Signature
Date:

ECE 2300 -- Quiz \#2
S.R. Brankovic Section - MW 11:30 AM

October 5, 2005

## KEEP THIS QUIZ CLOSED AND FACE UP UNTIL YOU ARE TOLD TO BEGIN.

1. This quiz is closed book, closed notes. You may use one 8.5 " x 11 " crib sheet, or its equivalent.
2. Show all work on these pages. Show all work necessary to complete the problem. A solution without the appropriate work shown will receive no credit. A solution which is not given in a reasonable order will lose credit. If your work continues on to another page, indicate clearly where your work can be found. Failure to indicate this clearly will result in a loss of credit.
3. Show all units in solutions, intermediate results, and figures. Units in the quiz will be included between square brackets.
4. Do not use red ink. Do not use red pencil.
5. You will have 25 minutes to work on this quiz.

## Problem \#1.

Imagine the situation where you are at your home, let say, doing your hobby project. You have a broken multimeter which you want to fix. You know the range of this multimeter from the original manufacturer specification. If connected in parallel to the circuit over terminals A and B it can measure voltages up to 1550 mV . When used as an ammeter the maximum range is 30 mA . You also know that essential part of this multimeter, the real d'Arsonval meter hidden inside of the device is working fine and has the maximum range of 5 mA and $/$ or 50 mV . The design of the multimeter is such that the extended range of this multimeter on terminals A and B is achieved by having the certain number of identical resistors, $R_{1}, R_{2}, \ldots . . R_{n-1}, R_{n}$ in parallel to d'Arsonval meter. The same number of these identical resistors is connected in serial to this parallel connection as well (see Figure 1). In order to fix your multimeter and restore its ability to have measuring range of 1550 mV and 30 mA you need to find:
a) what is the original number of resistors ( $n$ ),
b) what is the value of resistance $R_{\chi}$. (Note, that $R_{1},=R_{2,}, \ldots=R_{n-1}=R_{n}=R_{x}$ )
c) In addition to that, your friend has asked you to use this meter and he asked you to sketch for him the multiplying factor of ammeter and voltmeter as a function of the number of resistors $n$, assuming the original $R_{x}$ value.


Figure 1.

## Solution:

This is an easy but little tricky problem. Let's have a careful look to the text and Figure 1. If we have done that, we can see that we can simplify our circuit if we replace all resistors in parallel connection with meter by their equivalent resistance;
$\frac{1}{R_{E Q 1}}=\frac{1}{R_{1}}+\frac{1}{R_{2}}+\ldots \ldots . . \frac{1}{R_{n-1}}+\frac{1}{R_{n}} ;\left(R_{1}=R_{2}=. . R_{n-1}=R_{n}=R_{X}\right)$
$\Rightarrow \frac{1}{R_{E Q 1}}=\frac{n}{R_{X}} \Rightarrow R_{E Q 1}=\frac{R_{X}}{n}$

The same can be done for the resistors connected in serial,
$R_{E Q 2}=R_{1}+R_{2}+\ldots \ldots . . R_{n-1}+R_{n} ;\left(R_{1}=R_{2}=. . R_{n-1}=R_{n}=R_{X}\right)$
$\Rightarrow R_{E Q 2}=n \cdot R_{X}$
Also, it is straightforward to find d'Arsonval meter's resistance using Ohm's law and its maximum measuring range;
$R_{M}=\frac{50[\mathrm{mV}]}{5[\mathrm{~mA}]}=10[\Omega]$
Now, the circuit looks little simpler (Figure 2);
Multimeter


Figure 2. Simplified and labeled circuit from Figure 1.

If we connect this multimeter in serial to the possible circuit over terminals A and B, the current from the circuit $I_{t}$ will be split over the parallel connection on the current running $V_{A B}$ through d' Arsonval meter $I_{M}$ and the current running through the resistors in parallel $I_{p}\left(R_{E Q 1}\right.$ $=R_{\chi} / n$.) Using the current divider rule, or the formula for the extended range ammeters we can write expression for current $I_{t}$ as:

$$
\begin{equation*}
I_{t}=I_{M} \frac{R_{M}+R_{E Q 1}}{R_{E Q 1}}=I_{M} \frac{10[\Omega]+\frac{R_{x}}{n}}{\frac{R_{x}}{n}} \tag{4}
\end{equation*}
$$

Substituting values for current $I_{M}$ as the maximum measuring current of d' Arsonval meter ( 5 mA ) and for $I_{t}$ the maximum measuring current of our multimeter ( 30 mA ) we get:

$$
\begin{equation*}
30 m A=5 m A \frac{10[\Omega]+\frac{R_{x}}{n}}{\frac{R_{x}}{n}} \Rightarrow \frac{R_{X}}{n}=2[\Omega], \tag{5}
\end{equation*}
$$

Obviously this ratio does not give us an explicit solution for $R_{X}$ and $n$, and we need an additional equation in order to get this answer. Let's look the voltage $v_{A B}$ that would be on the terminals A and B if our multimeter is connected in parallel to some possible circuit. The voltage $v_{A B}$ will be divided on the voltage across the resistors in serial connection $v_{s}\left(R_{E Q 2}\right)$ and voltage on the terminals of the parallel connection $v_{p}$. Using the voltage divider rule or expression for extended range voltmeter we can write:

$$
\begin{equation*}
v_{A B}=v_{p} \cdot \frac{R_{E Q 2}+\left(R_{\mathrm{M}}| | R_{E Q 1}\right)}{\left(R_{\mathrm{M}}| | R_{E Q 1}\right)}=v_{p} \cdot \frac{n R_{x}+\left(10[\Omega]| | \frac{R_{x}}{n}\right)}{\left(10[\Omega]| | \frac{R_{x}}{n}\right)}=v_{p} \cdot \frac{n R_{x}+(10[\Omega]| | 2[\Omega])}{(10[\Omega] \mid 2[\Omega])} \tag{6}
\end{equation*}
$$

Substituting values for $v_{p}$ as the maximum voltage of $d^{\prime}$ Arsonval meter ( 50 mV ) and for $v_{A B}$ the maximum voltage of our multimeter ( 1550 mV ) we get:

$$
\begin{equation*}
1550 \mathrm{mV}=50 \mathrm{mV} \cdot \frac{n R_{x}+\left(\frac{20}{12}[\Omega]\right)}{\left(\frac{20}{12}[\Omega]\right)} \Rightarrow n R_{x}=50[\Omega] \tag{7}
\end{equation*}
$$

And combining (7) and (5) we get;

$$
\begin{aligned}
& n=+\sqrt{25} \\
& \hline n=5 \\
& \hline \text { and } \\
& \hline R_{x}=10[\Omega]
\end{aligned}
$$

Now, since we need to sketch the multiplying factor dependence on $n$ for our voltmeter and ammeter, using $R_{x}=10[\Omega]$, form equation (4) and (6) we get;

$$
\begin{array}{|l|}
\hline \frac{I_{t}}{I_{M}}=1+n, \\
\hline \frac{v_{A B}}{v_{p}}=n^{2}+n+1 \\
\hline
\end{array}
$$

The graphical presentation is shown below:


