Name: ____ **(Print)**

Signature ____

Date: ______________

ECE 2300 -- Quiz #6

S.R. Brankovic Section – MW 11:30 AM
Dec. 5th, 2005

KEEP THIS QUIZ CLOSED AND FACE UP UNTIL YOU ARE TOLD TO BEGIN.

1. This quiz is closed book, closed notes. You can have one crib sheet.
2. Show all work on these pages. Show all work necessary to complete the problem. A solution without the appropriate work shown will receive no credit. A solution which is not given in a reasonable order will lose credit. **<u>If your work continues on to another page, indicate clearly where your work can be found. Failure to indicate this clearly will result in a loss of credit.</u>**
4. Show all units in solutions, intermediate results, and figures. Units in the quiz will be included between square brackets.
5. Do not use red ink. Do not use red pencil.
6. You will have 25 minutes to work on this quiz.

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Problem #1.

The variable resistor in the circuit shown below is adjusted until the average power it absorbs is maximum.

a) Find the value of R
b) Find the maximum average power the resistor absorbs.

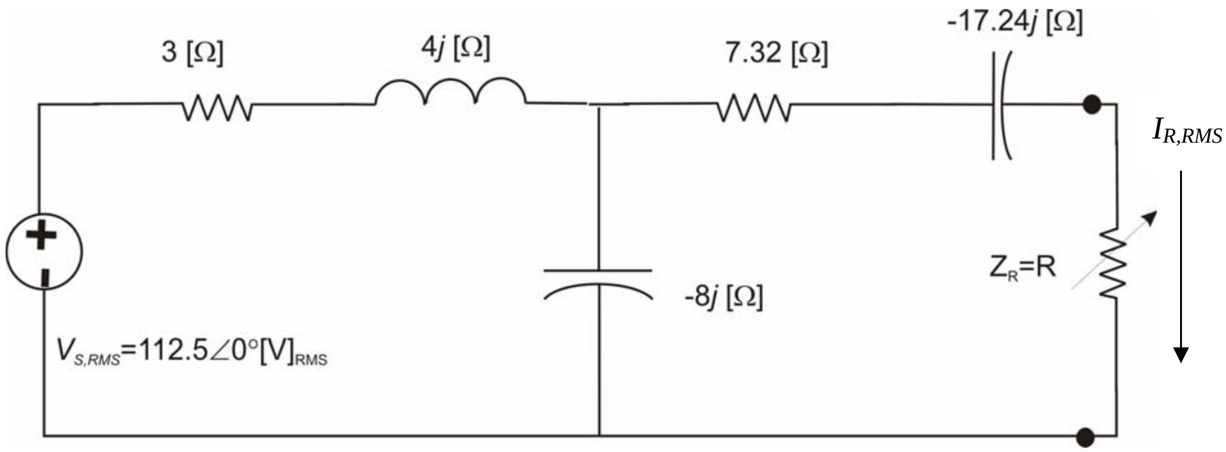

Figure 1.

Solution

There are several ways to solve this problem. The shortest one is presented in the book on pg 473-475. However, since that material was not covered in the class, we will have to go a little longer way to get to the solution. Also we will learn some new things too. So here we go:

First we will need to find the current $I_{R,RMS}$ that goes through the impedance $Z_R=R$, and, of course, this current will be expressed in terms of R. So, to do that, we need to simplify the circuit, or we could use some of the methods learned earlier, like mesh currents or node voltage. I will go by simplifying the circuit first.

The straight forward way to do that is to replace the part of the circuit connected to the Z_R by it's Thevenin's equivalent. The Z_{Th} we will calculate as we would do in the case of the regular DC circuit but combining the impedances with the voltage source taken out (Figure 2).

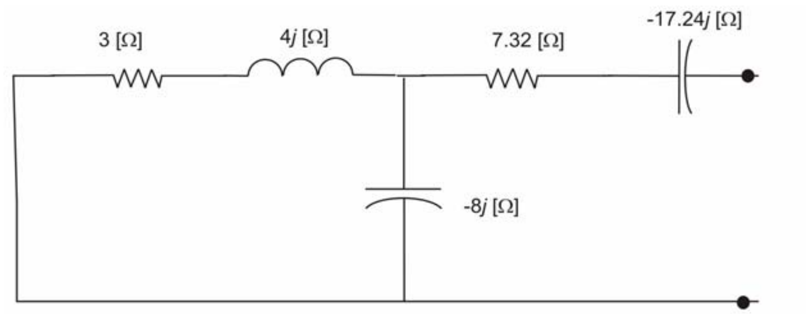

Figure 2.

$$Z_{Th} = \left[(3+4j) \| -8j\right] + 7.32 - 17.24j = \boxed{(15-15j)[\Omega]} \qquad (1)$$

The V_{Th} we can find just applying the voltage divider rule:

$$V_{Th\,RMS} = V_{S,RMS} \cdot \frac{-8j}{3+4j-8j} = 112.5\angle 0° \cdot \frac{-8j}{3-4j} = \boxed{(144-108j)[V]_{RMS}} \qquad (2)$$

Now, the simplified circuit is shown in the Figure 3.

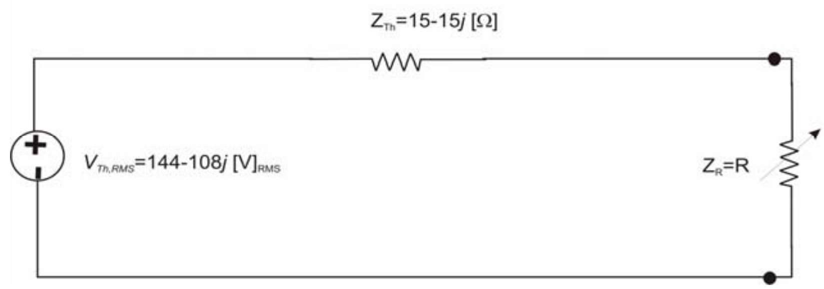

Figure 3

The average power absorbed by Z_R is given by:

$$P_{abs,by,Z_R} = \left|\bar{I}_{R,RMS}\right|^2 R \tag{3}$$

The current $I_{R,RMS}$ is given by (Figure 2);

$$I_{R,RMS} = \frac{V_{Th,RMS}}{Z_{Th} + Z_R} = \frac{V_{Th,RMS}}{Z_{Th} + R} = \boxed{\frac{144 - 108j}{15 + R - 15j}\ \text{[A]}_{\text{RMS}}} \tag{4}$$

The square of the module of the $I_{R,RMS}$ is then

$$\left|\bar{I}_{R,RMS}\right|^2 = \frac{144^2 + 108^2}{(15+R)^2 + 15^2} = \boxed{\frac{32400}{450 + 30R + R^2};\ \text{[A]}^2_{\text{RMS}}} \tag{5}$$

The average power absorbed by Z_R (resistor) is:

$$P_{abs,by,Z_R} = \frac{32400 \cdot R}{450 + 30R + R^2} \tag{6}$$

The maximum of this function in terms of R will be if condition

$$\frac{\partial P_{abs,by,Z_R}}{\partial R} = 0, \text{ is fulfilled.} \tag{7}$$

This is when

$$\frac{32400 \cdot (450 + 30R + R^2) - (30 + 2R) \cdot 32400R}{(450 + 30R + R^2)^2} = 0 \tag{8}$$

or,

$$32400 \cdot (450 + 30R + R^2) - (30 + 2R) \cdot 32400R = 0 \tag{9}$$

After dividing the equation (9) with 32400, and performing some algebra one gets

$$\begin{gathered} 450 - R^2 = 0 \Rightarrow R = +\sqrt{450} \\ \boxed{R = 21.21[\Omega]} \end{gathered} \tag{10}$$

Using the equation (6) and estimated value for R the maximum average power absorbed by resistor is:

$$P_{abs,by,Z_R} = \frac{32400 \cdot R}{450 + 30R + R^2} = \frac{32400 \cdot 21.21}{450 + 30 \cdot 21.21 + 21.21^2} \boxed{= 447.35[W]} \qquad (11)$$

Important thing here is to note, that the value of

$$R^2 = \left|\bar{Z}_R\right|^2 = 450[\Omega^2]$$

And that

$$\left|\bar{Z}_{Th}\right|^2 = 450[\Omega^2]$$

Obviously one could see that the maximum average power absorbed by the resistor is when the magnitude of the Thevenin's impedance is equal to the magnitude of the resistor impedance:

$$\left|\bar{Z}_{Th}\right| = \left|\bar{Z}_R\right|$$

In most general case when the impedance of the load involves resistance and reactance $Z_L=R+X\cdot j$, this condition is defined when the conjugate Thevenin's impedance is equal the load's impedance:

$$Z^*_{Th} = Z_L$$

So, we covered an extra lecture very useful in your future endeavors of Electrical Engineering.