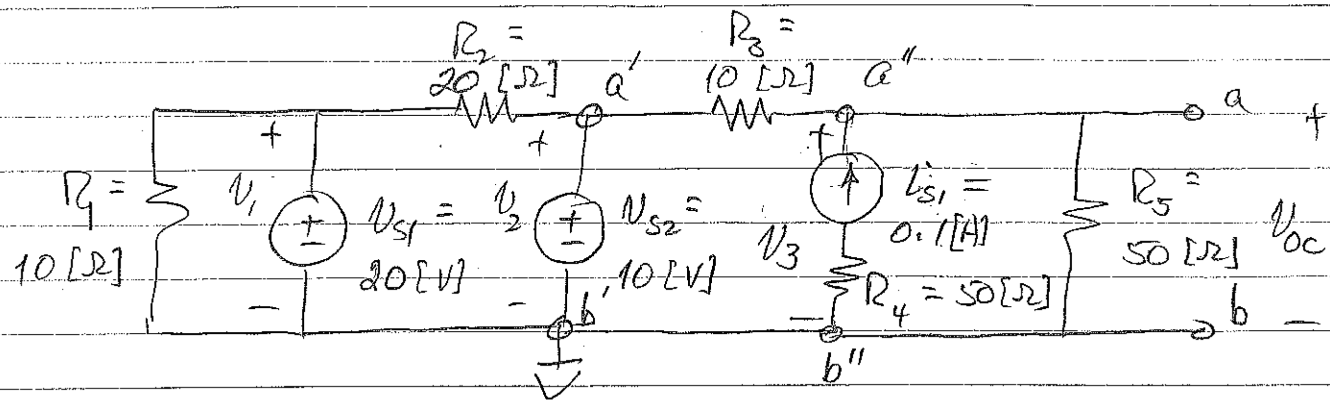
From Chapter 4 on Source Transformations:
In the notes we consider this problem:



Consider solving for v_{oc} using node voltage, we have:

$$v_1 = v_{S1} = 20\ [V]$$

$$v_2 = v_{S2} = 10\ [V]$$

v_3 is the only unknown:

$$-i_{S1} + \frac{v_3 - v_2}{R_3} + \frac{v_3}{R_5} = 0$$

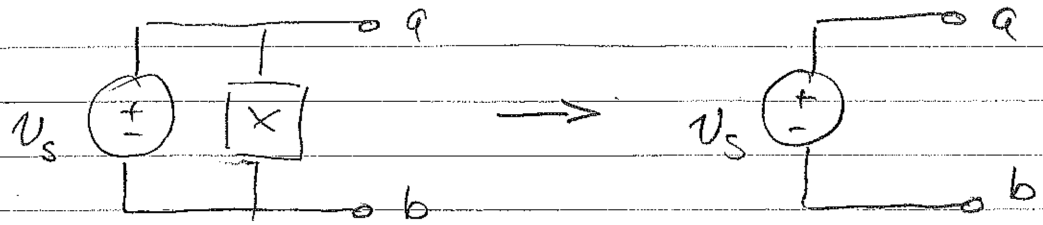
$$\Rightarrow v_3 = 9.167\ [V] = \underline{\underline{v_{oc}}}$$

1. The value of v_3, and hence v_{oc}, is completely unaffected by anything to the left of v_{S2}. This happens because the combination R_1, v_{S1}, and R_2 are in parallel with a voltage source. Thus anything to the left of a', b' is not "seen" by anything to the right of a', b'.

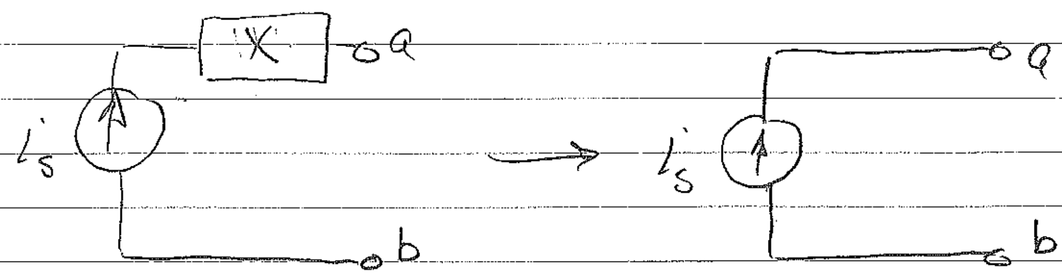
2. The value of v_3, and hence v_{oc}, is

completely unaffected by R_4. That happens because R_4 is in series with a current source. Thus anything outside of a", b" cannot "see" R_4.

So we have two special cases:



Something connected at a, b cannot "see" X.

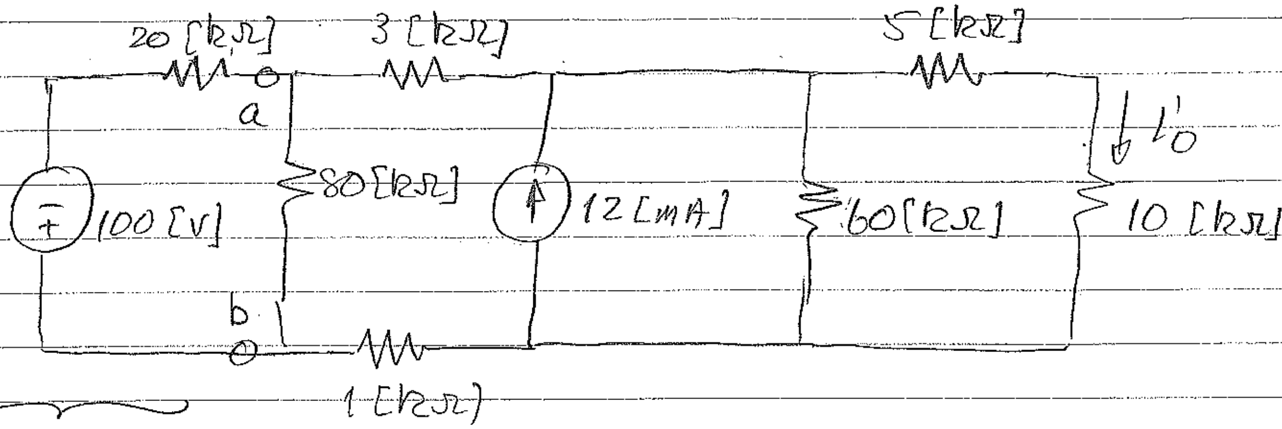


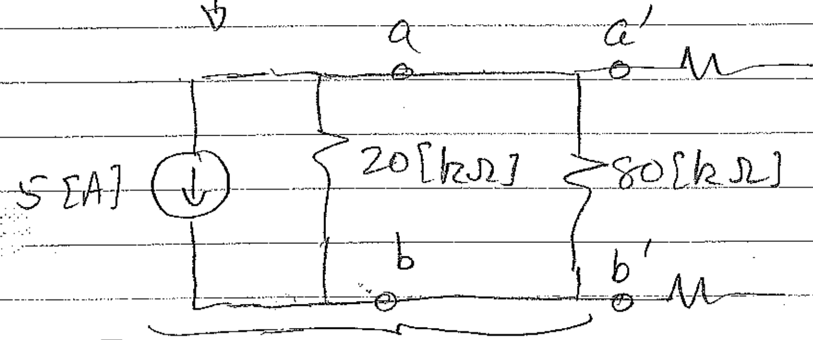
Something connected at a, b cannot "see" X.

BUT: If we need to know what's going on INSIDE X, for example the voltage across R_4, or the current through v_s, then we need to consider those components. But as long as we are interested in things outside X, we can ignore X.

PROBLEM 4.05

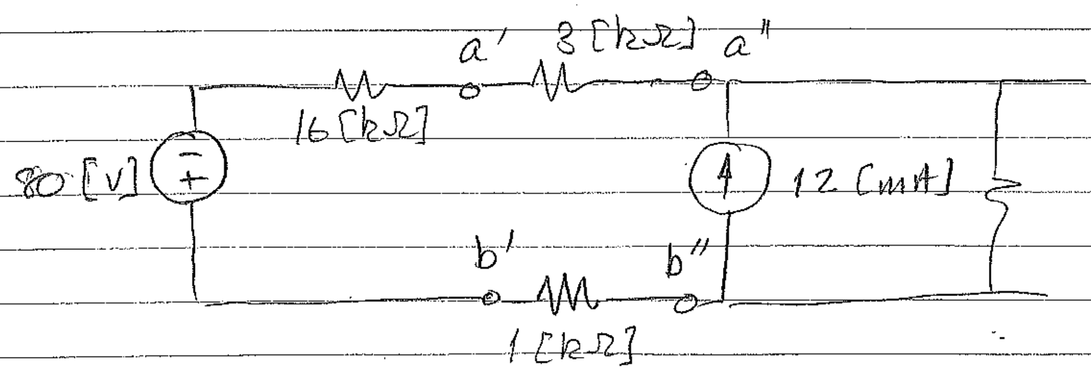
- Find i_o' using SOURCE TRANSFORMATIONS.

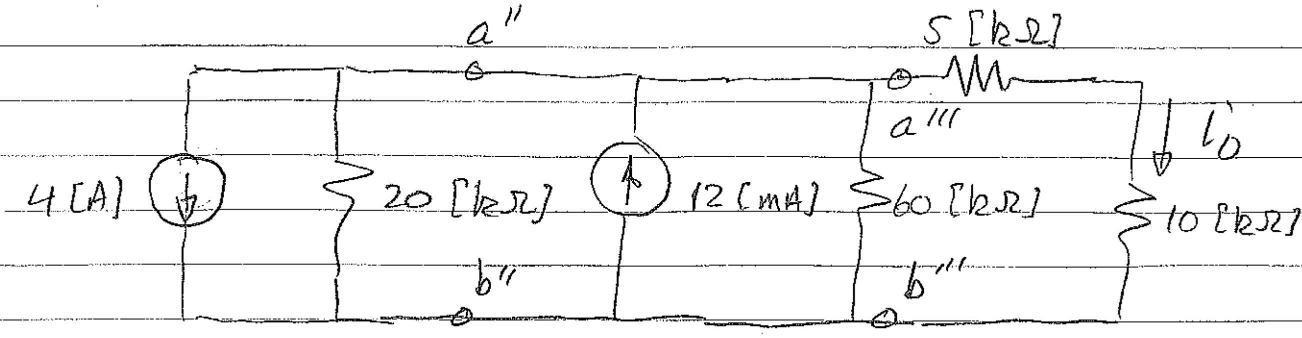




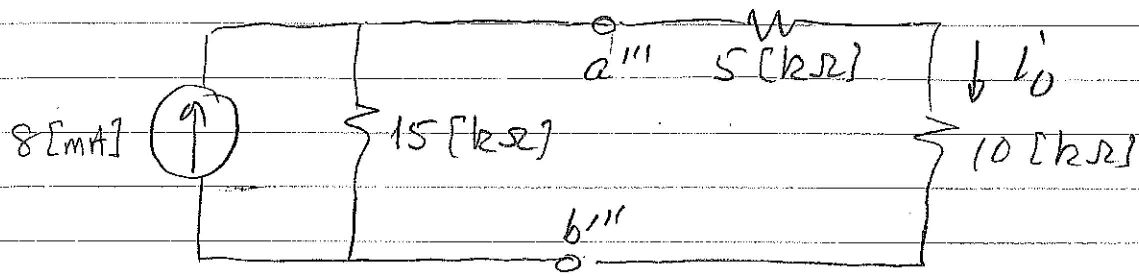
Watch polarities in this transformation!

20 [kΩ] // 80 [kΩ] = 16 [kΩ]





20 [kΩ] // 60 [kΩ] = 15 [kΩ]



We can use KCL to show that 12 [mA] "up" and 4 [mA] "down" is equivalent to 8 [mA] "up".

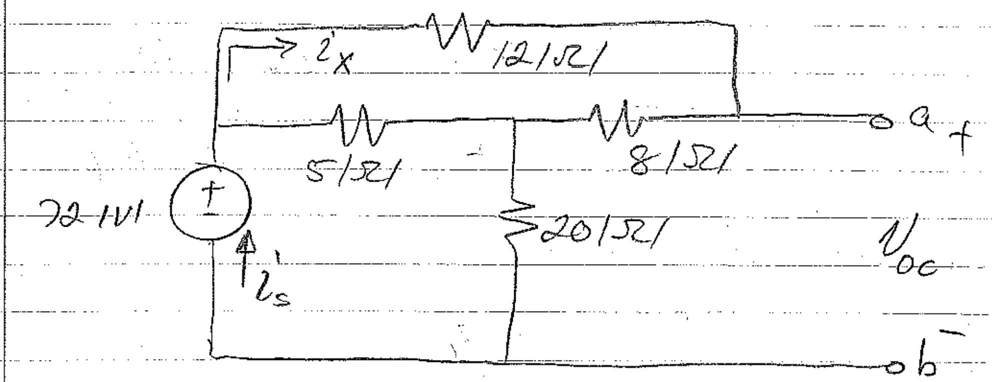
Alternatively, we could have used three more source transformations (2 current - voltage and 1 voltage → current) to arrive at the circuit above. In any case we can now use CDR:

$$\underline{\underline{i_0'}} = 8\,[mA] \cdot \frac{15\,[k\Omega]}{15\,[k\Omega] + 15\,[k\Omega]} = \underline{\underline{4\,[mA]}}.$$

It is important to note that source transformations are done with respect to specific terminals: components outside those terminals remain fixed. As we went through this example, we labelled the terminals we were working with: a, b; a'', b''; etc.

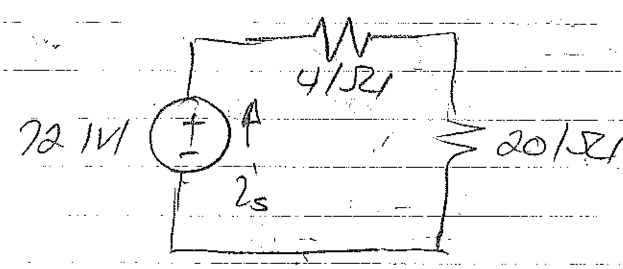
PROBLEM 4.1 (Nilsson & Reidel 8 ed.)

- Find the Thevenin Equivalent at a, b.



Note that a, b is open circuit ⇒ 12 |Ω| is in series with 8 |Ω|.

So ... $(12+8) \parallel 5 = \frac{20 \cdot 5}{25} = 4 |\Omega|$



Of course now we have lost terminals a, b so we need to "unfold" this circuit:

$$i_s' = 72/24 = 3 |A|$$

In original circuit we can use current divider to find i_x:

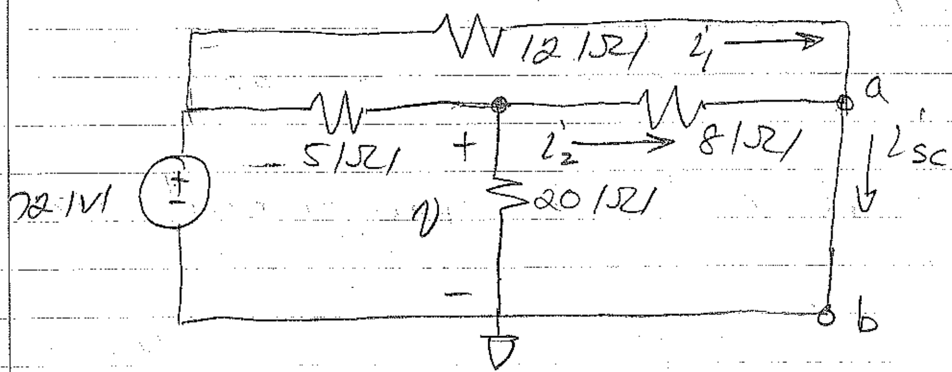
$$i_x = i_s' \cdot \frac{5}{(12+8)+5} = i_s' \cdot \frac{5}{25} = 0.6 |A|$$

Now $\quad v_{oc} - i_s' \cdot 20 - i_x' \cdot 8 = 0$

$$\underline{\underline{v_{oc}}} = 20\, i_s' + 8\, i_x' = \underline{\underline{64.8\ [V]}}$$

$$= v_{Th}$$

Short-circuit current:



Note that we must take i_{sc}' in the direction $a \to b$ if we take v_{oc} with positive polarity at a.

$$\frac{v}{20} + \frac{v}{8} + \frac{v - 72}{5} = 0$$

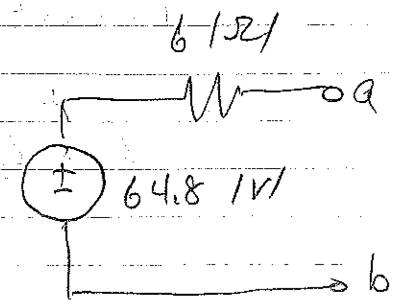
$$\Rightarrow \quad v = 38.4\ [V]$$

$$\therefore \quad i_2' = v/8 = 4.8\ [A]$$

$$i_1' = 72/12 = 6\ [A]$$

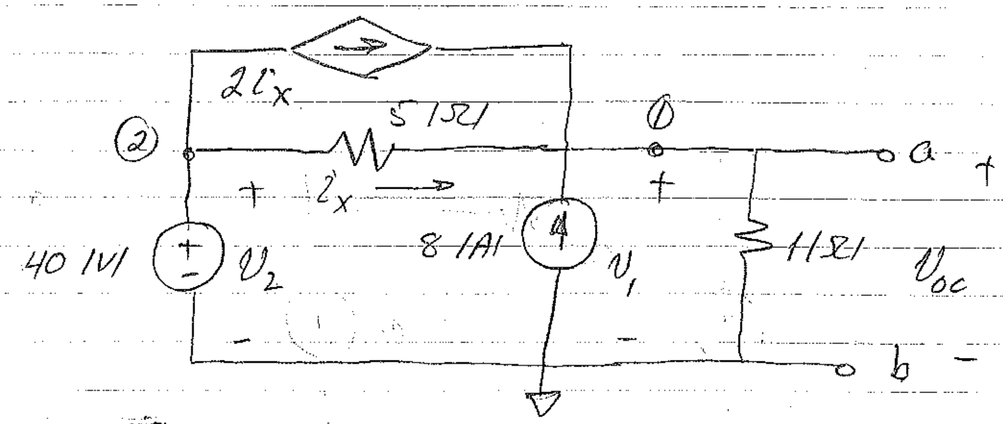
$$\Rightarrow \quad i_{sc}' = i_1' + i_2' = 10.8\ [A]$$

$$\therefore \quad \underline{\underline{R_{Th}}} = v_{oc}/i_{sc} = \underline{\underline{6\ [\Omega]}}$$



PROBLEM 4.2 (Nilsson & Reidel 8ed)

Find the Thevenin Equivalent at a, b.



<u>open-circuit voltage</u>

$$\frac{v_1}{1} - 8 + \frac{v_1 - v_2}{5} - 2i_x = 0$$

$$i_x = \frac{v_2 - v_1}{5}$$

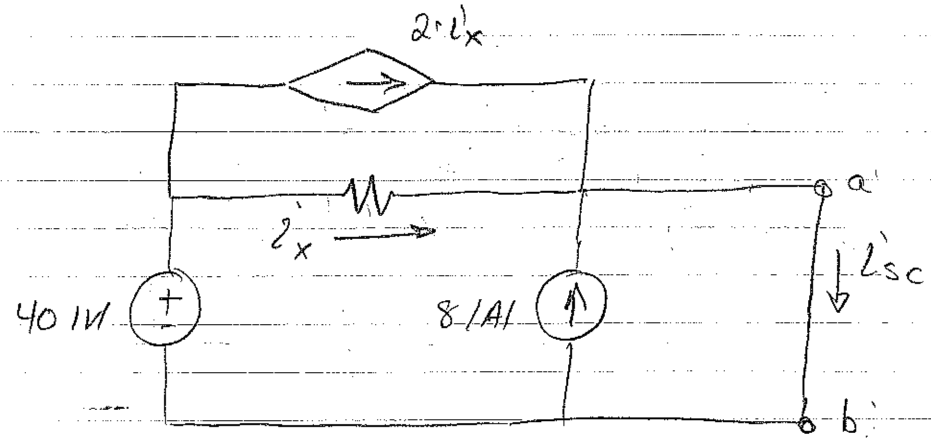
$$v_2 = 40 \ |V|$$

Solution: $v_1 = 20 \ |V|$

$i_x = 4 \ |A|$

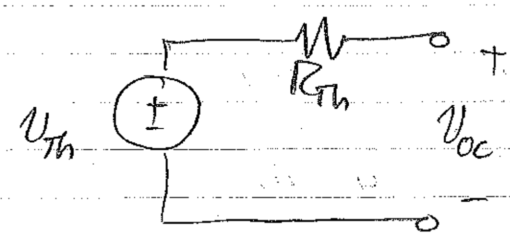
Then $v_{oc} = \underline{\underline{v_{Th}}} = v_1 = \underline{\underline{20 \ |V|}}$

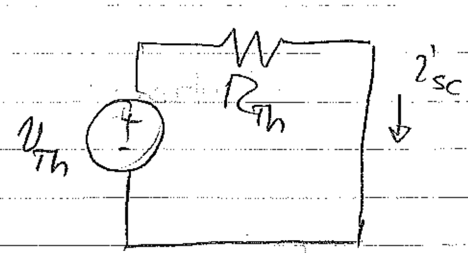
short-circuit current



Note that i_{sc} must be taken in the direction a → b if v_{oc} has positive polarity at a.

why is that? Go back to the Thevenin equivalent:





For $v_{Th} > 0$ and $R_{Th} > 0$, v_{oc} and i'_{sc} are positive as indicated.

Note also that we were able to remove the 1 Ω resistor because the short circuit ⇒ no current through it.

(PROBLEM 4.2 con't)

KVL: $-40 + 5i'_x = 0$

KCL: $i'_{sc} - 8 - i'_x - 2i'_x = 0$

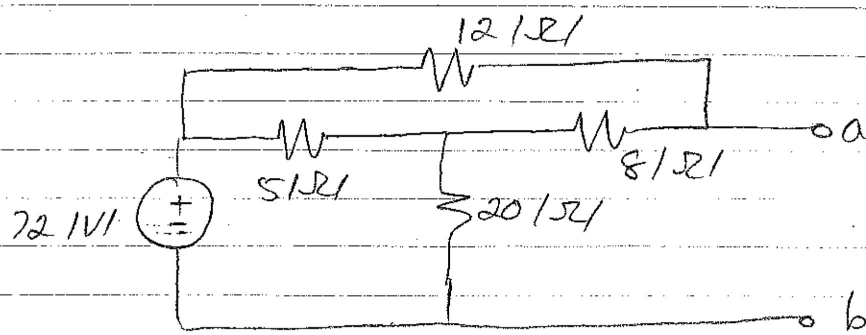
Solution: $i'_x = 8 \, |A|$

$i'_{sc} = 32 \, |A|$

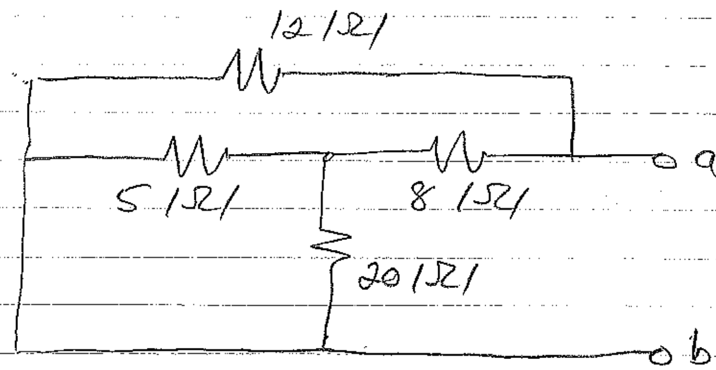
$\therefore \quad R_{Th} = v_{oc}/i'_{sc} = 20/32 = 5/8 \, |\Omega|$.

PROBLEM 4.1 REVISITED : R_{TH} calculation

Use test-source idea — but here is a case where we don't need to apply the test source!

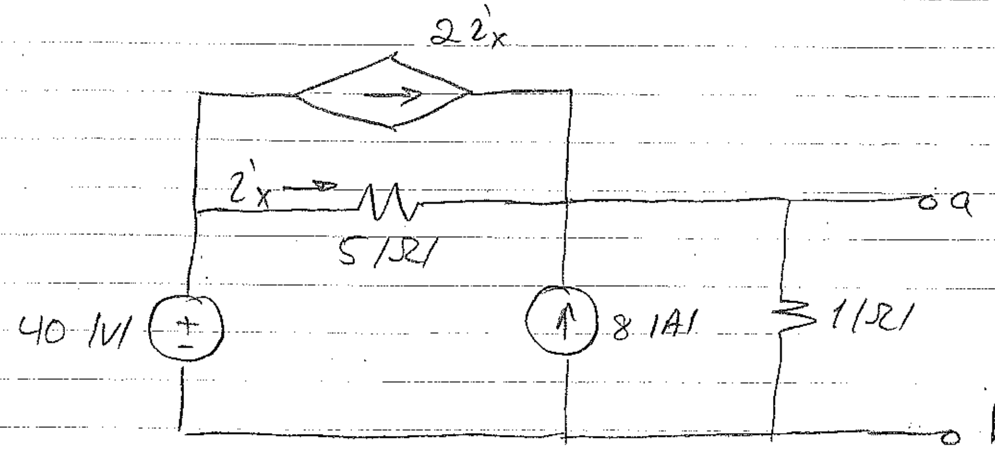


De-activate independent sources:

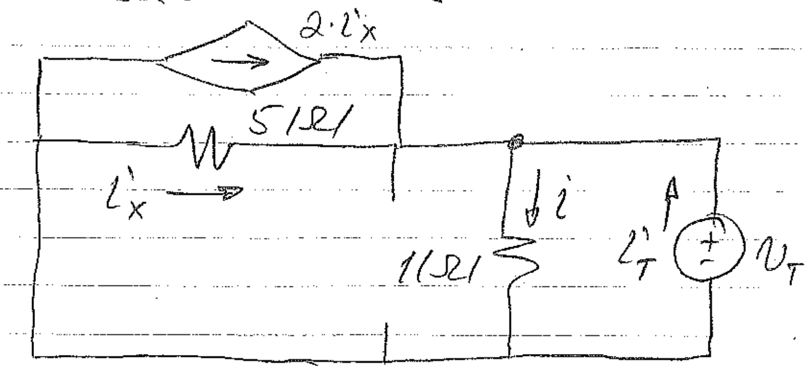


Now $R_{Th} = (20 \| 5 + 8) \| 12 = 6 |\Omega|$

PROBLEM 4.2 REVISITED: R_{TH} calculation



Apply test voltage and de-activate independent sources:



KCL: $-i_T - i_x - 2i_x + i = 0$

$i = v_T/1.$

KVL: $v_T + 5i_x = 0 \Rightarrow i_x = -v_T/5$

$\therefore \quad i_T = -3i_x + \frac{v_T}{1}$

$= 3/5\, v_T + v_T/1 = v_T(3/5 + 1)$

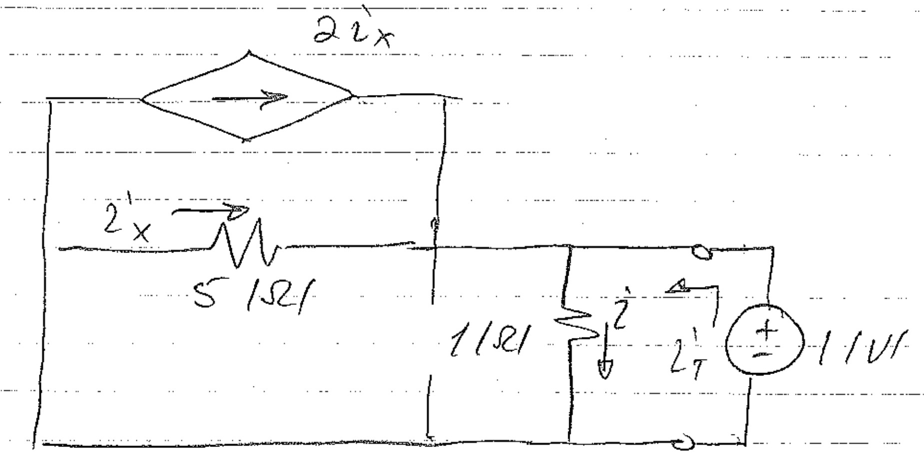
$\Rightarrow \quad v_T/i_T = 1/(8/5) = \underline{\underline{5/8\ \Omega = R_{Th}}}$

(PROBLEM 4.2 REVISITED con't)

Note that we need to find v_T/i_T so we maneuvered our equations to arrive at this in DRILL EX 4.22.

BUT: Since any test source will work, it is often useful to choose a specific value, say 1 [V] (or 1 [A]).

Looking again at DR. EX. 4.23: apply a 1 [V] test source:



Now

$$i = 1\ [A]$$

$$1 + 5i_x = 0 \Rightarrow i_x = -0.2\ [A]$$

$$\therefore \quad i_T = -3i_x + i$$

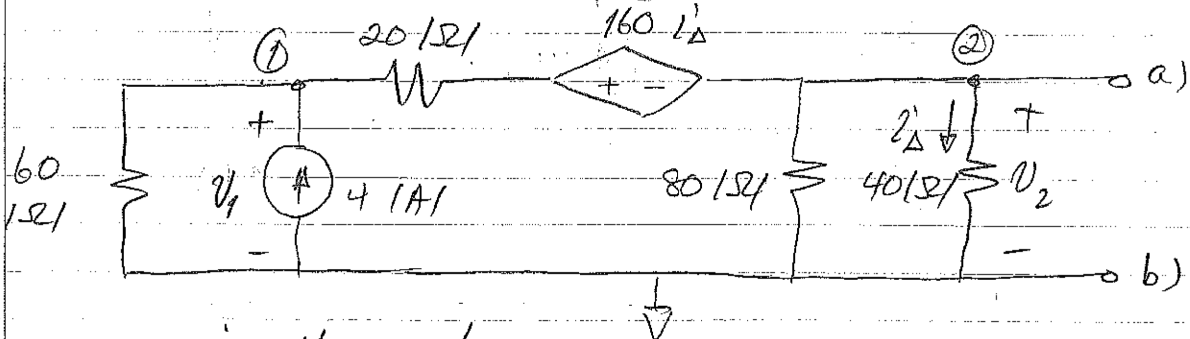
$$= +0.6 + 1 = 1.6\ [A]$$

$$\Rightarrow \quad R_{Th} = v_T/i_T = 1/1.6 = 0.625\ [\Omega]$$

$$(= 5/8\ [\Omega])$$

PROBLEM 4.3 (Nilsson & Riedel, 8 ed)

Find the Thevenin Equivalent at a), b).



open-circuit voltage:

$$\frac{v_2}{40} + \frac{v_2}{80} + \frac{v_2 - v_1 + 160\, i_\Delta'}{20} = 0$$

$$\frac{v_1}{60} - 4 + \frac{v_1 - v_2 - 160\, i_\Delta'}{20} = 0$$

$$i_\Delta' = v_2/40$$

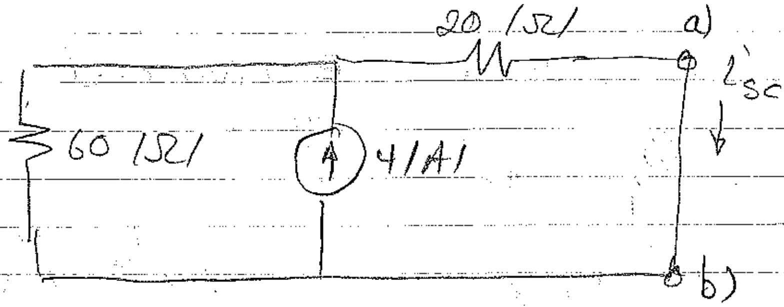
Solution: $v_2 = v_{oc} = \underline{\underline{v_{Th} = 30\ |V|}}$

$$i_\Delta' = 0{,}75\ |A|$$

$$v_1 = 172{,}5\ |V|$$

short-circuit current

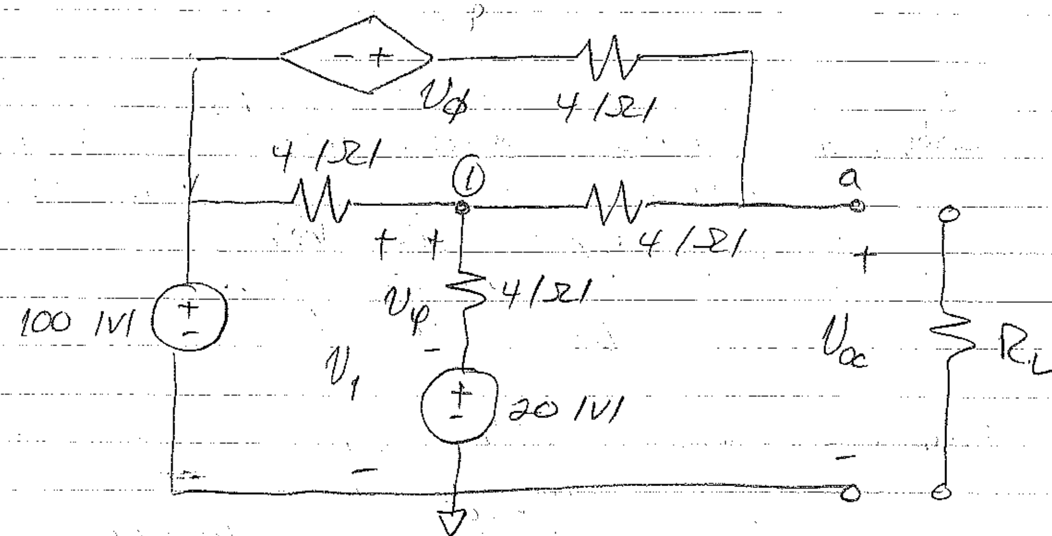
With a), b) short-circuited, there is no current in the 80 |Ω| or 40 |Ω| resistors. Also, $i_\Delta' = 0$, so the circuit simplifies considerably:



Then $i'_{sc} = 4 \cdot \frac{60}{60+20} = 3 |A|$

So $R_{Th} = v_{oc} / i'_{sc} = 10 |\Omega|$.

PROBLEM 4.4 (Nilsson & Reidel 8ed)



a) Find R_L for maximum power transfer
b) Find maximum power transferred.

Since we need both the value of R_L for max power and the max power, we need both R_{Th} and V_{Th}. For this we need two of the following: open circuit voltage; short circuit current; equivalent resistance.

Although only two are required, we will find all three for illustration purposes.

<u>open-circuit voltage</u>

$$\frac{v_1 - 100}{4} + \frac{v_1 - 20}{4} + \frac{v_1 - 100 - v_\phi}{8} = 0$$

(No current flows out through terminal a) under open-circuit conditions, so the two $4\,\Omega$ resistors are in series)

$$v_1 - 20 - v_\varphi = 0$$

Solving these equations:

$$v_1 = 80 \, [V]$$

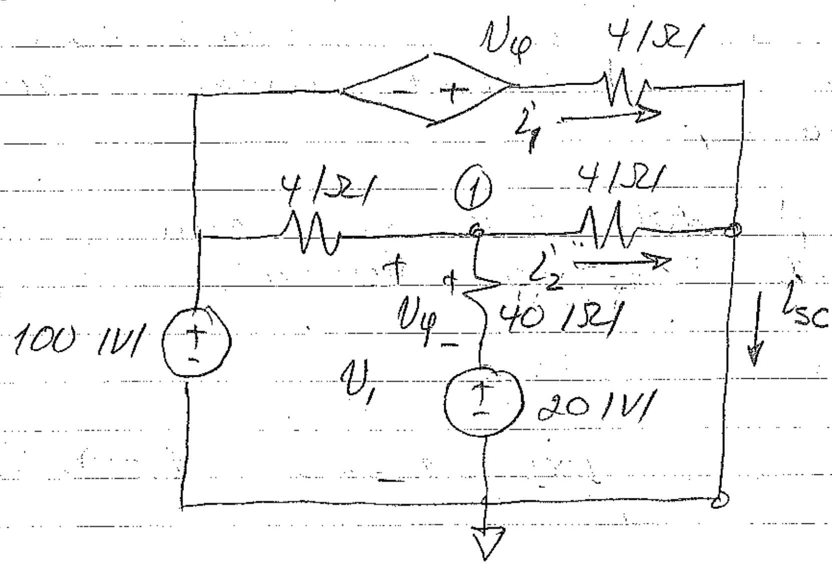
$$v_\varphi = 60 \, [V]$$

Then $\quad v_{oc} = i \cdot 4 + v_1$

$$i = -\frac{v_1 - 100 - 2v_\varphi}{8} = 10 \, [A]$$

$$\therefore \quad \boxed{v_{oc} = 120 \, [V]}$$

Short-circuit current



(PROBLEM 4.4 con't)

$$\frac{v_1 - 20}{4} + \frac{v_1 - 100}{4} + \frac{v_1}{4} = 0$$

$$\Rightarrow \quad v_1 = 40 \, |V|$$

Now

$$i'_{sc} = i'_1 + i'_2$$

KVL: $\quad -100 - v_\varphi + 4 i'_1 = 0$

$$v_\varphi = v_1 - 20 = 20 \, |V|$$

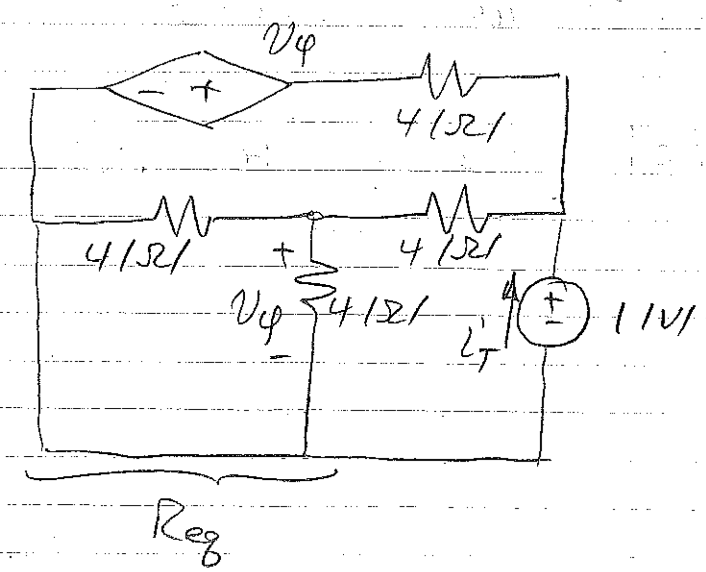
$$\therefore \quad i'_1 = \frac{100 + v_\varphi}{4} = 30 \, |A|$$

$$i'_2 = \frac{v_1}{4} = 10 \, |A|$$

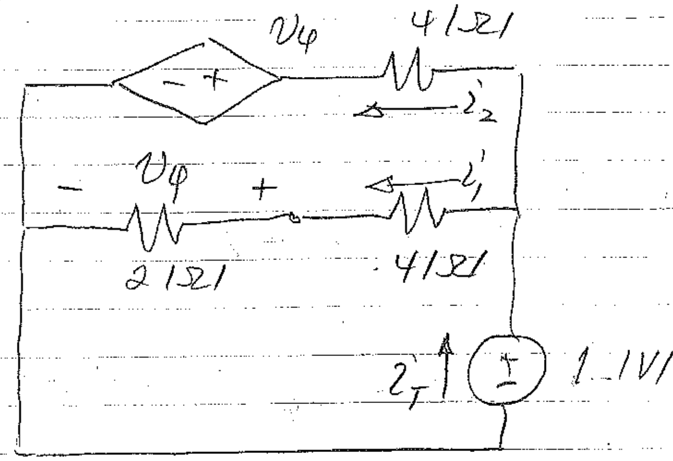
$$\Rightarrow \quad i'_{sc} = 40 \, |\Omega|$$

$$\therefore \quad R_L = \underline{R_{Th}} = v_{oc}/i'_{sc} = 120/40 = \underline{\underline{3 \, |\Omega|}}$$

Try test source for fun:



$R_{eq} = 4 \| 4 = 2\,|\Omega|$



voltage-divider: $v_\varphi = 1 \cdot \frac{2}{4+2} = 1/3\ |V|$

$i_1' = 1/6\ |A|$

$i_2' = \frac{1 - v_\varphi}{4} = \frac{1 - 1/3}{4} = \frac{2/3}{4} = 1/6\ |A|$

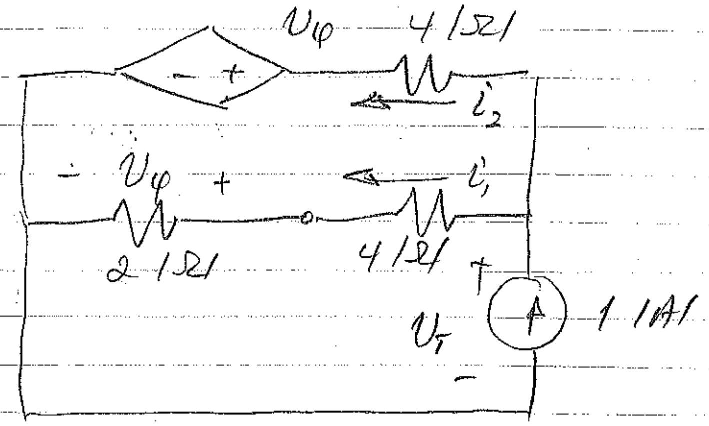
$\therefore \quad i_T' = i_1' + i_2' = 1/3\ |A|$

Finally $\quad R_{Th} = \frac{1\,|V|}{i_T'} = 3\,|\Omega|$

... as we expect.

Let's use a test current source too...

(PROBLEM 4.4 con't)



$$i_1' = v_T/6$$

$$i_2' = \frac{v_T - v_\varphi}{4}$$

$$v_\varphi = 2 \cdot i_1' = v_T/3$$

Then
$$i_1' + i_2' = 1 = v_T/6 + \frac{v_T - v_T/3}{4}$$
$$= v_T/6 + v_T/6 = v_T/3$$

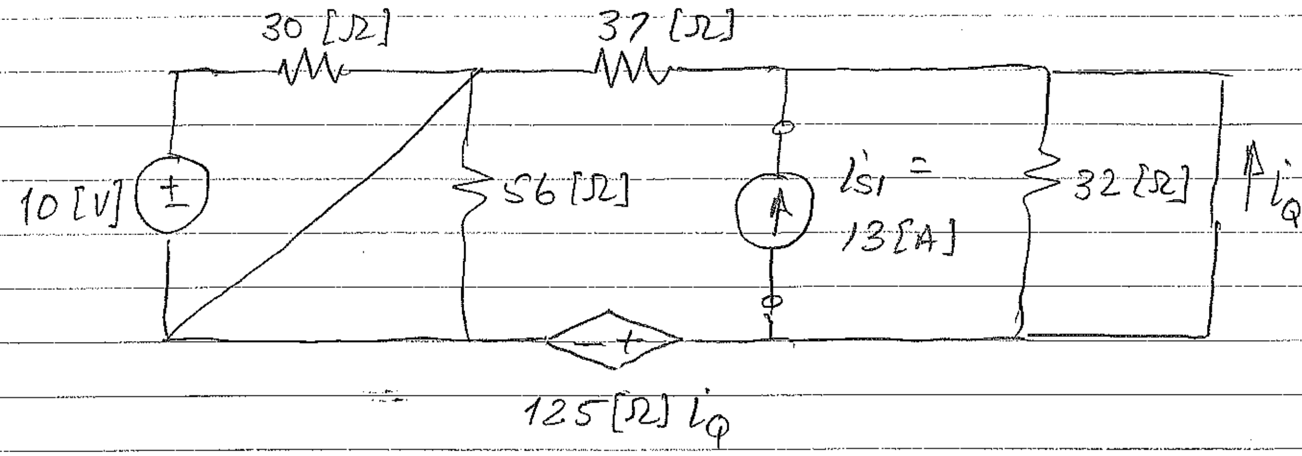
$$\therefore\ v_T = 3\ [V]$$

Finally $\underline{R_{Th}} = v_T/1 = \underline{3\ [\Omega]}$

b)
$$\underline{P_{max}} = \frac{V_{Th}^2}{4 \cdot R_{Th}} = \frac{(120)^2}{4 \cdot 3} = \underline{1200\ [W]}$$

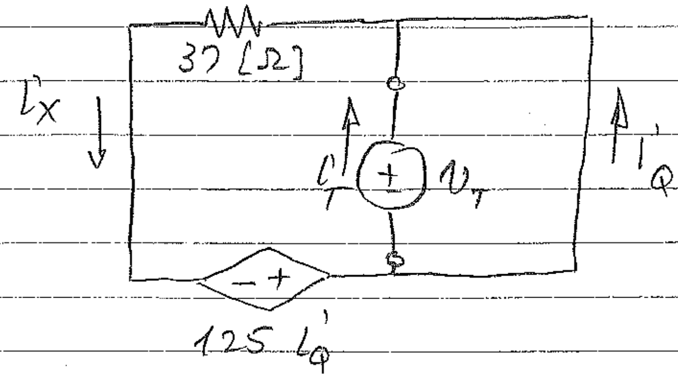
PROBLEM 4.5

- Find the Thevenin Equivalent seen by the current source.



Note: "seen by the current source" means exactly that: what equivalent circuit does the current source see? For that, we need to disconnect the current source because the current source is not part of the equivalent circuit.

For reasons we will see in a moment, we will find R_{TH} with a test source.



We have removed several components that were in parallel with a short.

KVL: $-v_T - 40 i_\varphi' = 0$

Set $v_T = 1\,[V]$ (arbitrary) $\Rightarrow i_\varphi' = -25\,[mA]$

KVL: $-v_T + 37 i_x' - 125 i_\varphi' = 0$

$$\Rightarrow i_x' = \frac{1 + 125(-0.025)}{37} = -57.43\,[mA]$$

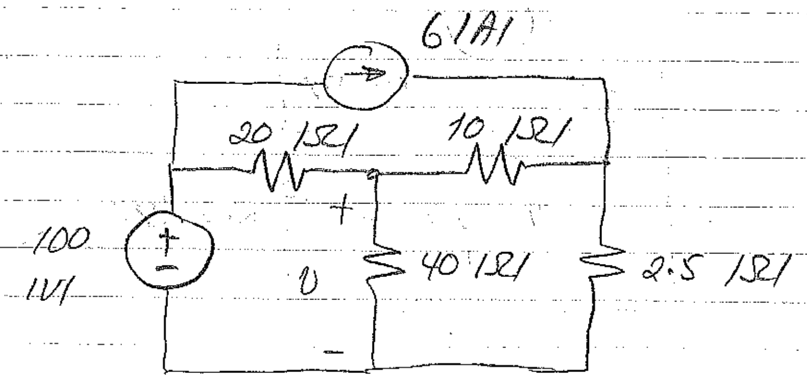
KCL: $i_T' = i_x' - i_\varphi' = -32.405\,[mA]$

$$\therefore \underline{R_{TH} = -30.86\,[\Omega]}$$

Note that the terminals of the Thevenin Equivalent do not "see" any independent sources, which means...

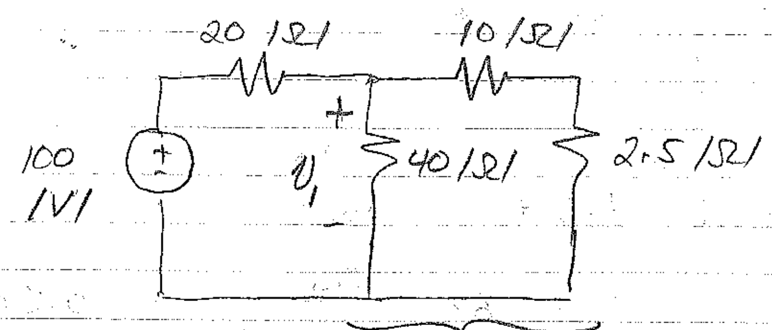
$$\underline{v_{TH} = 0}$$

PROBLEM 4.6



a) Find v using SUPERPOSITION.
b) Find the power dissipated in 40 |Ω| resistor.

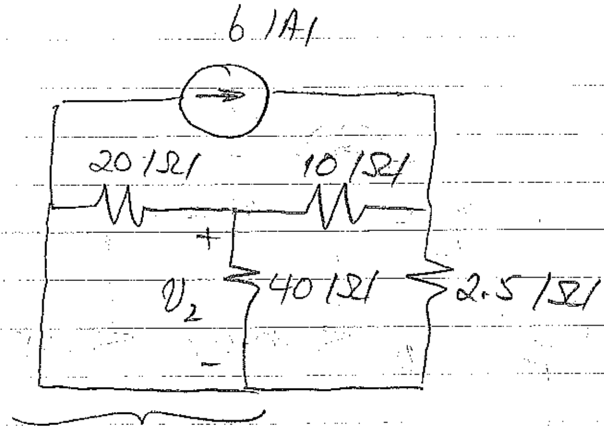
De-activate current source:



$$R_{eq} = (10 + 2.5) \parallel 40 = 9.52\ |\Omega|$$

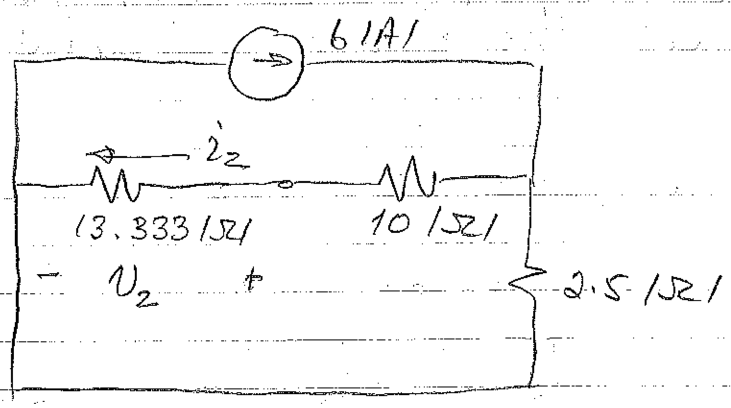
$$v_1 = 100 \cdot \frac{R_{eq}}{R_{eq} + 20} = 32.25\ |V|$$

De-activate voltage source:



$R_{eq} = 40 \| 20 = 13.333$ |Ω|

Now:



current divider rule:

$$i_2' = 6 \cdot \frac{2.5}{(23.333 + 2.5)} = 0.58065 \text{ |A|}$$

$$v_2 = 13.333 \cdot i_2' = 7.742 \text{ |V|}$$

Now

$$v = v_1 + v_2 = 39.992 \text{ |V|}$$

b) $P_{abs} = v^2/R = 39.984$ |W|

IMPORTANT: $P_{abs} \neq \frac{v_1^2}{R} + \frac{v_2^2}{R} = 27.5$ |W|

Reason: $p = v^2/R \neq v_1^2/R + v_2^2/R$