Name: ______________________________ (please print)

Signature: ___________________________

ECE 2202 – Exam 2

November 5, 2022

Keep this exam closed until you are told to begin.

1. This exam is closed book, closed notes. You may use one 8.5" x 11" crib sheet, or its equivalent.
2. Show all work on these pages. Show all work necessary to complete the problem. A solution without the appropriate work shown will receive no credit. A solution that is not given in a reasonable order will lose credit. Clearly indicate your answer (for example by enclosing it in a box).
3. Show all units in solutions, intermediate results, and figures. Units in the exam will be included between square brackets.
4. If the grader has difficulty following your work because it is messy or disorganized, you will lose credit.
5. Do not use red ink. Do not use red pencil.
6. You will have 90 minutes to work on this exam.

1. ________________/30

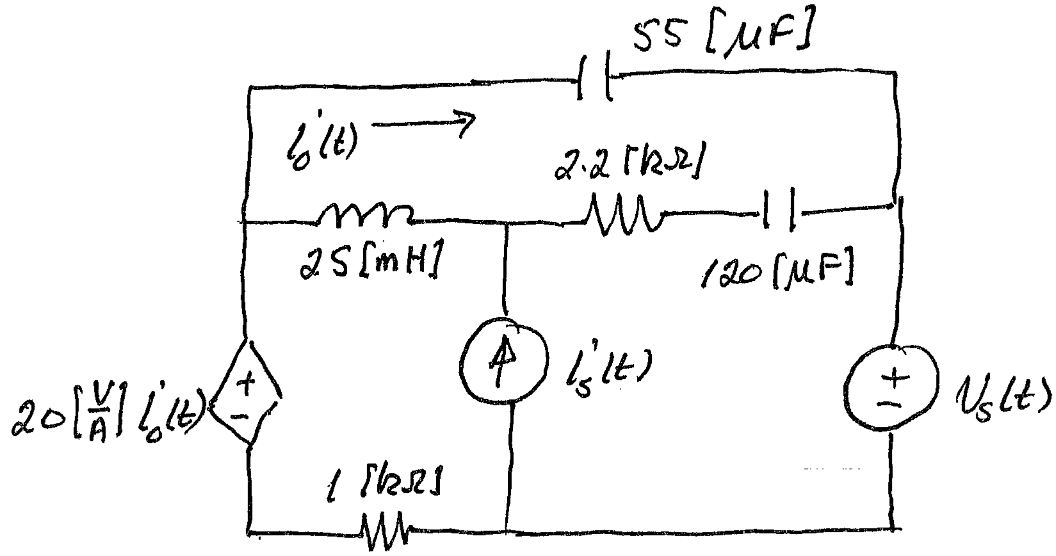
2. ________________/35

3. ________________/35

Total = 100

Room for extra work

1. {30 Points} For the circuit below, find the Thevenin equivalent seen by the current source $i_S(t)$. Draw and label the parameters of the Thevenin equivalent in the time domain, using resistors, capacitors, and inductors, as needed.



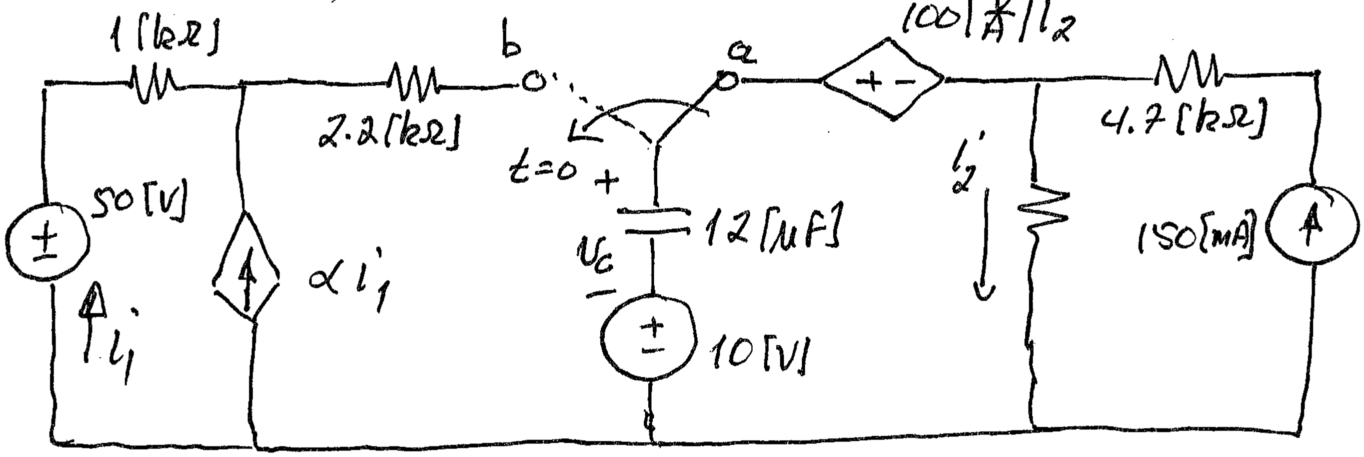
$i_s(t) = 20\,[mA] \cos(300t + 12°)$

$v_s(t) = 5.5\,[V] \sin(300t - 22°)$

Room for extra work

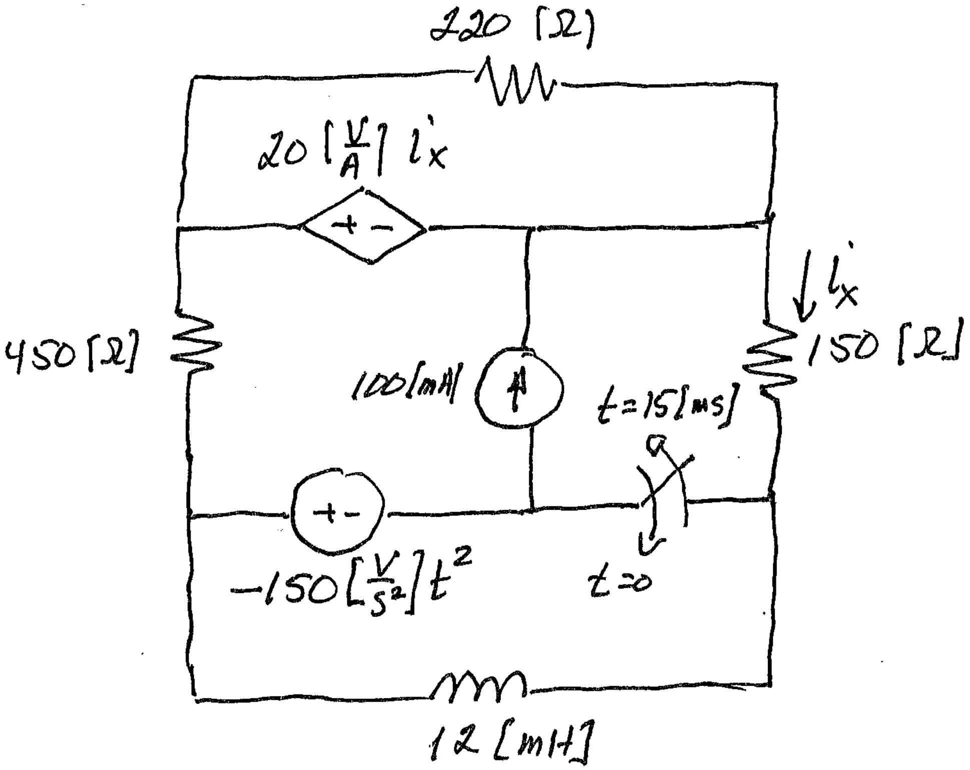
2. {35 Points} In the circuit below, the switch was in position ‘a’ for a long time, and then moved to position ‘b’ at t = 0.

   a) Find the value of α that will result in a time constant τ_C of 0.1 [ms] for t > 0.
   b) Using the value of α from part a), write an expression for v_C(t) as a function of time for t > 0. (If you cannot find a value for α in part a), use α = 5).



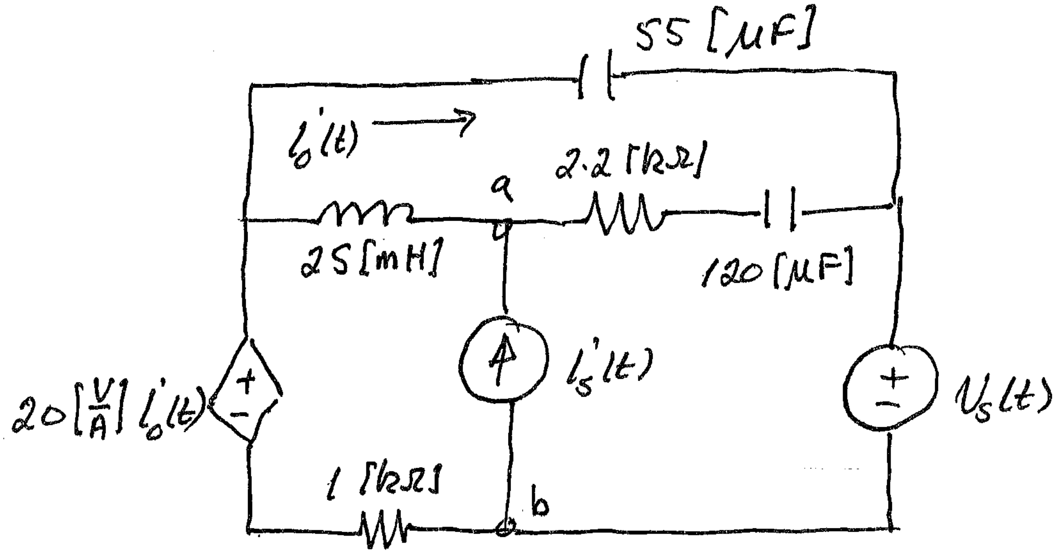
Room for extra work

3. {35 Points} In the circuit below, the switch was open for a long time, and then closed at t = 0. At t = 15 [ms] it opened again. Find $v_X(t)$ for all time t > 0.



Room for extra work

1. {30 Points} For the circuit below, find the Thevenin equivalent seen by the current source $i_S(t)$. Draw and label the parameters of the Thevenin equivalent in the time domain, using resistors, capacitors, and inductors, as needed.



$$i_s'(t) = 20\,[mA]\cos(300t + 12°)$$

$$v_s(t) = 5.5\,[V]\sin(300t - 22°)$$

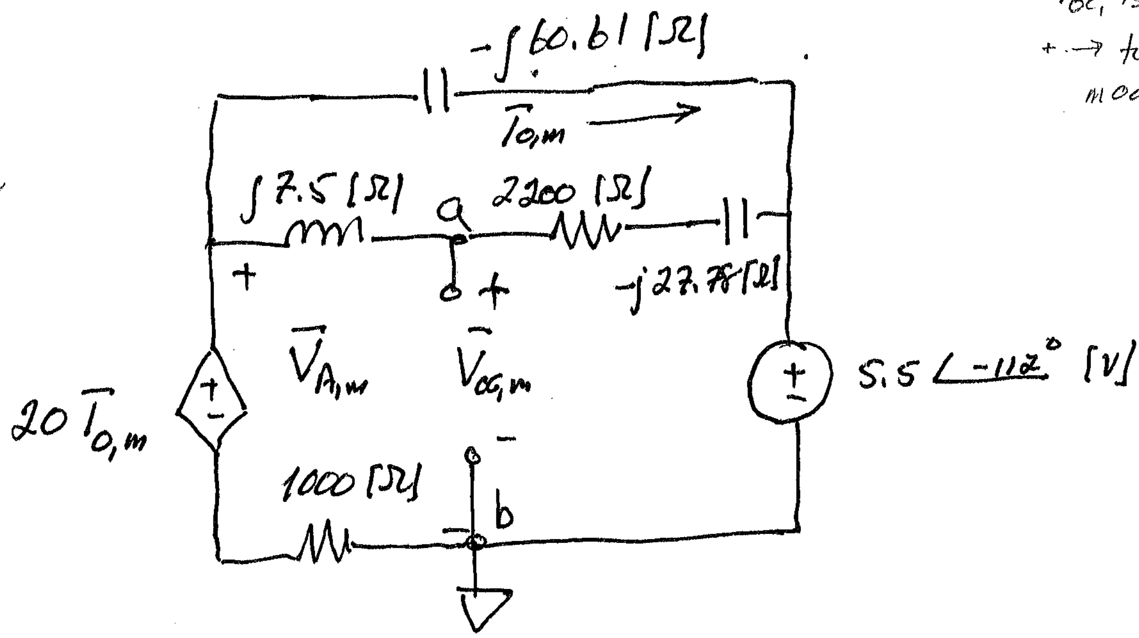
drawing +6

$\bar{V}_{oc}, \bar{I}_{sc}, Z_{TH}$ +10

+ → time model +4

Open-circuit voltage:

+6



+7

$$\frac{\bar{V}_{A,m} - 20\bar{I}_{o,m}}{1000} + \frac{\bar{V}_{A,m} - 5.5\angle -112°}{2200 + j(7.5 - 27.78)} + \frac{\bar{V}_{A,m} - 5.5\angle -112°}{-j60.61} = 0$$

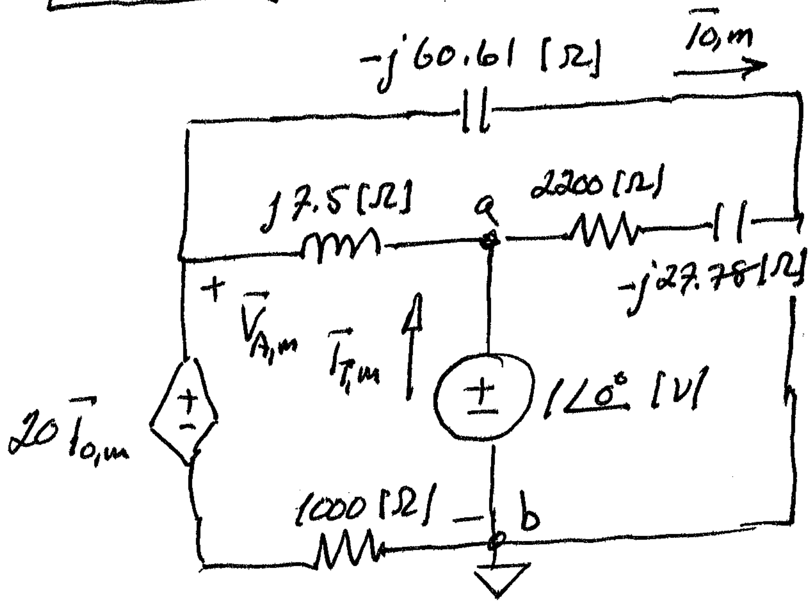
Room for extra work

$$\vec{I}_{o,m} = \frac{\vec{V}_{A,m} - 5.5\angle -112^\circ}{-j60.61} \; ; \quad \vec{V}_{oc,m} = \vec{V}_{A,m} - \frac{\vec{V}_{A,m} - 5.5\angle -112^\circ}{2200 + j(7.5 - 27.78)} \cdot j7.5$$

+1 $$\vec{V}_{oc,m} = \vec{V}_{TH,m} = -1.739 - j5.209\,[V] = 5.492\,[V]\angle -108.75^\circ$$

+2 $$\underline{v_{TH}(t) =} \; 5.492\,[V] \cos(300t - 108.75^\circ)$$

Z_{TH}:



pg 2

+7 $$\begin{cases} \dfrac{\vec{V}_{A,m} - 20\vec{I}_{o,m}}{1000} + \dfrac{\vec{V}_{A,m} - 1}{j7.5} + \dfrac{\vec{V}_{A,m}}{-j60.61} = 0 \\[2ex] \vec{I}_{o,m} = \dfrac{\vec{V}_{A,m}}{-j60.61} \qquad \vec{I}_{T,m} = \dfrac{1}{2200 - j27.78} - \dfrac{\vec{V}_{A,m} - 1}{j7.5} \end{cases}$$ (+3)

+1 $$\vec{I}_{T,m} = 1.749 + j18.39\,[mA] = 18.47\angle 84.57^\circ\,[mA]$$

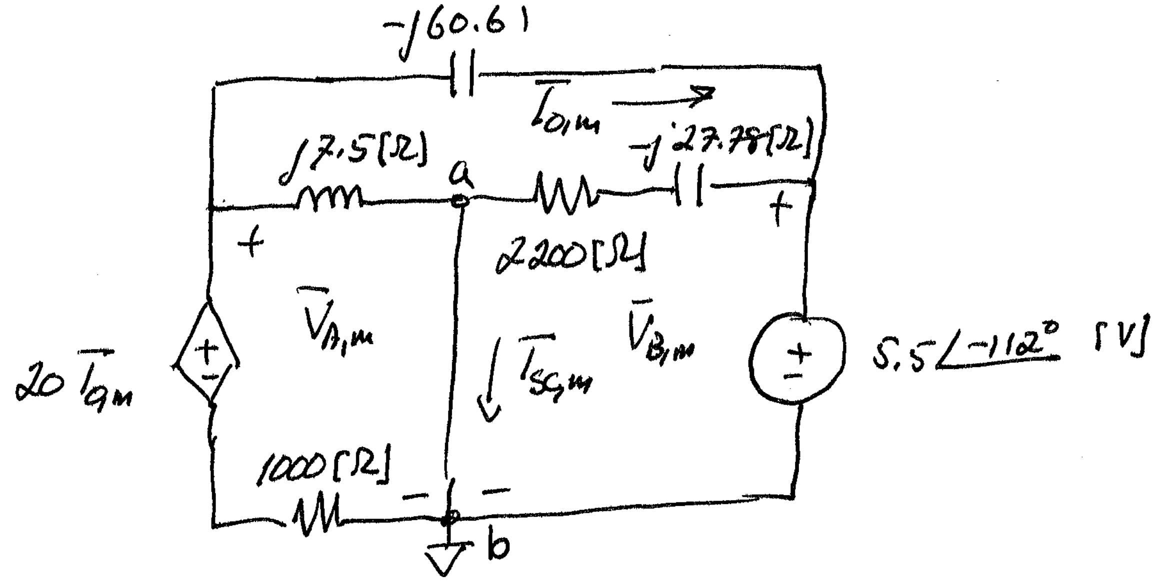
$$Z_{TH} = 1/\vec{I}_{T,m} = 5.125 - j53.88\,[\Omega] = 54.13\angle -84.57^\circ\,[\Omega]$$

+2 $$\therefore \; R = 5.125\,[\Omega] \qquad C = 61.87\,[\mu F]$$

$$(-j53.88 = -j/\omega C)$$

Room for extra work

Short-circuit current as a check:



+7

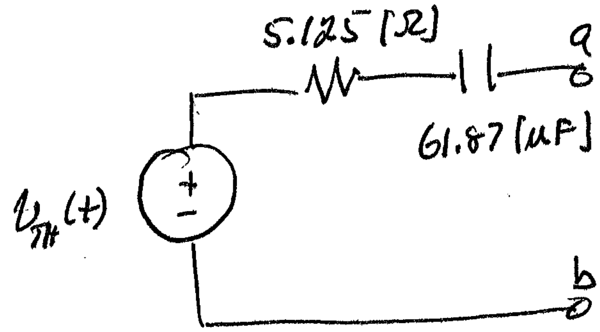
$$\left\{\begin{aligned} &\frac{\bar{V}_{A,m} - 20\,\bar{I}_{o,m}}{1000} + \frac{\bar{V}_{A,m}}{j7.5} + \frac{\bar{V}_{A,m} - 5.5\angle{-112^\circ}}{-j60.61} = 0 \\ &\bar{V}_{B,m} = 5.5\angle{-112^\circ}\ [V] \qquad \bar{I}_{o,m} = \frac{\bar{V}_{A,m} - 5.5\angle{-112^\circ}}{-j60.61} \\ &\bar{I}_{SC,m} = \frac{\bar{V}_{A,m}}{j7.5} + \frac{\bar{V}_{B,m}}{2200 - j27.78} = 92.6 - j\,41.04\ [mA] \end{aligned}\right.$$

(+3)

+1 $\dfrac{\bar{V}_{TH}}{Z_{TH}} = 92.56 - j\,41.56$ [mA] — so this checks.

+2

Model:

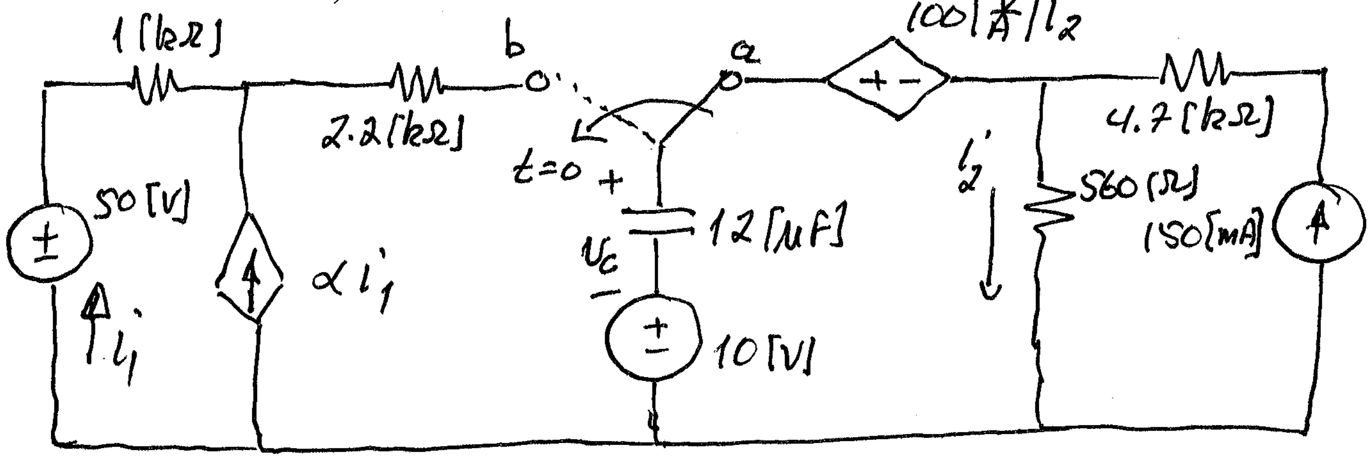


+2

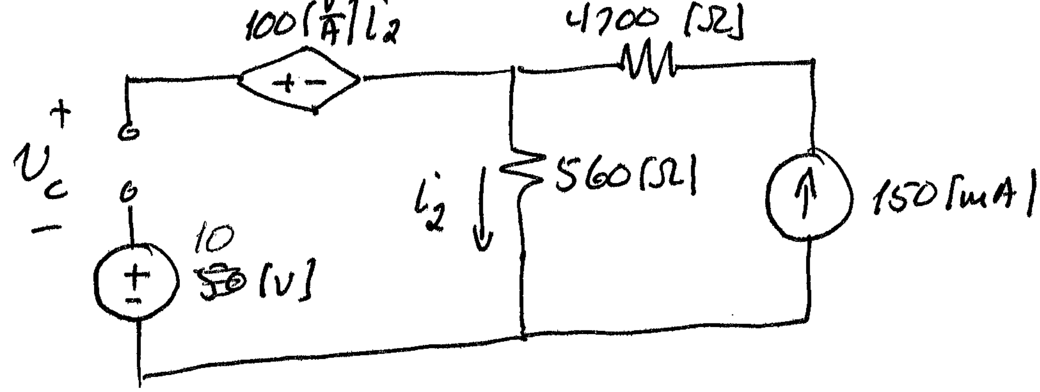
+2 labels

2. {35 Points} In the circuit below, the switch was in position ‘a’ for a long time, and then moved to position ‘b’ at t = 0.

   a) Find the value of α that will result in a time constant τ_C of 0.1 [ms] for t > 0.
   b) Using the value of α from part a), write an expression for $v_C(t)$ as a function of time for t > 0. (If you cannot find a value for α in part a), use α = 5).



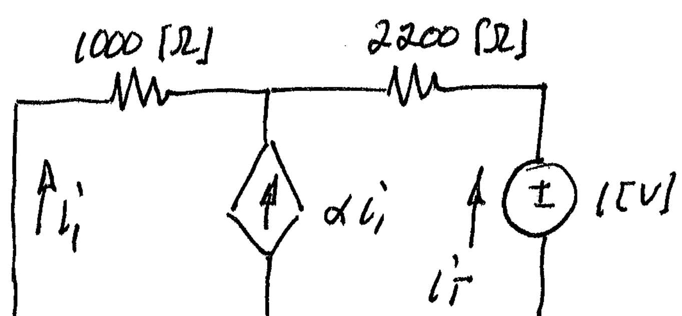
t<0



+5

$$v_C = -10 + 100(0.15) + 560(0.15) = 89 \text{ [V]} = v_C(0)$$

t>0: set up test source for R_{TH}



$$i_T = -(1+\alpha) i_1$$

$$\left(R_{TH} = \frac{-1}{(1+\alpha) i_1}\right)$$

a)

Room for extra work

We need $\tau_c = 0.1$ [ms] $= R_{TH} \cdot C$

$\therefore \quad R_{TH} = \dfrac{10^{-4}}{12\times 10^{-6}} = 8.333\ [\Omega] = \dfrac{1}{i_T'}$

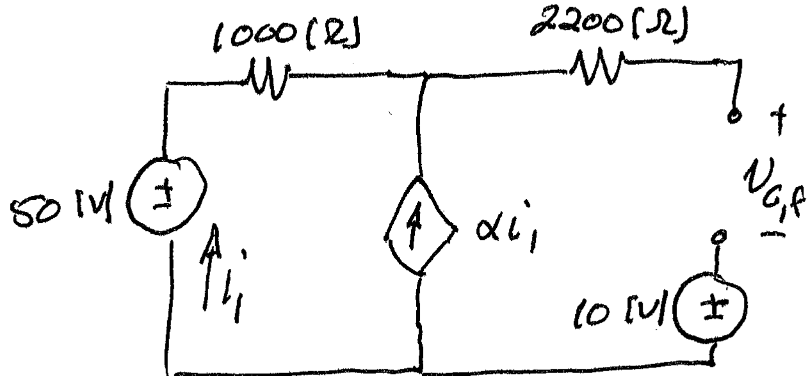
+5

$\therefore \quad i_T' = 120$ [mA] $= -(1+\alpha)\, i_1'$

$1000\, i_1' + 2200(-i_T') + 1 = 0 \Rightarrow i_1' = 263$ [mA]

+10 $\Rightarrow \boxed{\alpha = -1.456}$

b) We have $\tau_c = 10^{-4}$ [s]. We need $v_{C,f}$:



$i_1 + \alpha i_1 = 0$

$\Rightarrow i_1 = 0$

$-v_{C,f} + 50 - 10 = 0$

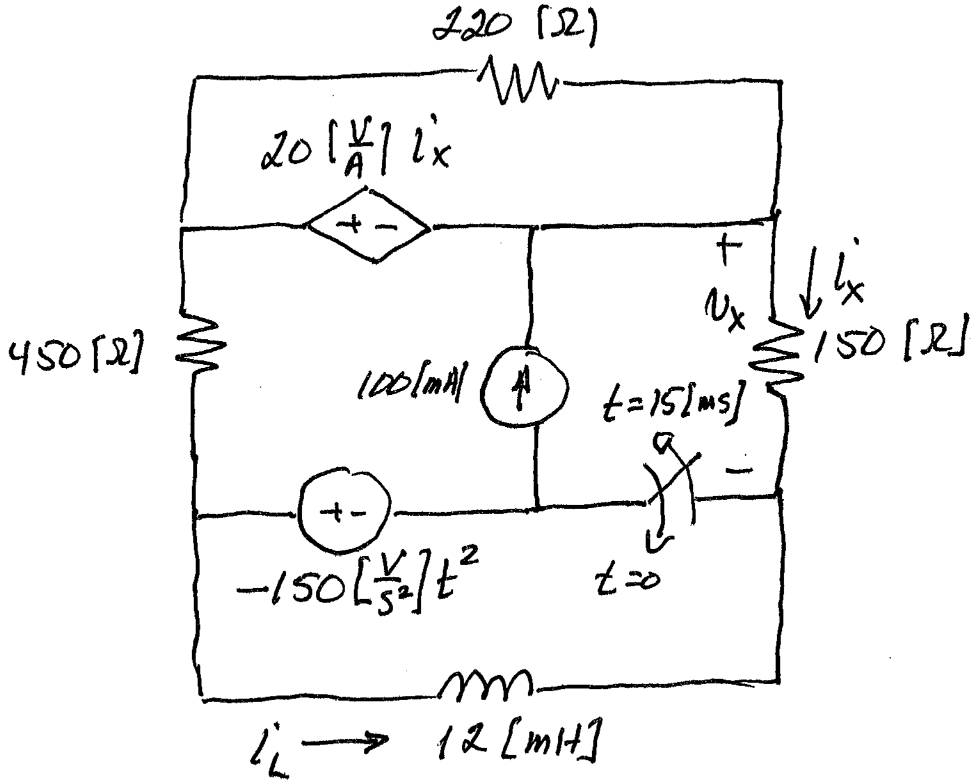
$v_{C,f} = 40$ [V]

+10

$v_C(t) = \overset{40}{v_{C,f}} + (\overset{89}{v_C(0)} - \overset{-40}{v_{C,f}})\, e^{-t/\tau_c}$

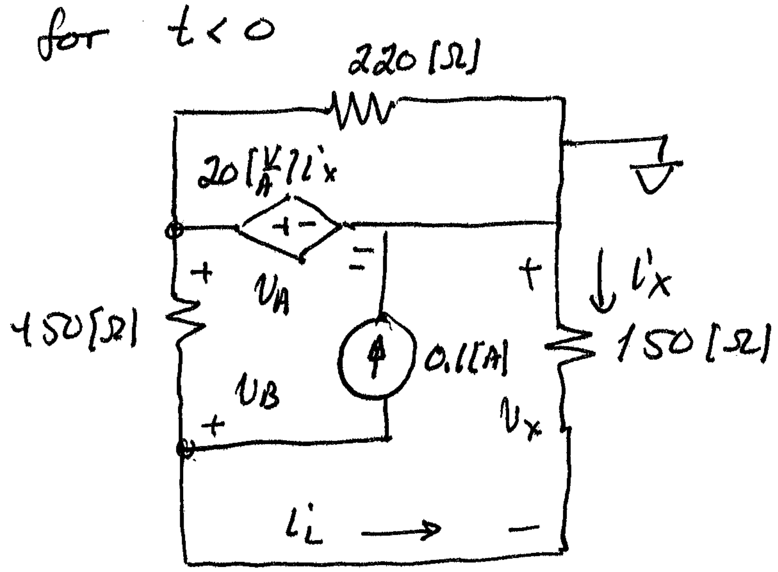
+5 $\quad v_C(t) = 40\ [V] + 49\ [V]\, e^{-t/0.1[ms]} \qquad t \ge 0$

3. {35 Points} In the circuit below, the switch was open for a long time, and then closed at t = 0. At t = 15 [ms] it opened again. Find $v_X(t)$ for all time t > 0.



we will want i_L later...

Draw for t<0



+5

$$v_A = 20 i_x$$

$$\frac{v_B - v_A}{450} + 0.1 + \frac{v_B}{150} = 0$$

$$i_x = -v_B/150$$

$$v_A = 1.452 \text{ [V]}$$

$$v_B = -10.89 \text{ [V]}$$

$$i_x = 72.58 \text{ [mA]}$$

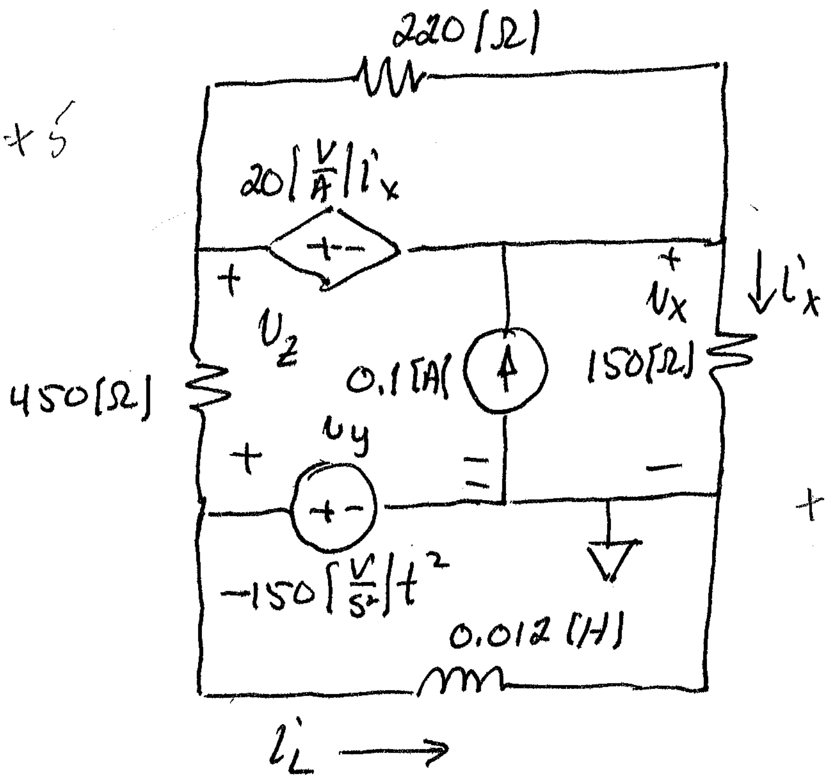
- Steady state ⇒ L → short.
- voltage source in series w/ 100 [mA] is ignored.
- we are asked for v_x for t>0 so we don't need v_x here.

+10 Room for extra work

$i_L'(0) = -i_x' = -72.58\ [mA]$

Re-draw for $0 < t < 15\ [ms]$

+5



$$i_L'(t) = \frac{1}{L}\int_0^t (-150)t^2\,dt + i_L'(0)$$

$$= \frac{-150}{0.012}\,\frac{1}{3}\,t^3\Big|_0^t - 72.58\ [mA]$$

4166.6

$$i_L'(15\,[ms]) = -86.64\ [mA]$$

+10

To find v_x:

1. $v_z = 20\,i_x' + v_x$
2. $v_y = -150\,t^2$
3. $\dfrac{v_x}{150} - 0.1 + \dfrac{v_z - v_y}{450} = 0$
4. $i_x' = \dfrac{v_x}{150}$

+10

From 3:

$$3v_x - 45 + v_z - v_y = 0$$

From 4 & 2:

$$v_z = 1.133\,v_x$$

$$\Rightarrow 4.133\,v_x - 45 - 150t^2 = 0$$

$$\therefore\ v_x = 10.89 + 36.29\,t^2\ [V]$$

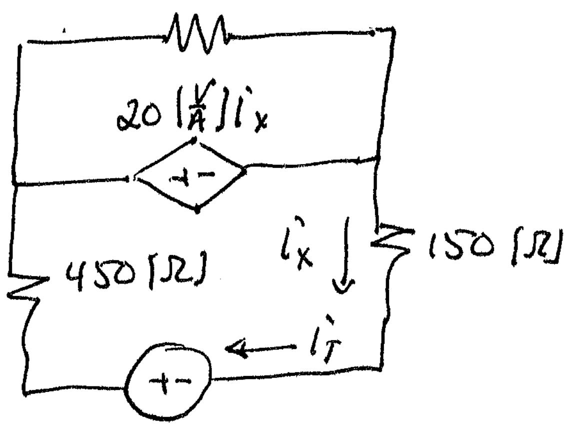
$0 < t < 15\ [ms]$

pg. 2'

Room for extra work

For $t > 15$ [ms], the configuration is the same as for $t < 0$. So $i_{L,f} = i_L(0) = -72.58$ [mA].

+6 Re-draw w/ test source to find R_{TH}.



$i_x = i_T$

$20 i_T + 600 i_T - 1 = 0$

$i_T = \frac{1}{620}$ [A]

$\Rightarrow R_{TH} = 620$ [Ω]

+8

+2 $\tau_L = L/R_{TH} = \frac{0.012}{620} = 19.36$ [μs] $= 0.0194$ [ms]

$$i_L(t) = i_{L,f} + (i_L[15\,[ms]] - i_{L,f})\, e^{-\frac{t - 15\,[ms]}{0.0194\,[ms]}}$$

$$= -72.58 + (-86.64 + 72.58)\, e^{-\frac{t-15\,[ms]}{0.0194\,[ms]}} \quad [mA]$$

$$= -72.58 - 14.06\, e^{-\frac{t-15\,[ms]}{0.0194\,[ms]}} \quad [mA] \qquad t \geq 15\,[ms]$$

+6

So $v_x(t) = +i_x(t) \cdot 150 = -i_L(t) \cdot 150$

+8 $\boxed{v_x(t) = 10.89\,[V] + 2.109\,[V]\, e^{-\frac{t-15\,[ms]}{0.0194\,[ms]}}} \qquad t > 15\,[ms]$