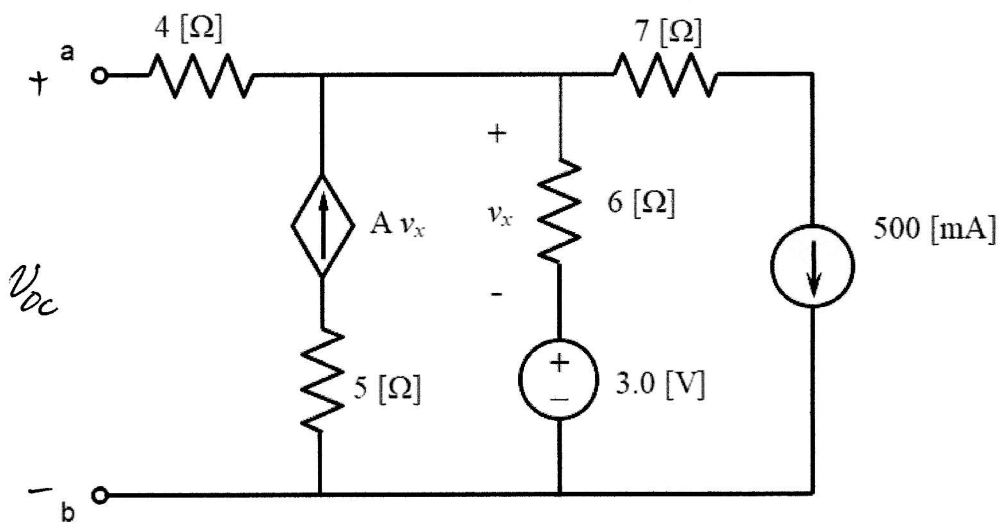
1. [35 points] For the circuit below, do the following.

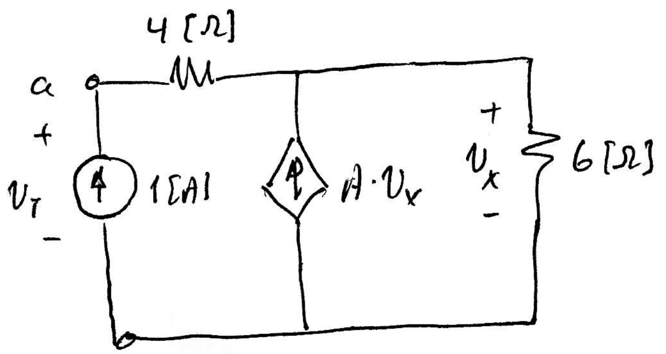
   a) Find the Thevenin equivalent of the circuit at terminals a, b, if the parameter A = 0.5.
   b) Find an expression for the Thevenin equivalent resistance as a function of the parameter A. That is, find R_{TH} in terms of A.
   c) Find the value of A that results in a Thevenin equivalent resistance of –1 [Ω].



Find v_{oc}: $\frac{v_x}{6} - 0.5\,v_x + 0.5 = 0 \Rightarrow v_x = 1.5$ [V]

a) $\boxed{v_{oc} = v_x + 3} = 4.5$ [V]

Apply a test source: $A = 0.5$

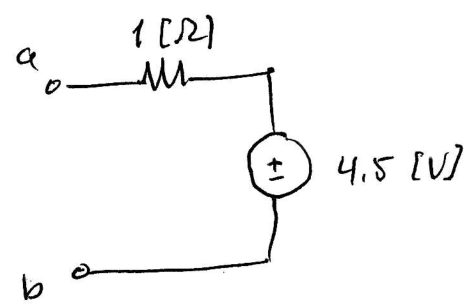


$-1 - 0.5\,v_x + \frac{v_x}{6} = 0$

$\Rightarrow v_x = -3$ [V]

$v_T = 1 \times 4 + v_x = 1$ [V]

$R_{TH} = 1$ [Ω]   So we have



Room for extra work

b) For arbitrary A, we have... (previous circuit)

$$-1 - A \cdot v_x + \frac{v_x}{6} = 0 \quad \text{and} \quad R_{TH} = 4 + v_x$$

So $v_x = R_{TH} - 4$ $\Rightarrow$ $-1 + \frac{R_{TH} - 4}{6} - A \cdot (R_{TH} - 4) = 0$

Solving for R_{TH} gives $\boxed{R_{TH} = \frac{1}{\frac{1}{6} - A} + 4}$

c) Solving for A we have

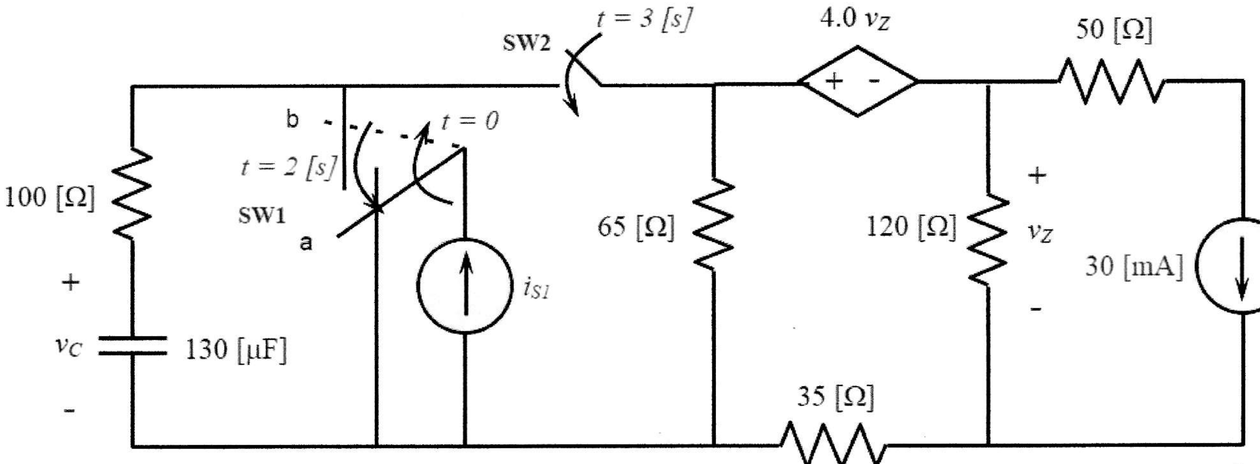
$$A = -\frac{1}{R_{TH} - 4} + \frac{1}{6}$$

If we want $R_{TH} = -1\ [\Omega]$, we need $\underline{A = 0.36667}$

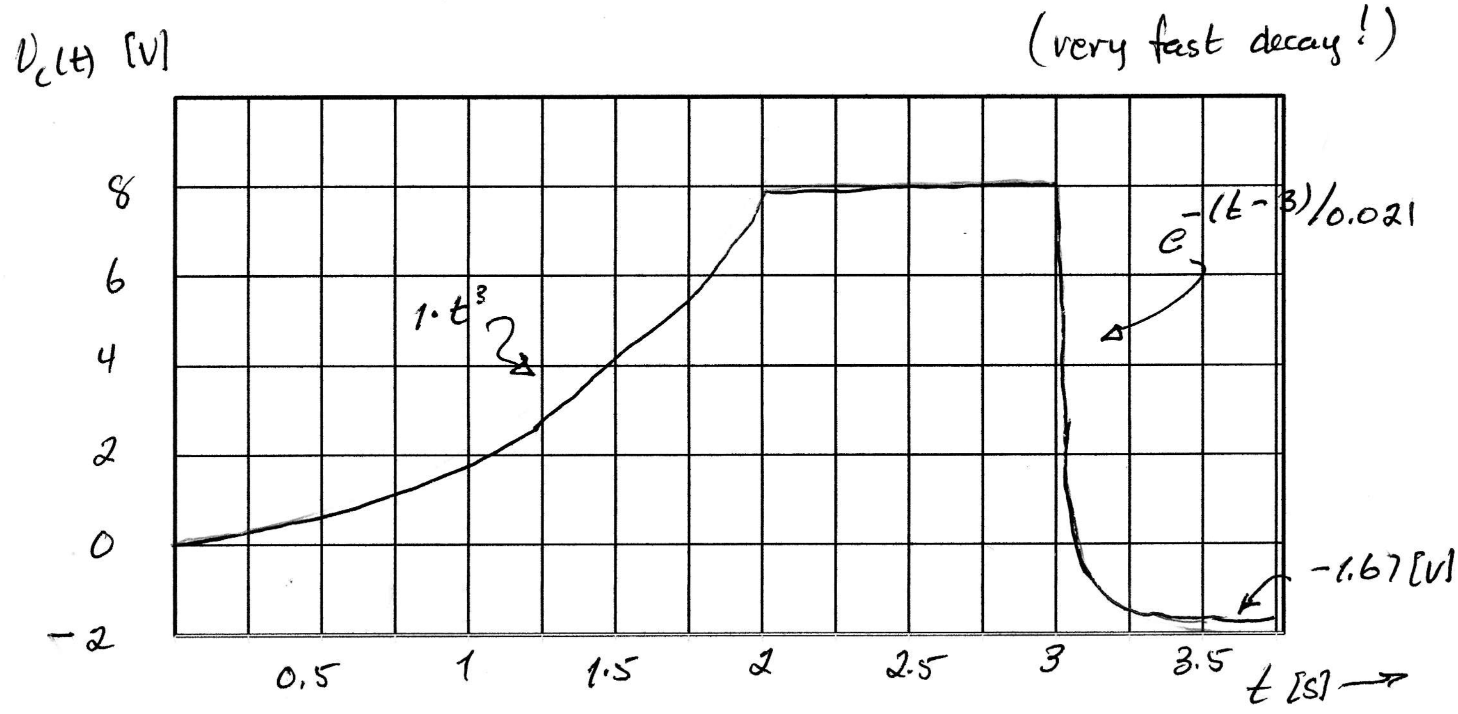
2. [60 points] In the circuit below, the switch SW1 was in position 'a' for a long time. At t = 0 it moved to position 'b', as indicated by the dashed line. At t = 2 [s] it moved back to position 'a'. Switch SW2 was open for a long time, and closed at t = 3 [s]. The current source i_{S1} has the value i_{S1} = ~~39~~ [mA/s^2] t^2.
0.39

a) Find the capacitor voltage $v_C(t)$ for $t \geq 0$ [s].

b) Make a rough plot of $v_C(t)$ for $t \geq 0$ [s]. You can use the graph given below or you can create your own graph. High accuracy and many calculated points are not required. You will get full credit if your graph is clearly labeled, and shows a few specific values.

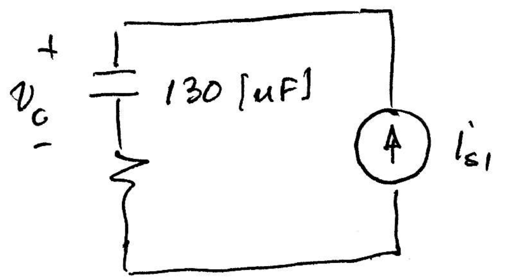


a) For t < 0, SW1 is at 'a' and SW2 is open. So the capacitor is isolated and $v_C(0) = 0$.



Room for extra work

$0 < t < 2$ [s] Sw1 at 'b', Sw2 open



$$v_c(t) = \frac{1}{C}\int_0^t i_{s1}(t)\,dt + v_c(0)$$

$$v_c(t) = \frac{1}{130\times10^{-6}}\int_0^t (0.39\times10^{-3})\,t^2\,dt + 0$$

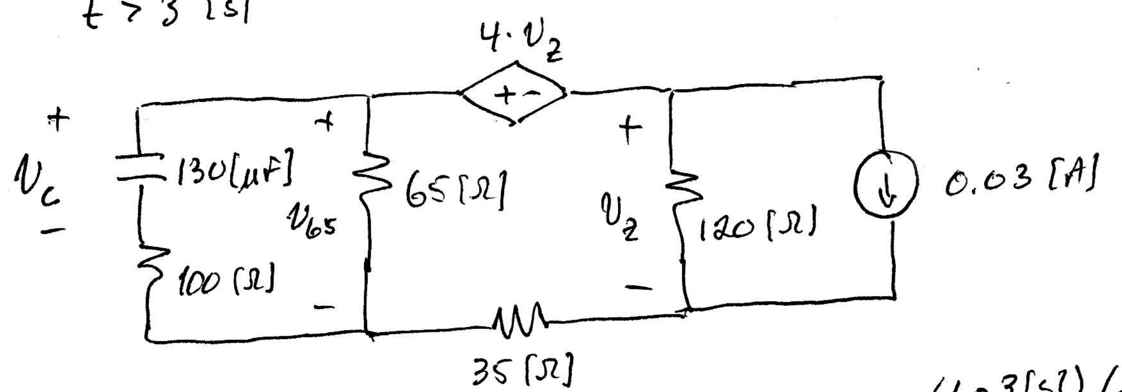
$$v_c(t) = \frac{1}{3}\,\frac{0.39\times10^{-3}}{130\times10^{-6}}\,t^3 = 1\cdot t^3 \ [V] \qquad 0 \le t \le 2 \ [s]$$

This has units $\left[\frac{V}{s^3}\right]$

$2 < t < 3$ [s] Sw1 at 'a', Sw2 open

Capacitor is again isolated so $v_c(t) = 1\cdot 2^3 = 8$ [V] $2 \le t \le 3$ [s]

$t > 3$ [s] Sw1 at 'a', Sw2 closed.



$$v_c(t) = v_{c,f} + (v_c(3[s]) - v_{c,f})\,e^{-(t-3[s])/\tau_c} \qquad t \ge 3 \ [s]$$

$v_c(3[s]) = 8$ [V]

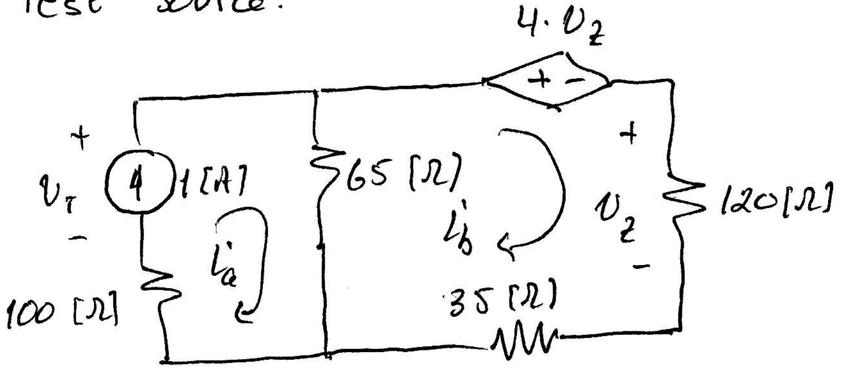
Room for Extra Work

$v_{c,f}$: At steady-state, $C \rightarrow$ open so $v_{c,f} = v_{65}$.

$$\frac{v_z}{120} + 0.3 + \frac{v_z + 4v_z}{100} = 0 \qquad v_z = -0.5143 \text{ [V]}$$

$$\boxed{v_{c,f} = v_{65} = 65 \cdot \frac{v_z + 4v_z}{100} = \underline{-1.67 \text{ [V]}}}$$

Test source:



$$i_a = 1 \text{ [A]}$$

$$65(i_b - i_a) + 5v_z + 35i_b = 0$$

$$v_z = 120\, i_b$$

$$v_T = 65(i_a - i_b) + 100 i_a$$

$$v_T = 158.96 \text{ [V]} \Rightarrow R_{TH} = 158.96 \text{ [Ω]}$$

$$\tau_c = R_{TH} C = 20.66 \text{ [ms]}$$

So...

$$v_c(t) = -1.67 + (8 + 1.67)e^{-(t-3\text{ [s]})/0.02066\text{ [s]}} \text{ [V]} \qquad t \geq 3 \text{ [s]}$$

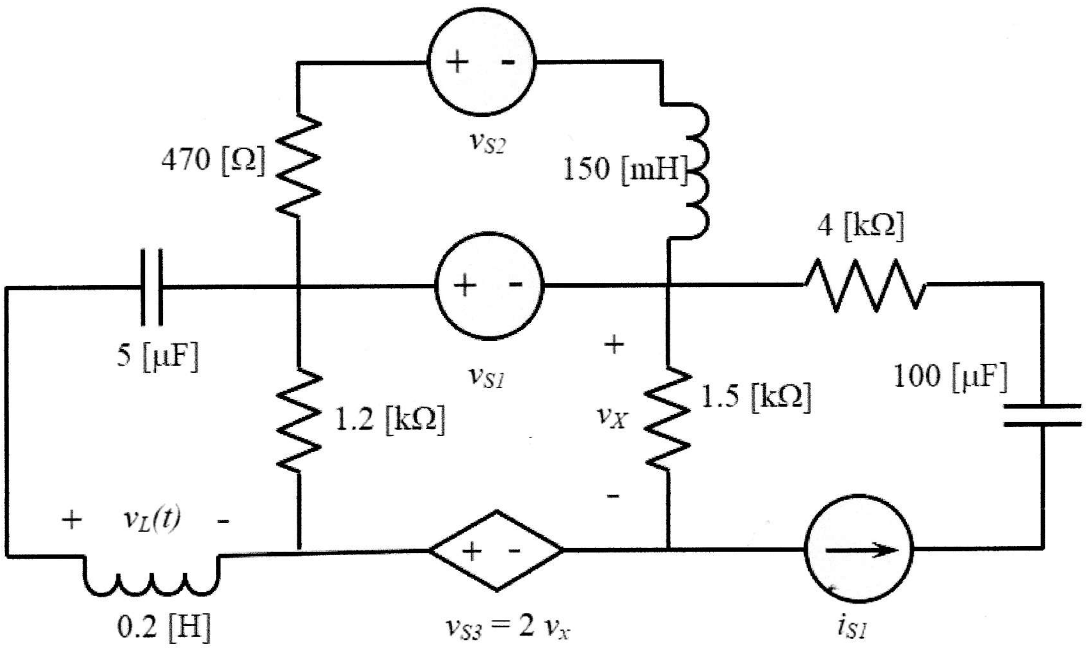
$$\boxed{v_c(t) = -1.67 + 9.67e^{-(t-3\text{[s]})/0.02066\text{ [s]}}} \text{ [V]} \qquad t \geq 3\text{[s]}$$

Summary:

$$v_c(t) = \begin{cases} 0 & t < 0 \\ 1 \cdot t^3 \text{ [V]} & 0 \leq t \leq 2 \text{ [s]} \quad \cancel{0 \leq t \leq 1} \\ 8 \text{ [V]} & 2 \leq t \leq 3 \text{ [s]} \\ -1.67 + 9.67e^{-(t-3)/0.02066} \text{ [V]} & t \geq 3 \text{ [s]} \end{cases}$$

3. [55 points] The circuit below is operating in steady state. Find the voltage v_L(t). The values of the sources are given as follows.

$v_{S1}(t) = 15$ [V] cos(600t) - 37[V] cos(1200t - 20°)
$v_{S2}(t) = 6.5$ [V]sin(1200t)
$i_{S1}(t) = 30$ [mA] sin(600t) = 0.03[A] cos (600 t - 90°)

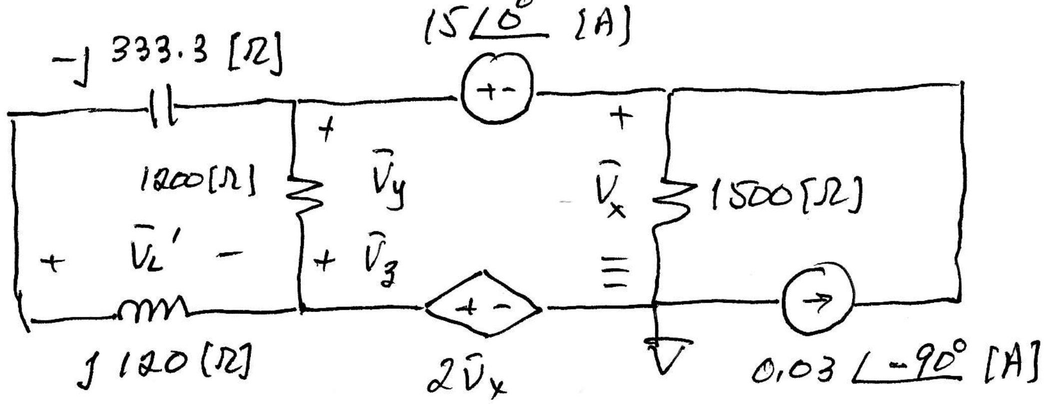


We need superposition, but note tha v_{S2}, 470[Ω], and 150 [mH] are in parallel with a voltage source and will not be "seen" by 0.2 [H], so we can ignore them.

We also note that v_{S1} has two frequency components, and we need to consider only one at a time.

Consider first $\omega = 600 [\frac{rad}{s}]$. We need that component of v_{S1} and i_{S1}:

Room for extra work



$$\frac{\bar{V}_x}{1500} - 0.03\angle -90° + \frac{\bar{V}_y - \bar{V}_z}{1200} + \frac{\bar{V}_y - \bar{V}_z}{j(120-333.3)} = 0$$

$$\bar{V}_x - \bar{V}_y = 15 \qquad \bar{V}_z = 2\bar{V}_x$$

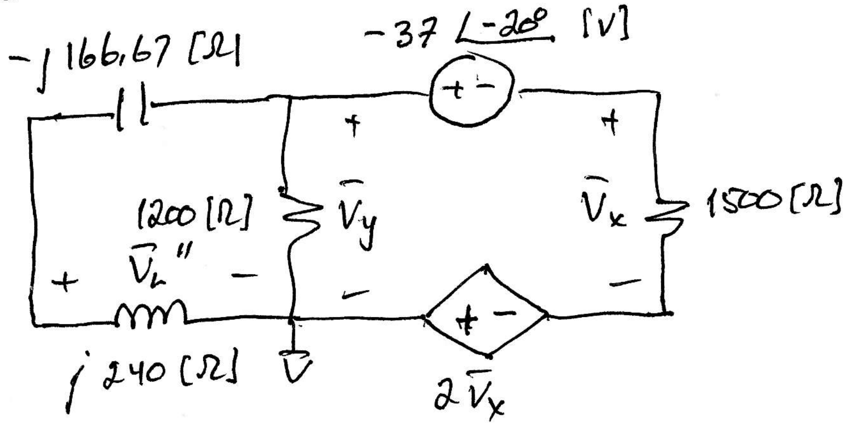
$$\bar{V}_x = -8.6839 + j\,2.3579 \text{ [V]} \quad 8.998\angle 164.81°$$

$$\bar{V}_y = -23.684 + j\,2.3579 \text{ [V]} \quad 23.801\angle 174.31°$$

$$\bar{V}_z = -17.368 + j\,4.7158 \text{ [V]} \quad 18.00\angle 164.81°$$

$$\boxed{\bar{V}_L' = \frac{\bar{V}_y}{j(120-333.3)} \cdot j\,120} = 13.322 - j\,1.3263 \text{ [V]} \quad 13.388\angle -5.685°$$

The $\omega = 1200\,[\frac{rad}{s}]$ component is in V_s:



p. 2

Room for extra work

$$\frac{\bar{V}_y}{j(240-166.67)} + \frac{\bar{V}_y}{1200} + \frac{\bar{V}_y + 37\angle{-20} + 2\bar{V}_x}{1500} = 0$$

$$\bar{V}_y + \bar{V}_x + 37\angle{-20^\circ} = 0$$

$$\bar{V}_x = 0.7470 + j2.1338 \quad [V] \qquad \bar{V}_y = -35.516 + j10.521 \quad [V]$$

$$2.2608\angle{70.706^\circ} \qquad 37.042\angle{163.5^\circ}$$

$$\bar{V}_L'' = \frac{\bar{V}_y}{j(240-166.67)} \cdot j240 = -116.24 + j34.434 \quad [V]$$

$$121.23\angle{163.50^\circ}$$

Converting to time domain for each $\bar{V}_L$ gives...

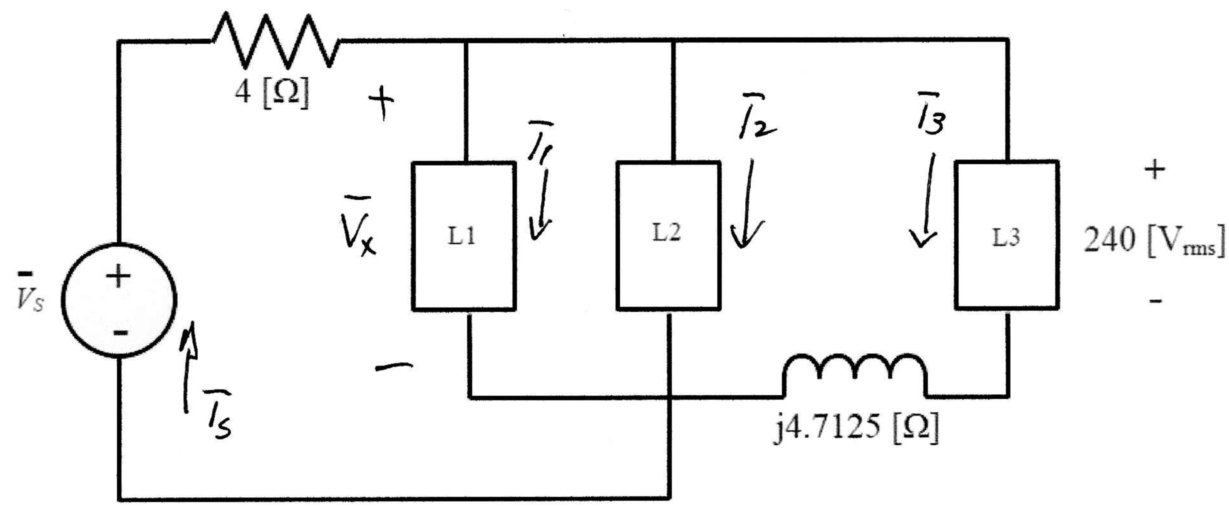
$$v_L(t) = 13.388\cos(\omega t - 5.685^\circ)\ [V]$$

$$+ 121.23\cos(\omega t + 163.50^\circ)\ [V]$$

4. [50 points] The circuit below is operating in steady state and at a frequency of 30 [Hz]. The voltage across load 3 is known and is shown in the diagram. The loads are specified as follows.

Load 1 absorbs 2.5 [kVAR] at a power factor of 0.8 leading.
Load 2 is a 30 [Ω] resistor in parallel with a 120 [μF] capacitor.
Load 3 absorbs 3.5 [kW] and 4.0 [kVAR].

a) Find the source voltage $\overline{V}_S$ necessary to produce the voltage indicated across L3.
b) Find the complex power delivered by the source.



L1: $pf = 0.8 \Rightarrow rf = \sqrt{1-0.8^2} = 0.6$

$|S_1| = \dfrac{2500}{0.6} = 4166.7$

leading

$\Rightarrow \quad S_1 = 4166.7 \cdot (0.8) - j\, 4166.7\,(0.6) = 3333.4 - j\,2500 \quad [VA]$

L2: $120\,[\mu F] \rightarrow -j/(2\pi \cdot 30 \cdot 120\times10^{-6}) = -j\,44.21\,[\Omega]$

$Z_2 = 30 \,//\, (-j\,44.21) = 20.541 - j\,13.939\ [\Omega]$

$24.824 \angle -34.160^\circ$

L3: $S_3 = 3500 + j\,4000 \quad [VA]$

Room for extra work

Game plan: We'll find the three load currents and sum them to find $\bar{I}_s$. This will require knowing $\bar{V}_x$ ...

$$\bar{I}_3^* = \frac{S_3}{240} = 14.583 + j16.667 \text{ [Arms]} \Rightarrow \bar{I}_3 = \begin{matrix} 14.583 - j\,16.667 \\ 22.146 \angle -48.815 \end{matrix} \text{ [Arms]}$$

$$\bar{V}_x = \bar{I}_3 \cdot j\,4.7125 + 240 = \begin{matrix} 161.46 + j\,68.724 \\ 175.48 \angle 23.057^\circ \end{matrix} \text{ [Vrms]}$$

$$\bar{I}_2 = \frac{\bar{V}_x}{Z_2} = \begin{matrix} 3.8274 + j\,5.9430 \\ 7.0688 \angle 57.218^\circ \end{matrix} \text{ [Arms]}$$

$$\bar{I}_1^* = \frac{S_1}{\bar{V}_x} = 11.899 - j\,20.548 \text{ [Arms]} \Rightarrow \bar{I}_1 = \begin{matrix} 11.899 + j\,20.548 \\ 23.745 \angle -59.926^\circ \end{matrix} \text{ [Arms]}$$

$$\bar{I}_s = \bar{I}_1 + \bar{I}_2 + \bar{I}_3 = \begin{matrix} 30.309 - j\,31.272 \\ 43.550 \angle -45.89^\circ \end{matrix} \text{ [Arms]}$$

KVL: $$\boxed{\bar{V}_s = \bar{V}_x + \bar{I}_s \cdot 4} = \begin{matrix} 282.70 - j\,56.363 \\ 288.26 \angle -11.275^\circ \end{matrix} \text{ [Vrms]}$$

$$\boxed{S_{\text{del by } \bar{V}_s}} = \bar{V}_s \cdot \bar{I}_s^* = \begin{matrix} 10332 + j\,7131.3 \\ 12554 \angle 34.615^\circ \end{matrix} \text{ [VA]}$$