Name: ______________________________ (please print)

Signature: ___________________________

ECE 2202 –Exam 2
November 14, 2020

Online

1. This quiz is open book, open notes.
2. Show all work necessary to complete the problem. A solution without the appropriate work shown will receive no credit. A solution which is not given in a reasonable order will lose credit.
3. Show all units in solutions, intermediate results, and figures. Units in the quiz will be included between square brackets.
4. If the grader has difficulty following your work because it is messy or disorganized, you will lose credit.
5. Do not use red ink. Do not use red pencil.
6. You will have 90 minutes to work on this exam, and 15 minutes to download/print, scan and submit.
7. You MUST use LOWER-CASE letters for time domain variables, and UPPER-CASE LETTERS WITH AN OVERBAR for phasor domain variables. Significant credit will be subtracted if you do not follow this rule.

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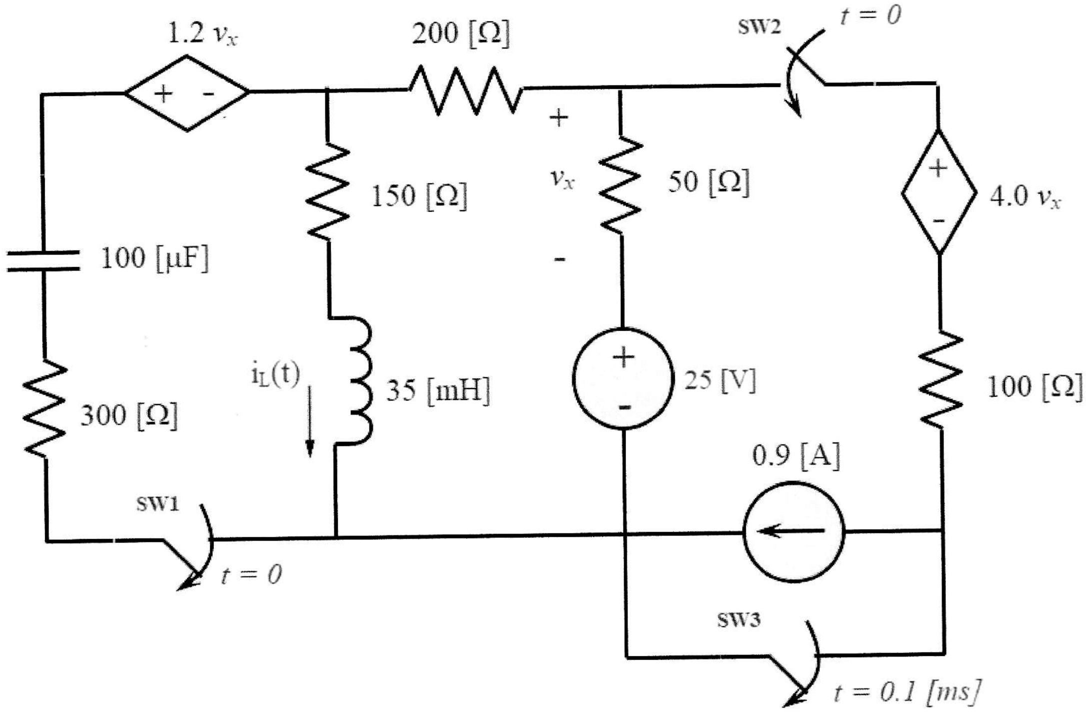
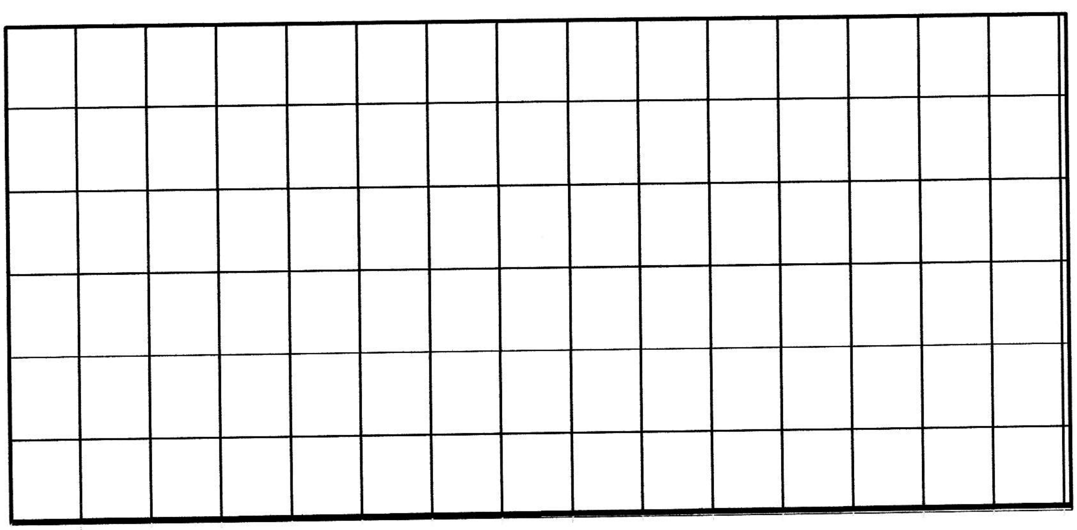
Room for extra work

1. [40 points] In the circuit below, switch SW1 was closed for a long time and then opened at t = 0. Switch SW2 was open for a long time and then closed at t = 0. Switch SW3 was closed for a long time and then opened at t = 0.1 [ms].

a) Find expressions for the inductor current as a function of time for $t \geq 0$.
b) Using the graph shown below the figure, plot $i_L(t)$ for $t \geq 0$. To receive full credit, your plot must be neat, and include axis titles and numbers with appropriate units.



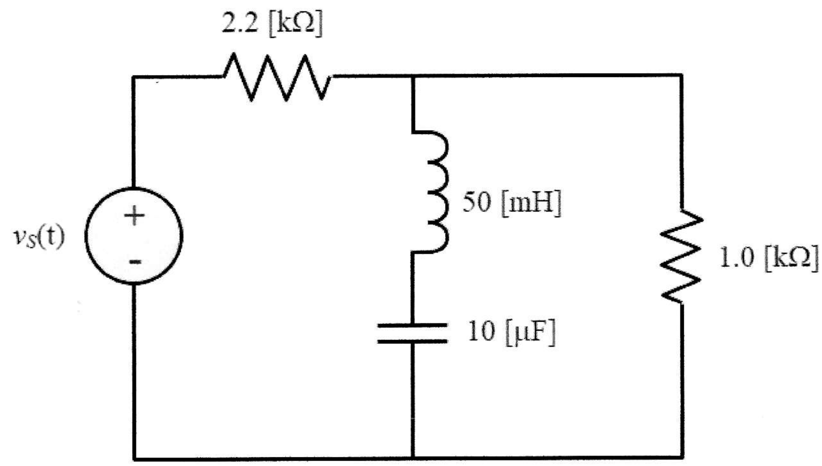


Room for extra work

Room for Extra Work

2. [25 points] The source in the circuit below is given by $v_S(t) = 15$ [V] $\cos(\omega t)$. The circuit is operating in the steady state.

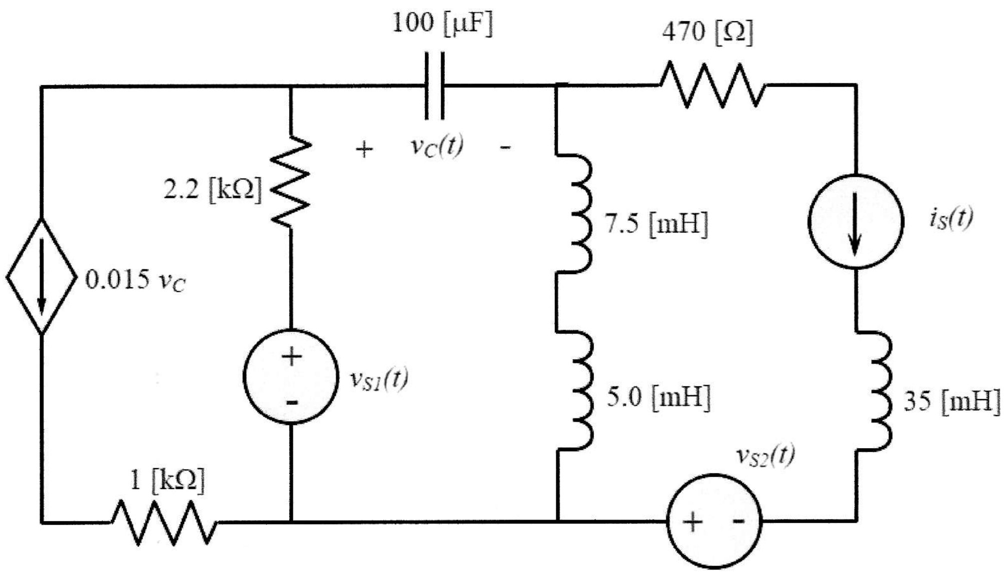
a) Find the Thevenin equivalent impedance seen by $v_S(t)$ at a frequency of $\omega = 1500$ [rad/s].
b) Convert the Thevenin impedance you found in a) to the time domain by choosing appropriate circuit elements in the time domain.
c) Find the frequency ω at which the Thevenin equivalent impedance seen by $v_S(t)$ is purely real.
d) Convert the Thevenin impedance at the frequency you found in c) to the time domain by choosing appropriate circuit elements in the time domain.



Room for extra work

3. [35 points] The circuit below is operating in steady state. Find the voltage v_C(t). The values of the sources are given as follows.

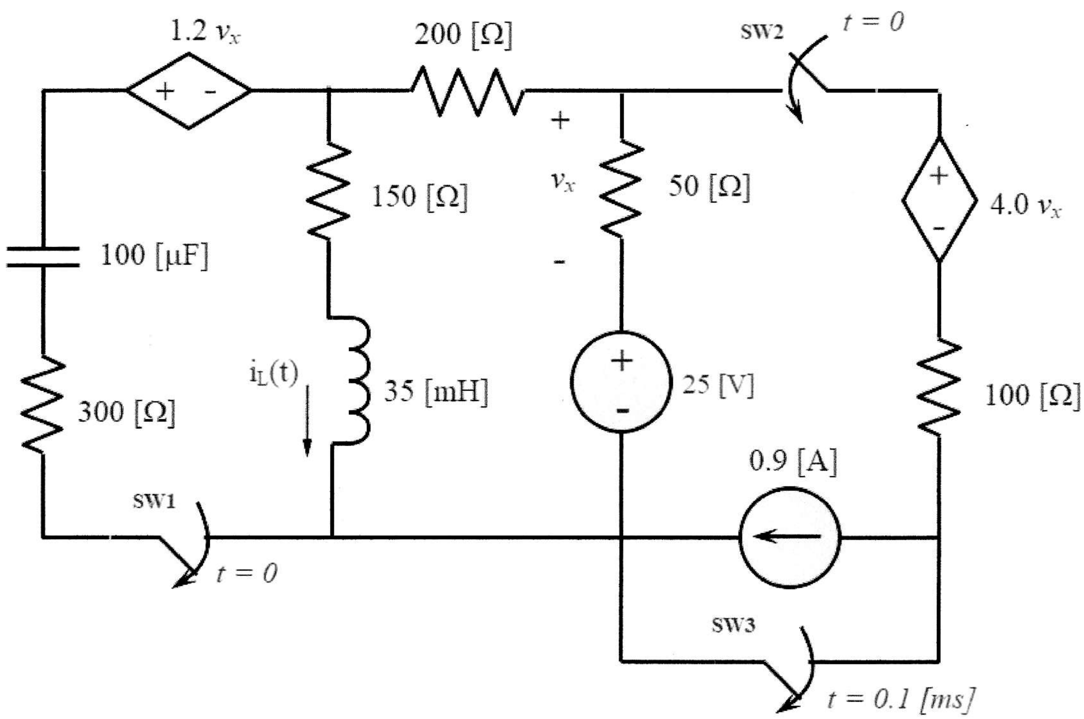
$$v_{S1}(t) = 15 \text{ [V]}$$
$$v_{S2}(t) = 30 \text{ [V]}\cos(750t)$$
$$i_S(t) = 2 \text{ [A] } \sin(1500t + 30^o)$$

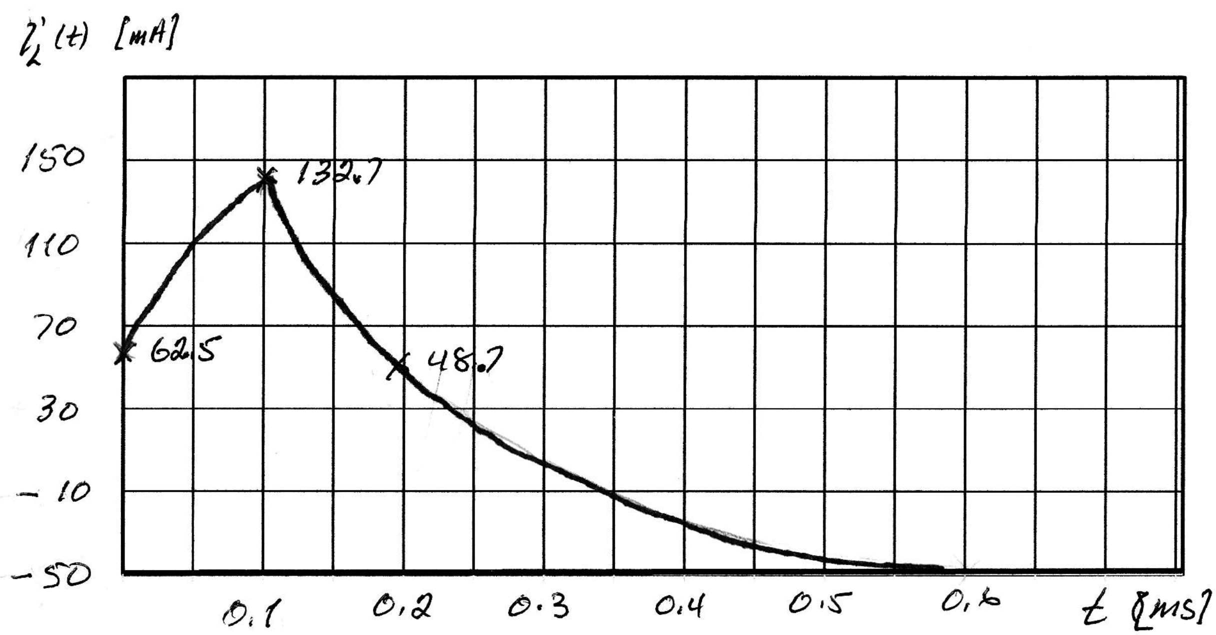


Room for extra work

1. [40 points] In the circuit below, switch SW1 was closed for a long time and then opened at t = 0. Switch SW2 was open for a long time and then closed at t = 0. Switch SW3 was closed for a long time and then opened at t = 0.1 [ms].

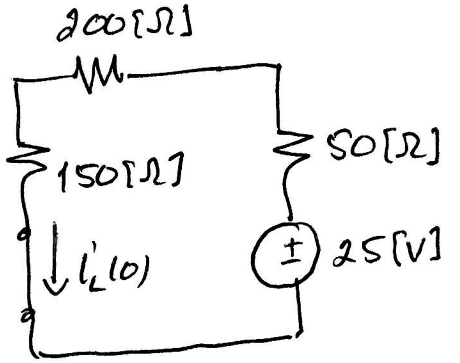
a) Find expressions for the inductor current as a function of time for $t \geq 0$.
b) Using the graph shown below the figure, plot $i_L(t)$ for $t \geq 0$. To receive full credit, your plot must be neat, and include axis titles and numbers with appropriate units.





Room for extra work

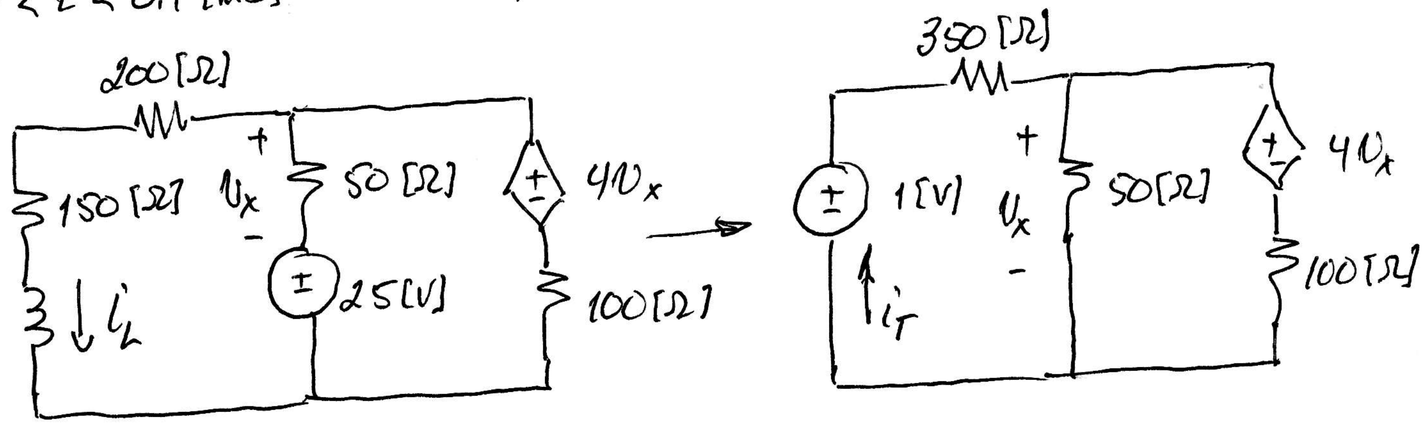
a) Consider the circuit for $t<0$: SW1 is closed but we are in steady state so the capacitor is open-ckt. SW2 is open...



L → short

$$i_L'(0) = \frac{25}{400} = 62.5\ [mA]$$

$0 < t < 0.1$ [ms] SW1 opens, SW2 closes. Apply test source:



$$\frac{v_x}{50} + \frac{v_x - 1}{350} + \frac{v_x - 4v_x}{100} = 0 \Rightarrow v_x = -0.4\ [V]$$

$$i_T = -\frac{v_x - 1}{350} = 4\ [mA] \Rightarrow R_{TH} = 250\ [Ω] \Rightarrow \tau_L = L/R_{TH} = 0.14\ [ms]$$

For final value, L → short.

$$\frac{v_x + 25}{350} + \frac{v_x}{50} + \frac{v_x + 25 - 4v_x}{100} = 0 \Rightarrow v_x = 45\ [V]$$

$$i_{L,f}' = \frac{v_x + 25}{350} = 0.2\ [A]$$

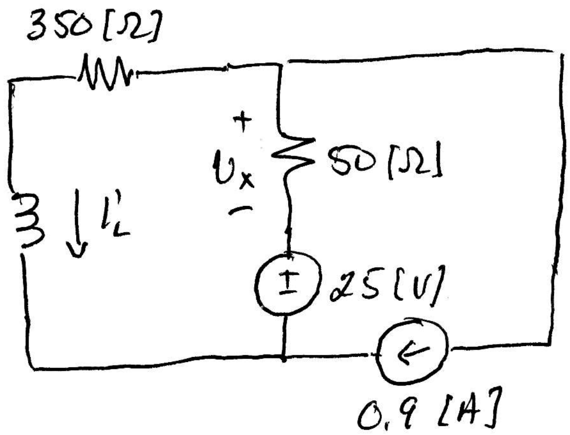
Room for Extra Work

$$i_L(t) = i_{L,f} + (i_L(0) - i_{L,f})e^{-t/\tau_L}$$

$$\boxed{i_L(t) = 0.2 + (0.0625 - 0.2)e^{-t/0.14\,[ms]}\ [A]} \quad 0 < t < 0.1\ [ms]$$

$$i_L(t = 0.1\,[ms]) = 0.2 + (-0.1375)e^{-0.1/0.14}\ [A] = ~~0.1327~~\ 132.7\ [mA]$$

$t > 0.1$ [ms] SW3 opens $L \to$ short $\Rightarrow$



$$\frac{v_x + 25}{350} + \frac{v_x}{50} + 0.9 = 0$$

$$v_x = -42.5\ [V]$$

$$i_{L,f} = \frac{v_x + 25}{350} = -0.05\ [A]$$

By inspection, $R_{TH} = 400\ [\Omega] \Rightarrow \tau_L = L/R_{TH} = 87.5\ [\mu s]$

$$i_L(t) = (i_L(0.1\,[ms]) - i_{Lf})e^{-(t - 0.1\,[ms])/0.0875\,[ms]} + (-0.05)\ [A] \quad t > 0.1\ [ms]$$

$$\boxed{i_L(t) = -0.05 + 0.1827e^{-(t-0.1)/0.0875}\ [A]} \quad t > 0.1\ [ms]$$

$$i_L(t) = \begin{cases} 0.2 - 0.1375e^{-t/0.14}\ [A] & 0 < t < 0.1\ [ms] \\ -0.05 + 0.1827e^{-(t-0.1)/0.0875}\ [A] & t \geq 0.1\ [ms] \end{cases}$$

$\nearrow$ p. 2

Room for extra work

How should we approach a plot of these functions?
We can note a few important points:

$t = 0 \Rightarrow i_L'(t) = 62.5$ [mA]

$t = 0.1$ [ms] $\Rightarrow i_L'(t) = 132.7$ [mA]

$t \to \infty \Rightarrow i_L(t) = -0.05$ [mA]

This is really all we need to make a rough plot, but we can identify one more point to help. After 1 time constant, $i_L'(t)$ for $t > 0.1$ [ms] has decayed to

$132.7e^{-1} \simeq 49$ [mA] This point happens at

$t = 0.1 + 0.0875 = 0.1875 \sim 0.2$ [ms].

We have indicated these points on the graph. Any reasonable approximation to this drawing receives full credit.

2. [25 points] The source in the circuit below is given by $v_S(t) = 15$ [V] $\cos(\omega t)$. The circuit is operating in the steady state.

a) Find the Thevenin equivalent impedance seen by $v_S(t)$ at a frequency of $\omega = 1500$ [rad/s].
b) Convert the Thevenin impedance you found in a) to the time domain by choosing appropriate circuit elements in the time domain.
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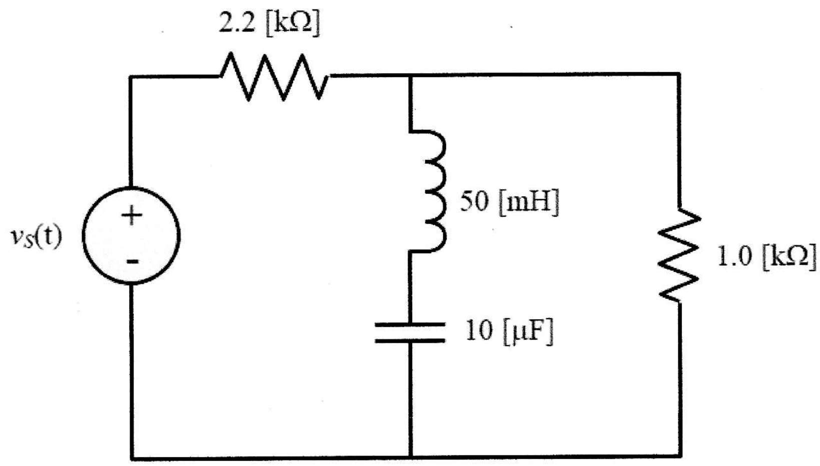
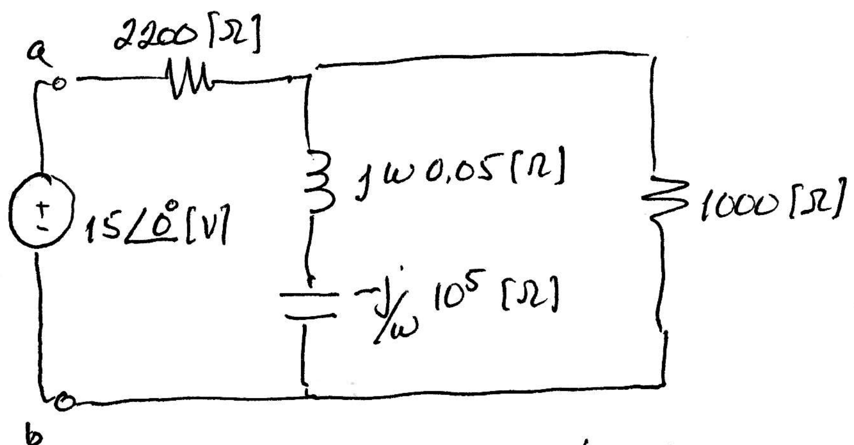
d) Convert the Thevenin impedance at the frequency you found in ~~b~~ c) to the time domain by choosing appropriate circuit elements in the time domain.



a) We draw the circuit in the phasor domain with impedances expressed as a function of ω.



So $Z_{TH} = 2200 + \left(j\omega 0.05 - \frac{j}{\omega}10^5\right) // 1000$

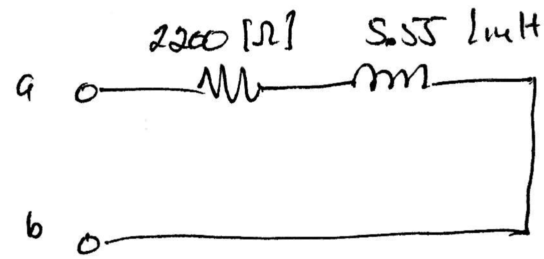
Room for extra work

For $\omega = 1500 \, [\text{rad/s}]$, this is $Z_{TH} = 2200 + (j75 - j66.67) \,||\, 1000$

$$Z_{TH} = 2200.1 + j\,8.329 \; [\Omega]$$
$$= 2200.1 \angle +0.217^\circ \; [\Omega]$$

b) $j\omega L = j8.329 \Rightarrow L = 5.55 \, [mH]$

The impedance is positive so we model it as an inductor.



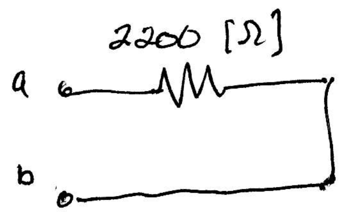
c) Looking at the expression for $Z_{TH}(\omega)$, it is clear that Z_{TH} will be real if ...

$$j\omega\, 0.05 = \frac{j}{\omega} 10^5 \Rightarrow \omega = \pm 1414.2 \left[\frac{\text{rad}}{s}\right]$$

More generally, if $j\omega L = +\frac{j}{\omega C}$, $\omega = \frac{1}{\sqrt{LC}}$.

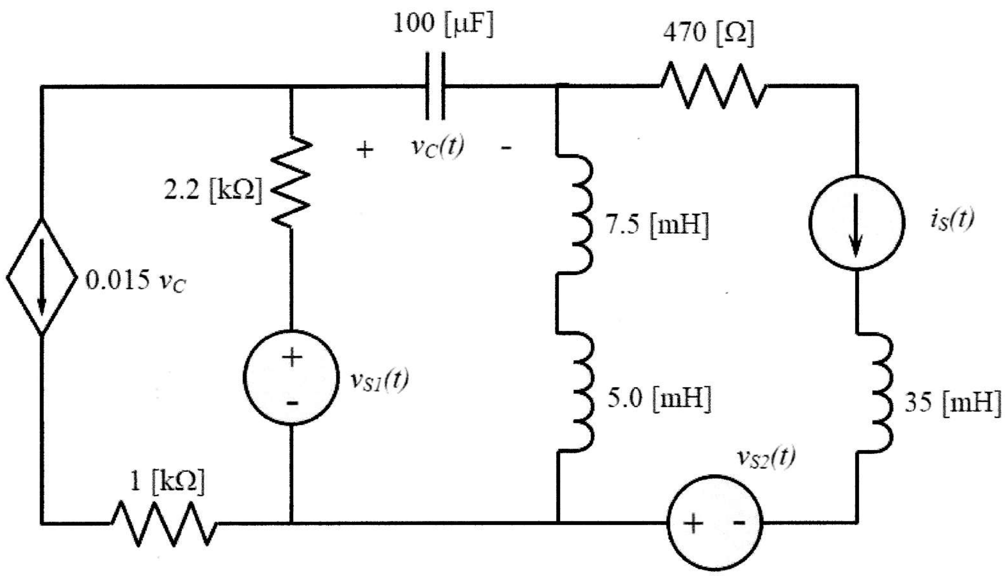
This is the "resonant frequency".

d) $$Z_{TH}\Big|_{\omega = 1414.2 \left[\frac{\text{rad}}{s}\right]} = 2200 \; [\Omega]$$



3. [35 points] The circuit below is operating in steady state. Find the voltage v_C(t). The values of the sources are given as follows.

$v_{S1} = 15$ [V]
$v_{S2} = 30$ [V]$\cos(750t)$
$i_S = 2$ [A] $\sin(1500t + 30^o)$



we will need superposition, but only for v_{S1} and i_S's because V_{S2} is in series with a current source and can be ignored, as can 35 [mH] and 470 [Ω].

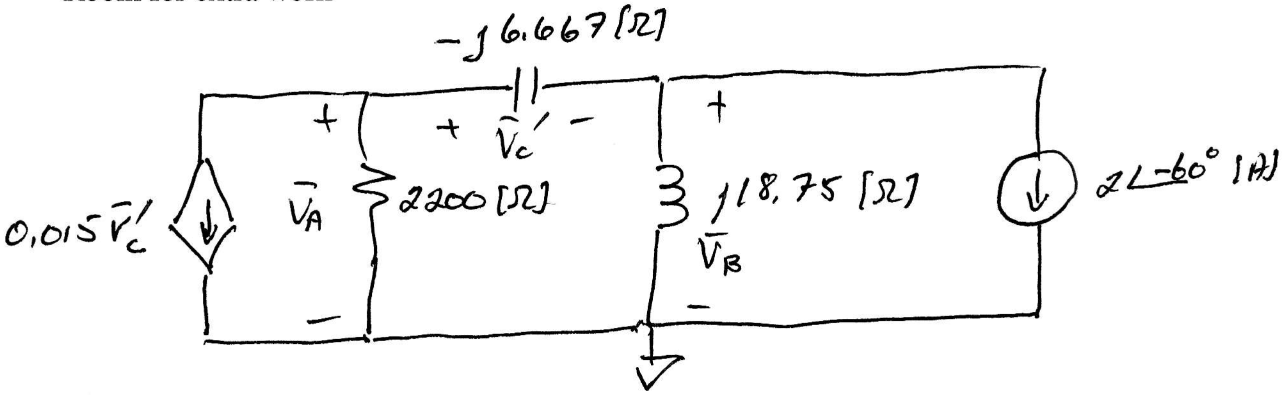
we first deactivate V_{S1} and consider only $i_S(t)$,

$i_S'(t) = 2[A]\sin(1500t + 30^0) = 2[A]\cos(1500t - 60^o)$

$\Rightarrow \bar{I}_S = 2\angle -60^o$ [A]

$\omega = 1500$ [rad/s]

Room for extra work



$$0.015\,\bar{V}_c + \frac{\bar{V}_A}{2200} + \frac{\bar{V}_A - \bar{V}_B}{-j6.667} = 0$$

$$\frac{\bar{V}_B - \bar{V}_A}{-j6.667} + \frac{\bar{V}_B}{j18.75} + 2\angle{-60^\circ} = 0$$

$$\bar{V}_c' = \bar{V}_A - \bar{V}_B$$

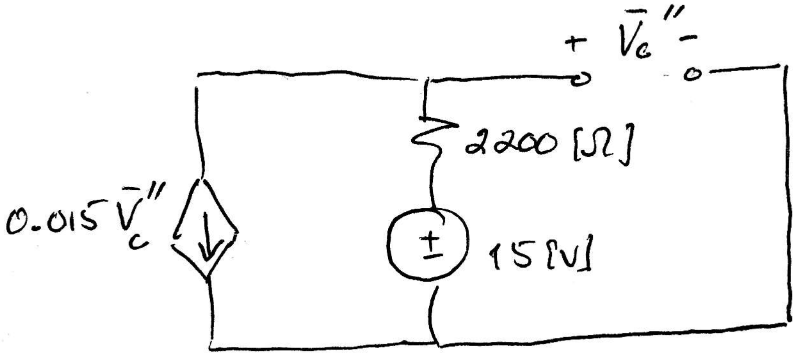
$\bar{V}_A = -32.59 - j18.58\ [V]$

$\bar{V}_B = -32.66 - j18.49\ [V]$

$\bar{V}_c' = 0.0655 - j0.0922\ [V]$

$= 0.113\angle{-54.6^\circ}\ [V]$

Now we de-activate i_S. Since $v_{S1} = 15\ [V] \Rightarrow \omega = 0$, capacitors are open ckt and inductors are short-ckts.



$$\frac{\bar{V}_c - 15}{2200} + 0.015\,\bar{V}_c = 0$$

$\bar{V}_c'' = 0.441\angle{0^\circ}\ [V]$

So $v_c(t) = 0.441\ [V] + 0.113\ [V]\cos(1500t - 54.6^\circ)$