Name: ______________________________ (please print)

Signature: ___________________________

ECE 2202 – Final Exam

December 6, 2022

Keep this exam closed until you are told to begin.

1. This exam is closed book, closed notes. You may use one 8.5" x 11" crib sheet, or its equivalent.
2. Show all work on these pages. Show all work necessary to complete the problem. A solution without the appropriate work shown will receive no credit. A solution that is not given in a reasonable order will lose credit. Clearly indicate your answer.
3. Show all units in solutions, intermediate results, and figures. Units in the exam will be included between square brackets.
4. Do not use red ink. Do not use red pencil.
5. You will have 160 minutes to work on this exam.

1. ________________/45
2. ________________/40
3. ________________/45
4. ________________/45
5. ________________/45

Total = 220

Room for extra work

1. {45 Points} The circuit shown in Figure 1 is a model for device TD, the well-known Tromberator Device, which is shown in Figure 2. Figure 2 also shows the voltage $\overline{V}_{ab}$, which has the value $\overline{V}_{ab} = 5.012 + j1.824$ [V] when nothing is connected to terminals a, b. In Figure 1, the impedance $Z = 200\angle -40^o$ [Ω].

   a) Find the Norton equivalent of this device in the phasor domain.
   b) If the voltage source operates at a frequency of 500 [rad/s], find a model for the Norton impedance in the time domain using resistors, capacitors, and inductors as needed.

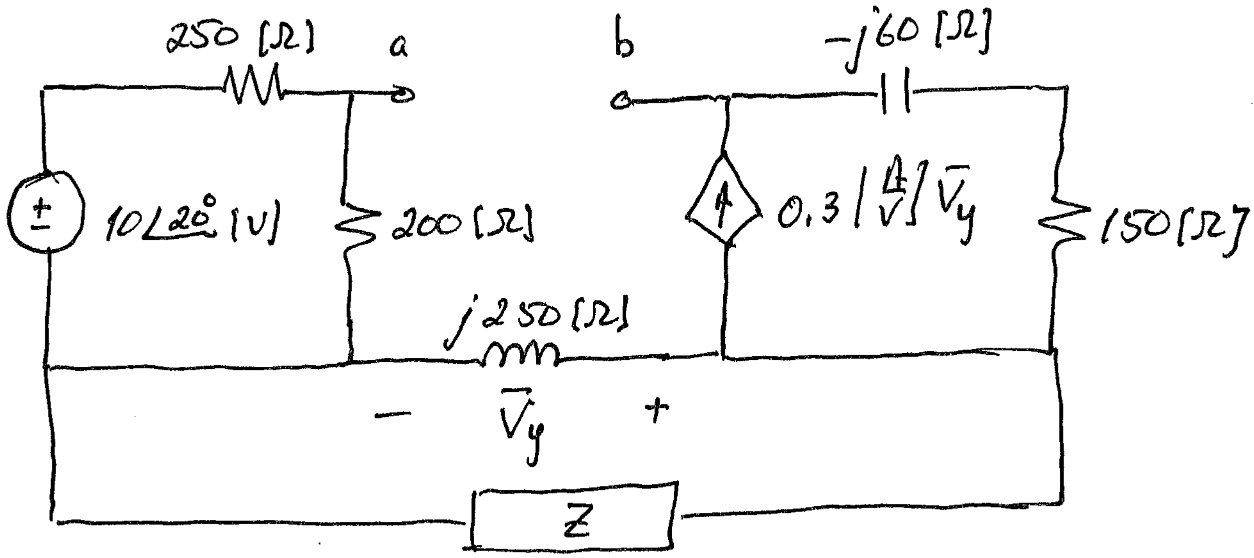


FIGURE 1.

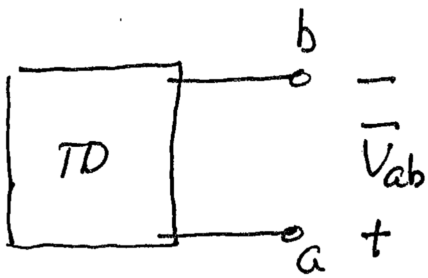
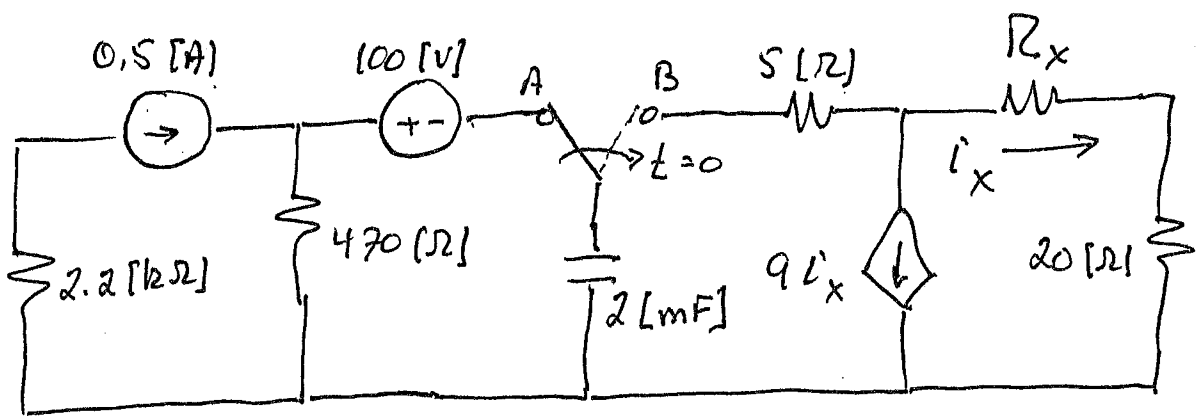


Figure 2.

Room for extra work

2. {40 Points} In the circuit below, the switch was at position 'A' for a long time, and then moved to 'B' at t = 0. The switch is designed so that after ten time constants in position 'B' it moves back to position 'A'.

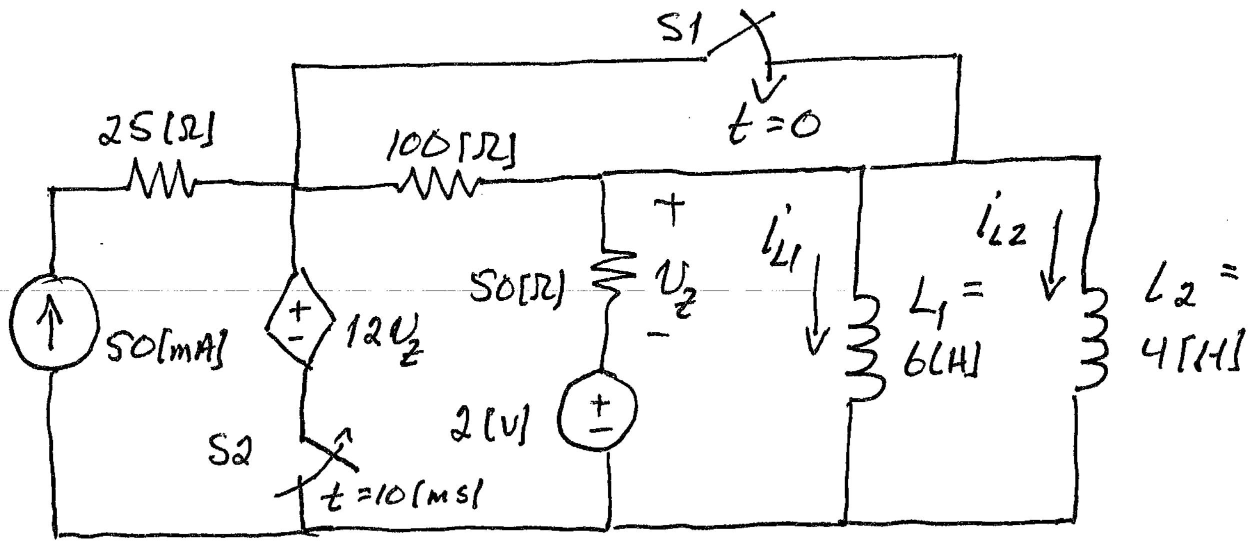
   a) How long does it take the switch to move back to position 'A' when the resistor R_X has the value 100 [Ω]?
   b) What value does R_X need to have in order to double the time you found in part a)?



Room for extra work

3. {45 Points} In the circuit below, switch S1 was open for a long time, and switch 2 was closed for a long time. At t = 0, S1 closed. After 10 [ms], S2 opened. At t = 0^-, the current i_{L2} in inductor L_2 was 15 [mA].

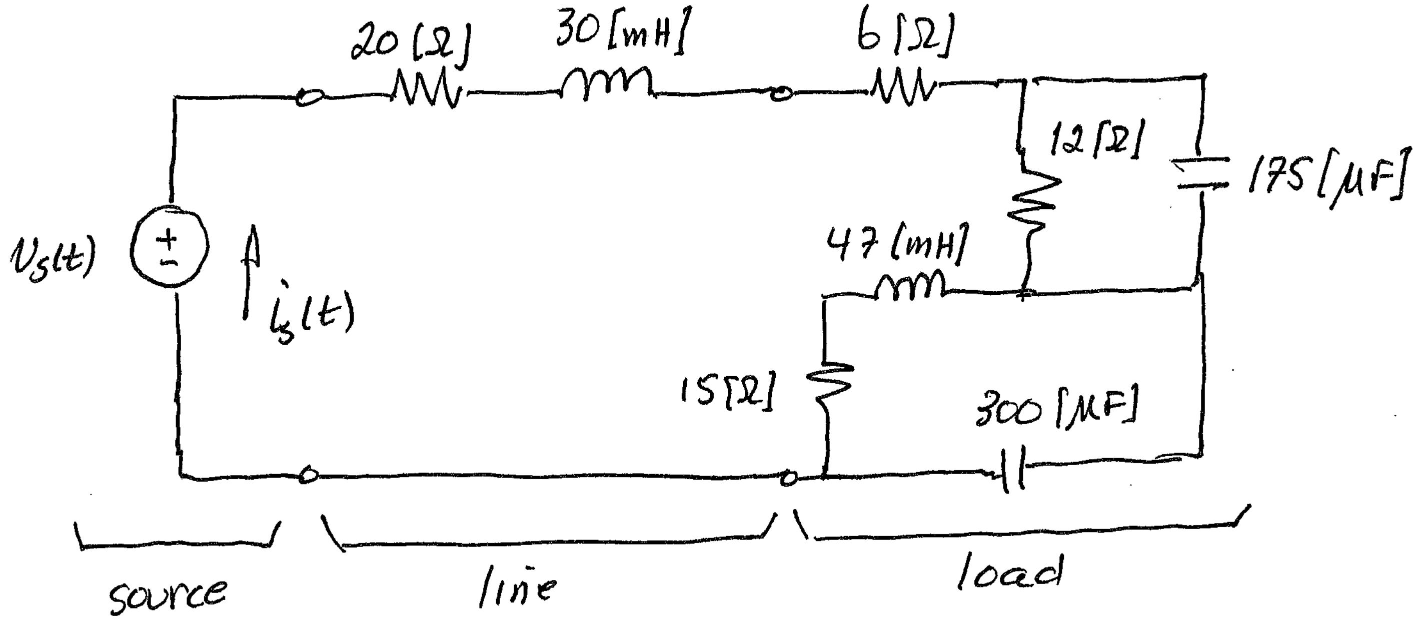
   a) Find the current i_{L1} in inductor L_1 at t = 0^-.
   b) Find the current i_{L1} as a function of time for $t \geq 10\ [ms]$.



Room for extra work

4. {45 Points} The circuit below models a power source, a transmission line, and a load, as labeled. The source voltage $v_S(t) = 360[V]\sin\left(377\left[\frac{rad}{s}\right]t - 200^o\right)$.

a) Find the complex power absorbed by the load.
b) Find the current $i_S(t)$.
c) Find the power absorbed by the load if the source voltage is changed to $v_S = 360[V]$.

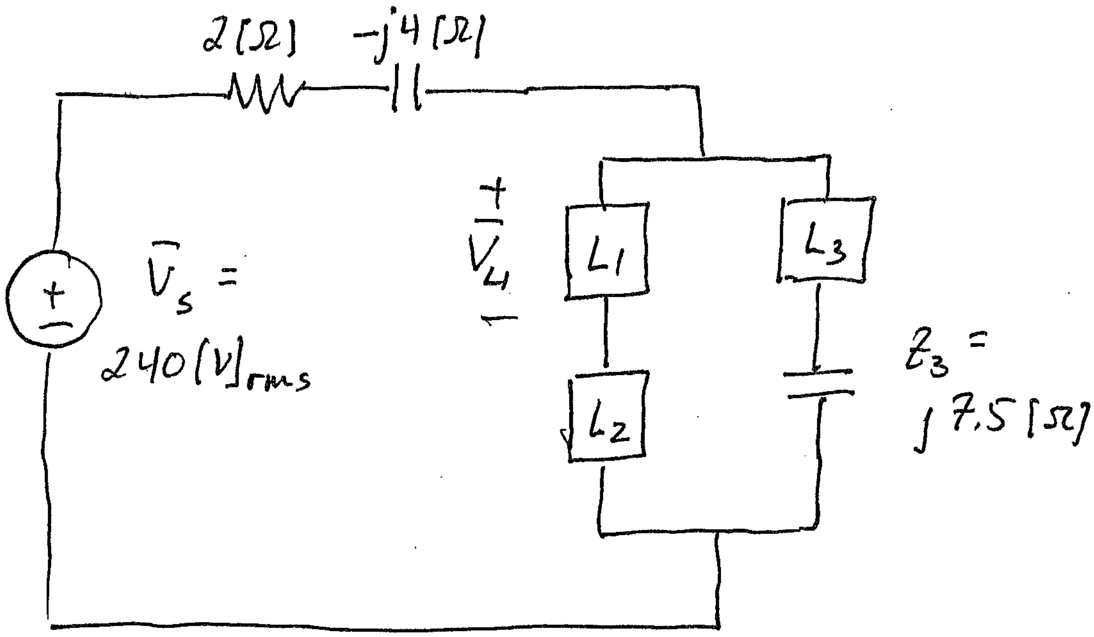


Room for extra work

5. {45 Points} The loads in the circuit below are described as follows.

Load L_1 absorbs 2500 [W] at a power factor of 0.7 lagging.
Load L_2 absorbs 3000 [W] and 1500 [VAR].
Load L_3 absorbs 5500 [kVA] at a power factor of 0.8 leading.
$\overline{V}_{L1} = 120\angle 60^o\ [V]_{rms}$.

a) Find the total power delivered by the source $\overline{V}_S$.
b) Find the apparent power delivered by the source $\overline{V}_S$.
c) Find the power factor of the load consisting of L_3 in combination with Z_3.



Room for extra work

Room for extra work

1. {45 Points} The circuit shown in Figure 1 is a model for device TD, the well-known Tromberator Device, which is shown in Figure 2. Figure 2 also shows the voltage $\overline{V}_{ab}$, which has the value $\overline{V}_{ab} = 5.012 + j1.824$ [V] when nothing is connected to terminals a, b. In Figure 1, the impedance $Z = 200\angle -40^o$ [Ω].

   $= 153.2 - j128.6$

   a) Find the Norton equivalent of this device in the phasor domain.
   b) If the voltage source operates at a frequency of 500 [rad/s], find a model for the Norton impedance in the time domain using resistors, capacitors, and inductors as needed.

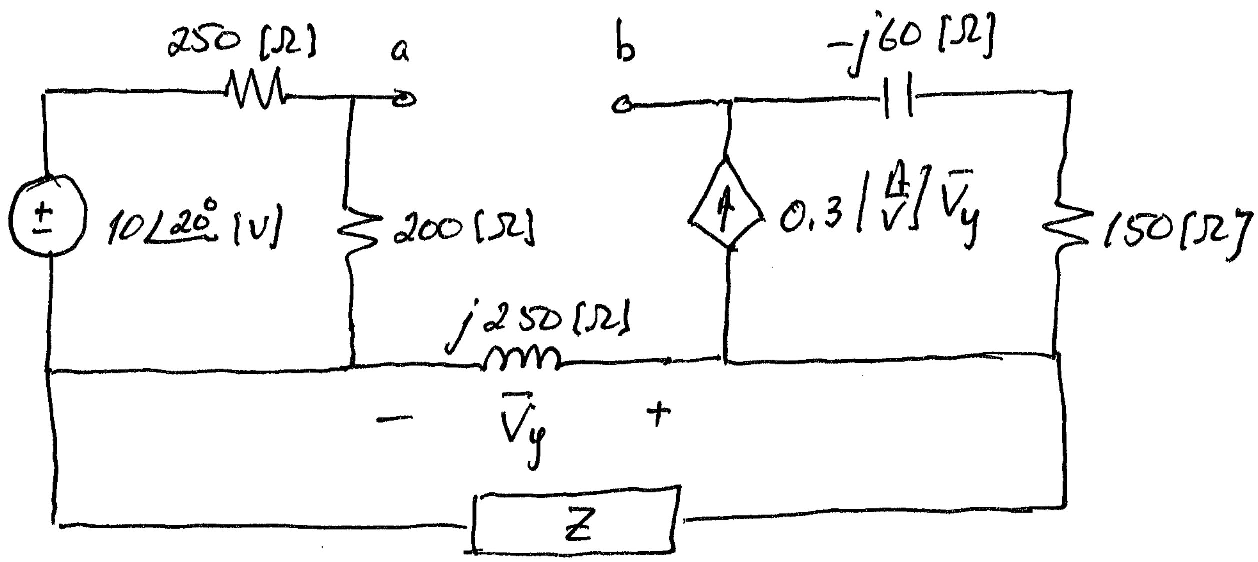


FIGURE 1.

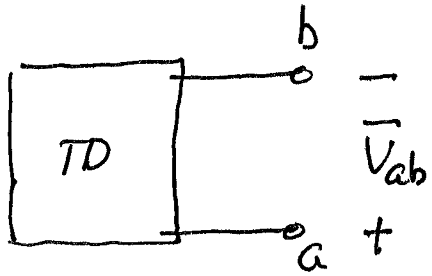


Figure 2.

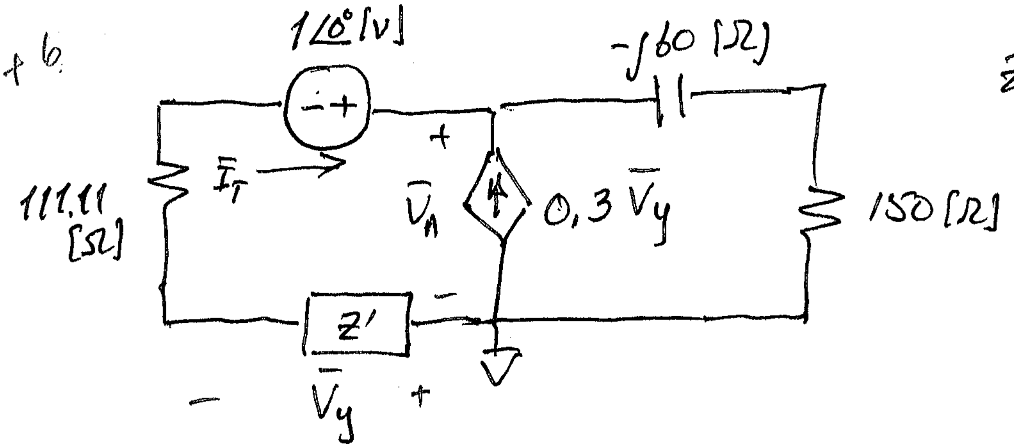
What we have been given is the open-circuit voltage $\overline{V}_{TH}$. So we need either the short-circuit current or the Thevenin/Norton resistance. We will find both.

Simplify: $250 \| 200 = 111.11$ [Ω]

$$Z \| j250 = 200\angle{-40^o} \| j250$$
$$= 250.5 + j51.41 \text{ [Ω]}$$
$$= 255.75\angle 11.60^o \text{ [Ω]} \equiv Z'$$

Room for extra work

+6



$$Z' = 255.8\angle 11.60°\ [\Omega]$$
$$= 250.5 + j51.41\ [\Omega]$$

soln

Eqns:

$$-0.3\,\bar{V}_y + \frac{\bar{V}_A}{150 - j60} + \frac{\bar{V}_A - 1}{111.11 + 255.8\angle 11.60°} = 0$$

+15

$$\bar{V}_y = -\frac{\bar{V}_A - 1}{111.11 + 255.8\angle 11.60°} \cdot 255.8\angle 11.60°$$

$$\bar{I}_T = \frac{\bar{V}_y}{255.8\angle 11.60°}$$

+4

$$\bar{V}_A = 973 - j8.717\ [V]$$
$$\bar{V}_y = 18.45 + j7.245\ [V]$$
$$\bar{I}_T = 76.37 + j13.24\ [\mu A]$$
$$= 77.5\angle 9.84°\ [\mu A]$$

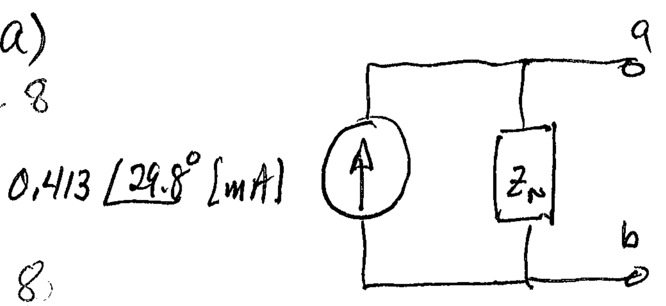
+2

$$Z_N = \frac{1}{\bar{I}_T} = 12.71 - j2.20\ [k\Omega]$$
$$= 12.90\angle -9.82°\ [k\Omega]$$

+2

$$\bar{I}_N = \bar{I}_{sc} = \frac{\bar{V}_{TH}}{Z_N} = 0.359 + j0.206\ [mA]$$
$$= 0.413\angle 29.8°\ [mA]$$

a)
+8

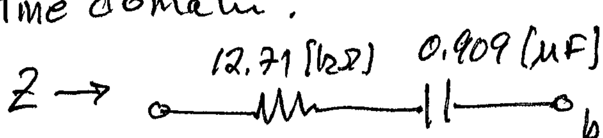


$$Z_N = 12.71 - j2.20\ [k\Omega]$$
$$= 12.90\angle -9.8\ [k\Omega]$$
$$R_N = 12.71\ [k\Omega]$$

$$Z_C = -j2.20$$

+8

b) time domain:



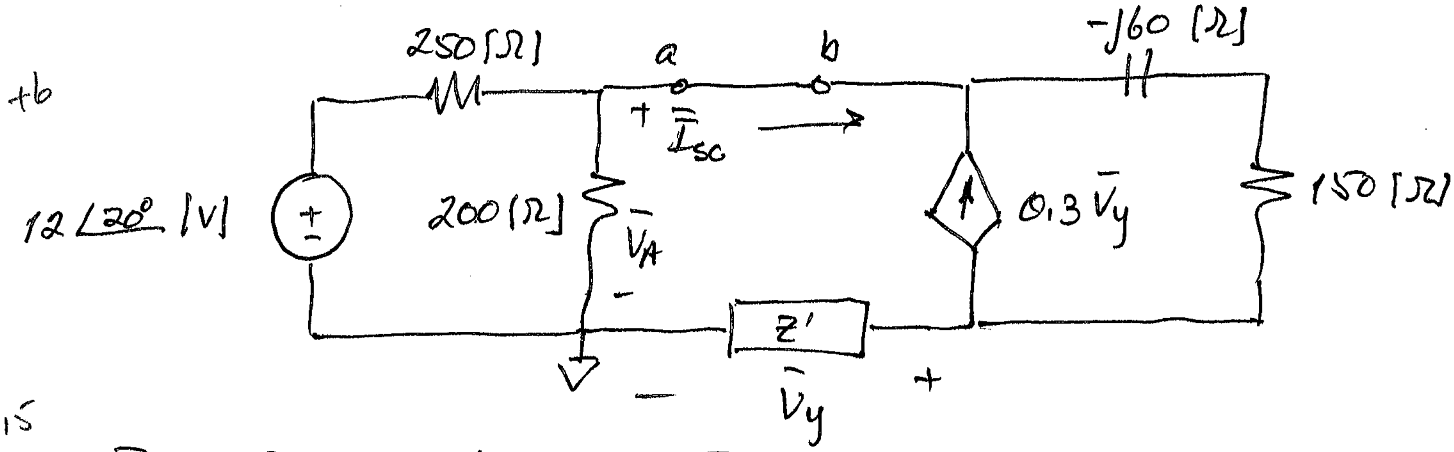
$$\Rightarrow -j2.20 = \frac{-j}{\omega C}$$

$$\therefore C = 0.909\ [\mu F]$$

Room for extra work

We could also have found $\bar{I}_{sc} = \bar{I}_N$:

+6



+15

$$\frac{\bar{V}_A}{200} + \frac{\bar{V}_A - 12\angle 20^\circ}{250} + \frac{\bar{V}_A - \bar{V}_y}{150 - j60} - 0.3\,\bar{V}_y = 0$$

$$\frac{\bar{V}_y}{255.8\angle 11.60^\circ} + 0.3\,\bar{V}_y + \frac{\bar{V}_y - \bar{V}_A}{150 - j60} = 0$$

$$\bar{I}_{sc} = -\frac{\bar{V}_A}{200} - \frac{\bar{V}_A - 12\angle 20^\circ}{250}$$

+4

$$\bar{V}_A = 4.97 + j\,1.80 \text{ [V]}$$

$$\bar{V}_y = 79.26 + 69.97 \text{ [mV]}$$

$$\bar{I}_{sc} = \bar{I}_N = 0.372 + j0.217 \text{ [mA]}$$

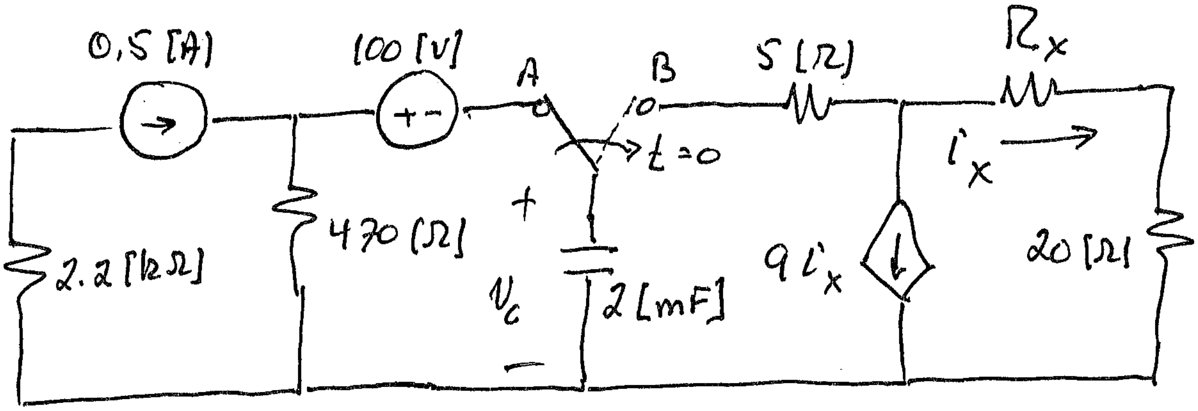
$$= 0.434\angle 30^\circ \text{ [mA]}$$

There is some round-off error but this is the same $\bar{I}_N$ we got earlier.

Z // j250 is wrong	-2	Z in series	-6	undefined	-2
R for Z	-2	inconsistent overbars	-4	no deactivation	-5
mixed domains	-8	wrong $\bar{I}_N$ polarity	-4	units	-1 ea.
I_N = test current		f = 500	-2		

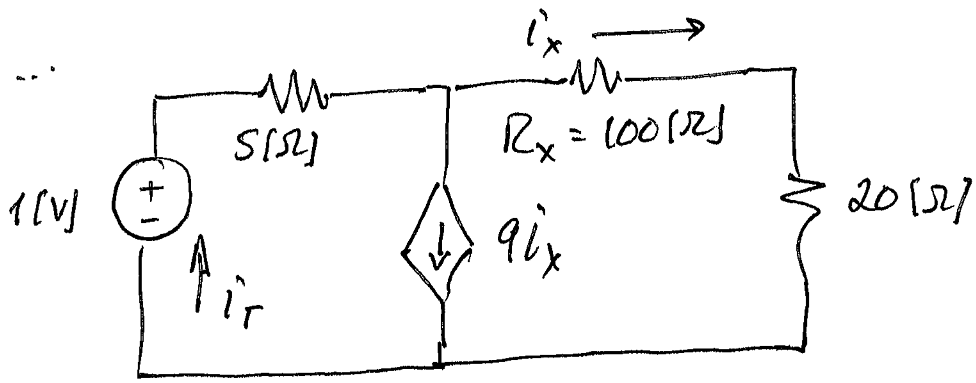
2. {40 Points} In the circuit below, the switch was at position ‘A’ for a long time, and then moved to ‘B’ at t = 0. The switch is designed so that after ten time constants in position ‘B’ it moves back to position ‘A’.

   a) How long does it take the switch to move back to position ‘A’ when the resistor R_X has the value 100 [Ω]?
   b) What value does R_X need to have in order to double the time you found in part a)?



All we need here is the time constant, so we will not take the time to find $v_c(0)$.

So for t > 0...



When $R_x = 100$ (Ω), we have...

$i_T = 10 i_x$ $\qquad -1 + 5 i_T + 120 i_x = 0$

$$\Rightarrow i_x = \frac{1}{170} = 5.882 \text{ [mA]}$$

+12 $\quad i_T = 58.82$ [mA] $\quad \Rightarrow R_{TH} = 17$ [Ω]

$$\Rightarrow \tau_c = R_{TH} \cdot C = 34 \text{ [ms]}$$

Room for extra work

a) So 10 time constants is $\underline{10\,\tau_c = 340\ [ms]}$.

+3

b) Now we need τ_c (or R_{TH}) as a function of R_x.

Referring to the diagram above, we have:

$$i_T = 10\, i_x \qquad -1 + 5 i_T + (R_x + 20) i_x = 0$$

$$\Rightarrow i_x = \frac{1}{70 + R_x}$$

+20

$$i_T = \frac{10}{70 + R_x} \Rightarrow R_{TH} = \frac{70 + R_x}{10}.$$

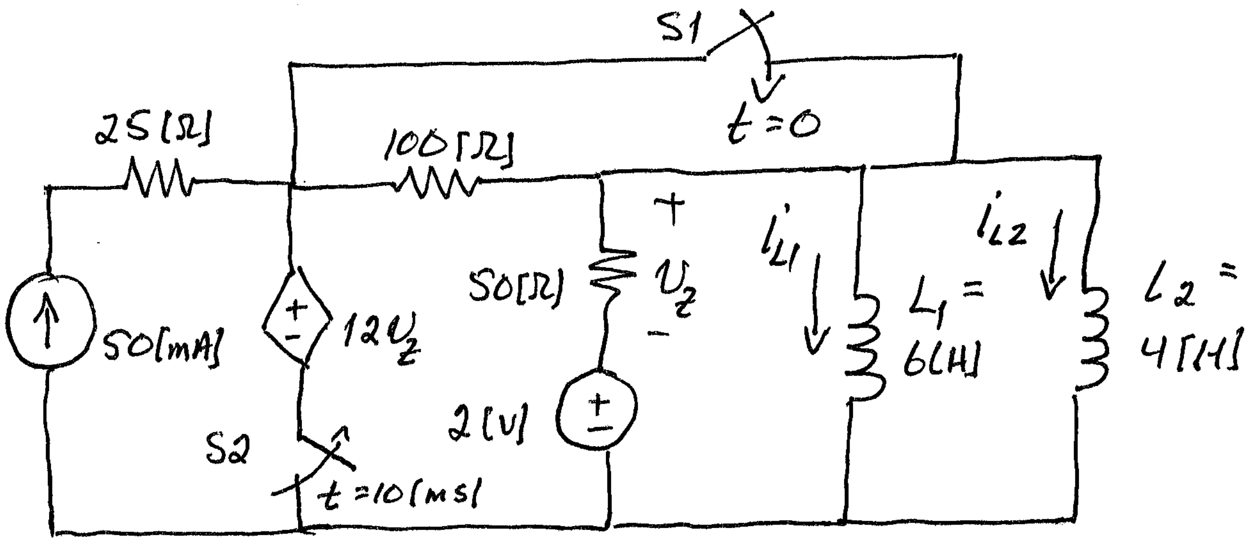
To double the time, we need twice R_{TH}. So

+5

$$\frac{70 + R_x}{10} = 34 \Rightarrow \underline{R_x = 270\ [\Omega]}.$$

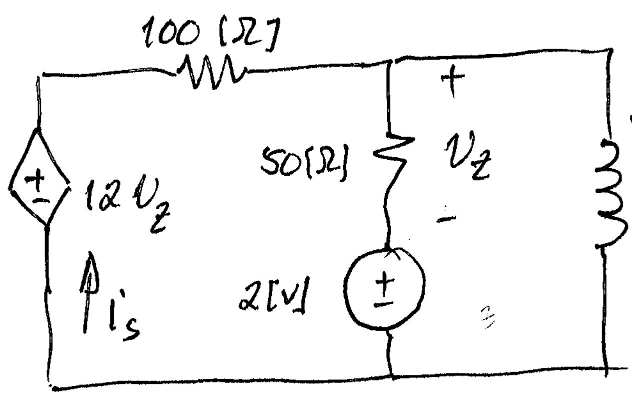
3. {45 Points} In the circuit below, switch S1 was open for a long time, and switch 2 was closed for a long time. At t = 0, S1 closed. After 10 [ms], S2 opened. At t = 0⁻, the current i_{L2} in inductor L_2 was 15 [mA].

   a) Find the current i_{L1} in inductor L_1 at $t = 0^-$.
   b) Find the current i_{L1} as a function of time for $t \geq 10$ [ms].



We need the initial current in L_1. To do that, we find the initial current in the equivalent inductor.

t<0



$L_{eq} = 4 \| 6 = 2.4$ [H]

$L_{eq} \rightarrow$ short $\Rightarrow v_Z = -2$ [V]

$\therefore \; i_s = \dfrac{12 v_Z}{100} = -0.24$ [A]

Room for extra work

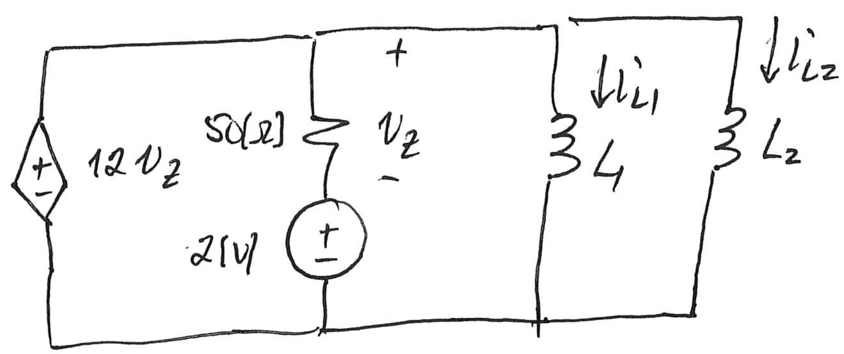
a)

$$i'_{Leq}(0) = i'_s - \frac{v_z}{50} = -200[mA]$$

+5 $$i'_{L1} + i'_{L2} = i'_{Leq} \Rightarrow i'_{L1}(0) = i'_{Leq}(0) - i'_{L2}(0) = \cancel{-35}[mA] \quad -215\ [mA]$$

$0 < t < 10\ [ms]$

+3



$$-12v_z + v_z + 2 = 0$$

$$v_z = \frac{2}{11} = 0.1818\ [V]$$

$$12v_z = 2.182\ [V]$$

+5 $$i'_{L1}(10[ms]) = \frac{1}{6}\int_0^{0.01[s]} 12v_z\, dt + i'_{L1}(0) \quad \#$$

$$= \frac{1}{6}\int_0^{0.01[s]} 2.182\, dt + (\cancel{-35}\ {-215}\ [mA]) = -\cancel{31.36}\ [mA] \quad -211.36\ [mA]$$

$0.364t$

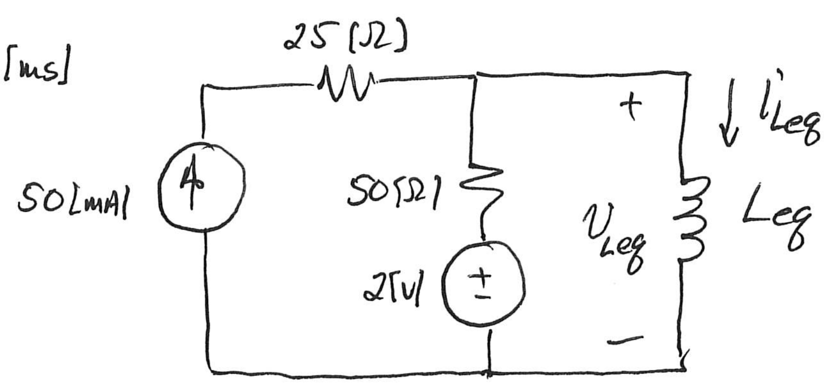
+5 $$i'_{L2}(10[ms]) = \frac{1}{4}\int_0^{0.01[s]} 2.182\, dt + 15[mA] = 20.46\ [mA]$$

0.546

+2 $$i'_{Leq}(10[ms]) = i'_{L1}(10[ms]) + i'_{L2}(10[ms]) = -\cancel{10.91}\ [mA] \quad \cancel{-0.650[mA]} \quad -190.9\ [mA]$$

page 2.

$t > 10[ms]$



Room for extra work

+3 $$i'_{Leq,f} = 0.05 + \frac{2}{50} = 90\,[mA]$$

+2 $$R_{TH} = 50\,[\Omega] \qquad \tau_L = \frac{2.4\,[H]}{50\,[\Omega]} = 48\,[ms]$$

$$i'_{Leq}(t) = i'_{Leq,f} + \left(i'_{Leq}(10[ms]) - i'_{Leq,f}\right) e^{\frac{-(t-10[ms])}{48[ms]}} \qquad t \geq 10[ms]$$

+5 $$i'_{Leq}(t) = 90 + (\overset{-190.91}{\cancel{-10.91}} - 90)\, e^{\frac{-(t-10[ms])}{48[ms]}}\ [mA] \qquad t \geq 10[ms]$$

+5 $$v_{Leq}(t) = L_{eq}\frac{di'_{Leq}(t)}{dt} = \overset{14.04}{\cancel{5.046}}\, e^{\frac{-(t-10[ms])}{48[ms]}}\ [V] \qquad t > 10[ms]$$

b)

+8 $$i'_{L1}(t) = \frac{1}{L_1}\int_{10[ms]}^{t} v_{eq}(t)\,dt + i'_{L1}(10[ms])$$

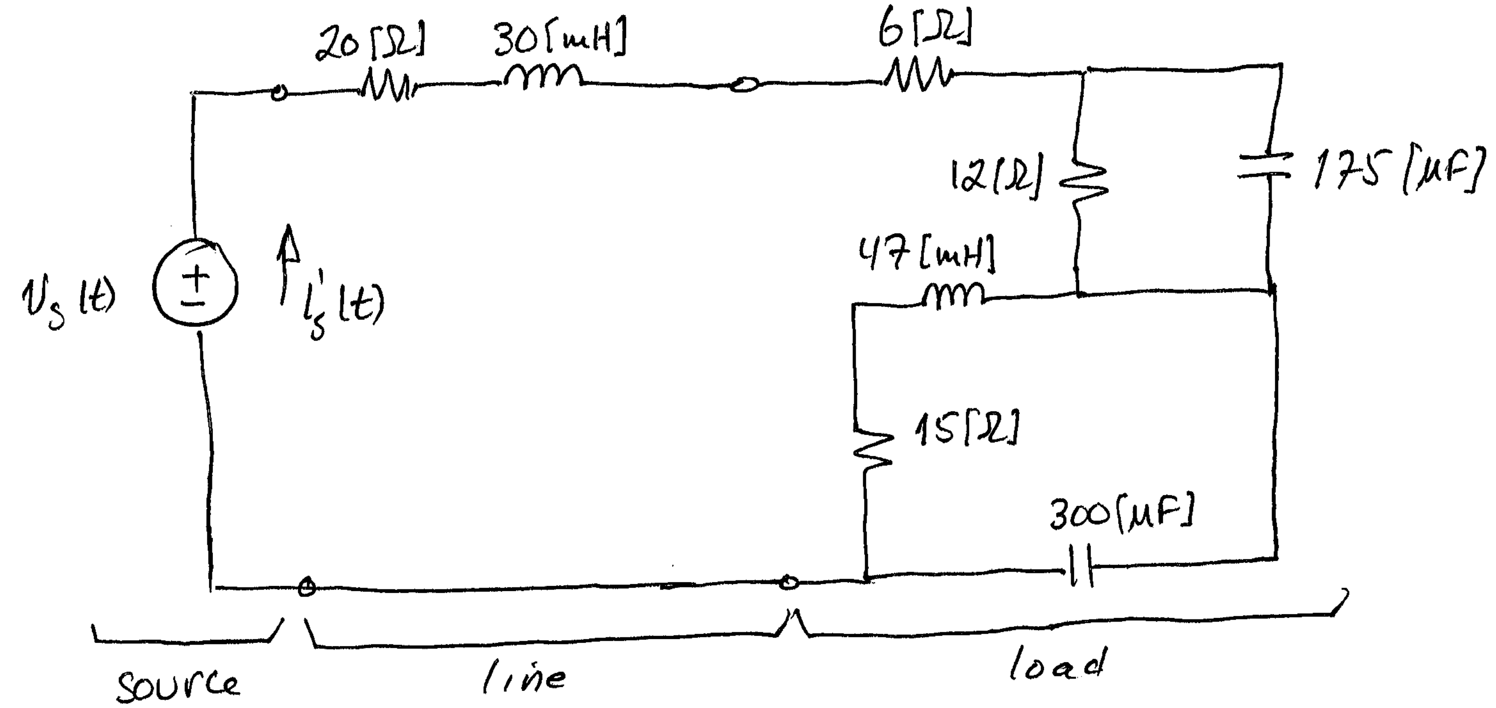
$$= \frac{1}{6}\int_{0.01}^{t} \overset{14.04}{\cancel{5.046}}\, e^{-\frac{(t-10[ms])}{48[ms]}}\,dt - \overset{211.14}{\cancel{31.36}}\ [mA]$$

$$= \frac{\overset{14.04}{\cancel{5.046}}}{6}(-0.048)\cdot e^{-\frac{(t-0.01)}{0.048}}\Bigg|_{0.01}^{t} - \overset{0.21114}{\cancel{0.03136}}\ [A]$$

+2 $$\boxed{i'_{L1}(t) = \overset{-0.1123}{\cancel{-0.04037}}\left(e^{-\frac{(t-0.01)}{0.048}} - 1\right) - \overset{0.21114}{\cancel{0.03136}}}\ [A] \quad t \geq 10[ms]$$

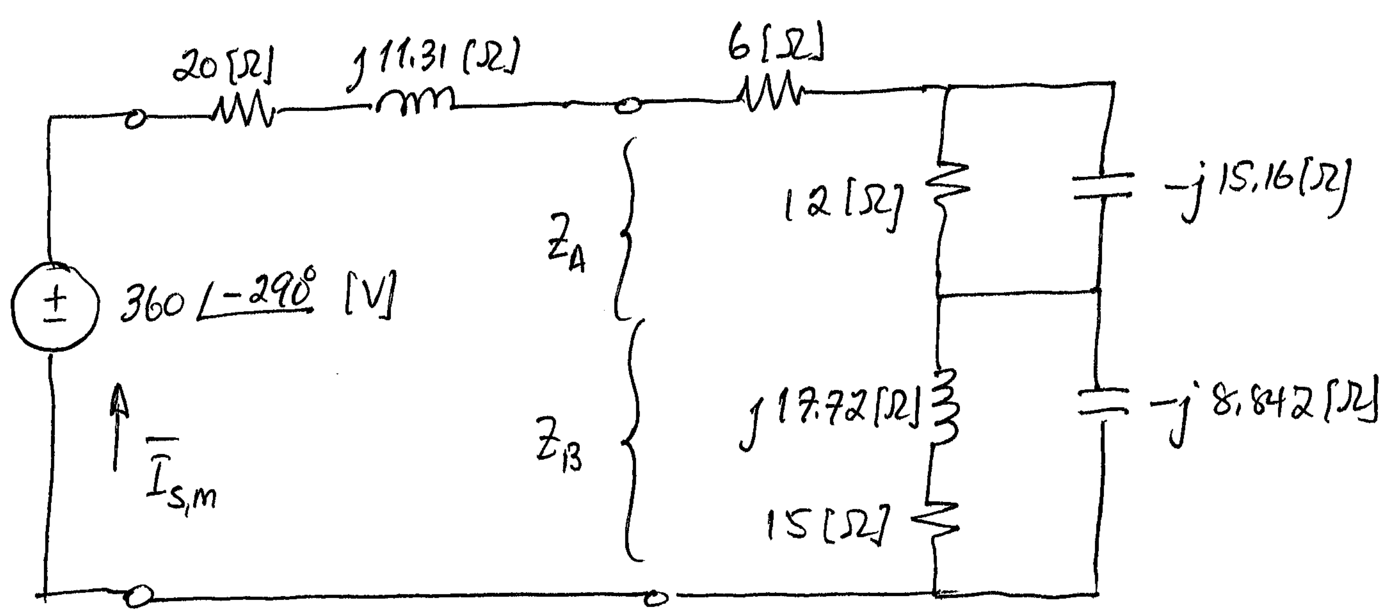
4. {45 Points} The circuit below models a power source, a transmission line, and a load, as labeled. The source voltage $v_S(t) = 360[V]\sin\left(377\left[\frac{rad}{s}\right]t - 200^o\right)$.

   a) Find the complex power absorbed by the load.
   b) Find the current $i_S(t)$.
   c) Find the power absorbed by the load if the source voltage is changed to $v_S = 360[V]$.



+3 Convert to phasor domain: $v_s(t) = 360\,[V]\cos\left(377\left[\frac{rad}{s}\right]t - 290^\circ\right)$

+8



$\bar{V}_{S,rms} = 254.56 \angle -290^\circ$ [V]$_{rms}$

$Z_{TOTAL} = 37.25 - j5.645$ [Ω]
$37.68 \angle -8.616$ [Ω]

Room for extra work

$$Z_A = 6 + 12 // (-j15.16) = 13.378 - j5.8397 \; [\Omega]$$

$\hookrightarrow 7.381 - j5.840$ [Ω] $9.412 \angle -38.35^\circ$ [Ω]

$$= 14.60 \angle -23.58^\circ \; [\Omega]$$

$$Z_B = -j8.842 // (15 + j17.72) = 3.8600 - j11.127 \; [\Omega]$$

$$= 11.78 \angle -70.87^\circ \; [\Omega]$$

Use "m"

$Z_A + Z_B = 17.24 - j16.97$ [Ω]
$= 24.19 \angle -44.5$

+10

$$\bar{I}_s = \frac{360 \angle -290^\circ}{20 + j11.31 + Z_A + Z_B} = 1.8822 + j9.3697 \; [A]$$

rms $1.331 + j6.625$

$$= 9.5570 \angle 78.64^\circ \; [A]$$

rms 6.758

b) $\therefore \; i_s'(t) = 9.557 \, [A] \cos(377 \, [\tfrac{rad}{s}] t + 78.64^\circ) \, [A]$

+4

a) $$S_{abs, Load} = \frac{1}{2} |\bar{I}_s|^2 (Z_A + Z_B)$$

45.67

$$= 788.16 - j774.69 \; [VA] = 1105.1 \angle -44.51^\circ \; [VA]$$

+12

c) $V_s = 360$ [V] implies a frequency of 0, so $C \rightarrow$ open and $L \rightarrow$ short. Then

+8

$$i_s' = \frac{360}{20 + 6 + 12 + 15} = \cancel{5.714} \; 6.792 \; [A]$$

$$\boxed{P_{abs, Load} = i_s'^2 \cdot (6 + 12 + 15) = \cancel{1077.6} \; 1522.5 \; [W]}$$

$\bar{V}_L = \bar{I}_s \cdot Z_L = 191.46 + j129.58$ [V]
$= 231.2 \angle 34.1^\circ$ [V]

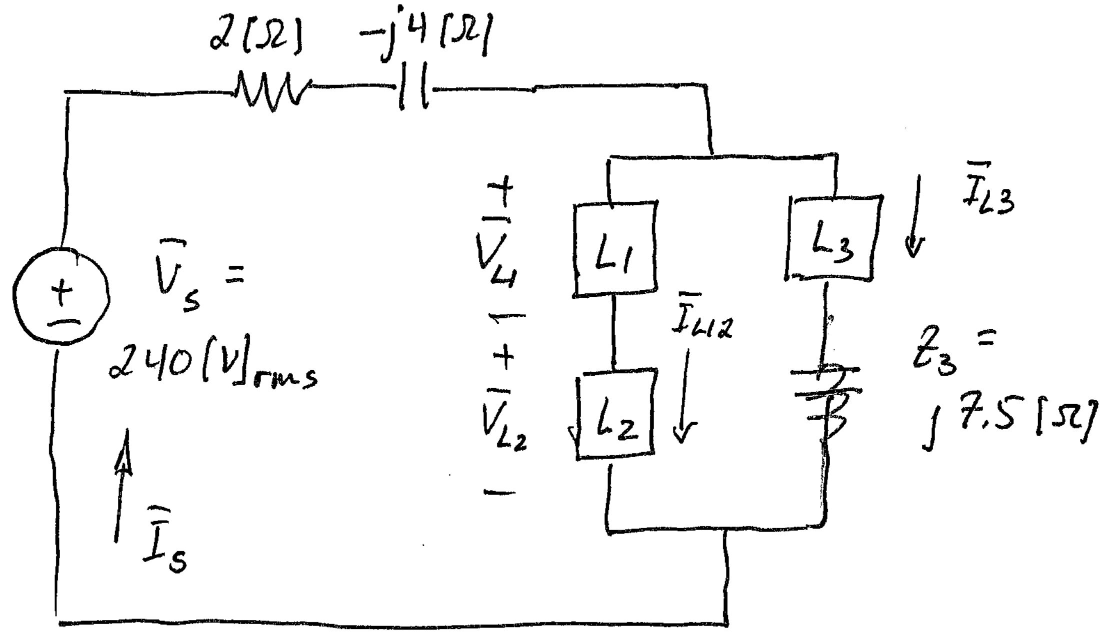
rms: $163.48 \angle 34.1^\circ$ [V]
$= 135.4 + j91.65$ [V]

$Z_A + Z_B + Z_{LINE} = 37.25 - j5.645$ [Ω]
$= 37.68 \angle -8.616$ [Ω]

5. {45 Points} The loads in the circuit below are described as follows.

Load L_1 absorbs 2500 [W] at a power factor of 0.7 lagging.
Load L_2 absorbs 3000 [W] and 1500 [VAR].
Load L_3 absorbs 5500 [VA] at a power factor of 0.8 leading.
$\overline{V}_{L1} = 120\angle 60^o\ [V]_{rms}$.

a) Find the total power delivered by the source $\overline{V}_S$.
b) Find the apparent power delivered by the source $\overline{V}_S$.
c) Find the power factor of the load consisting of L_3 in combination with Z_3.



Load 1: $P = 2500\ [W] \Rightarrow |S| = \dfrac{2500}{0.7} = 3571.4\ [VA]$

∴ Load 1 is absorbing

$$S_{abs,L1} = \frac{2500}{0.7}(0.7) + j\,\frac{2500}{0.7}\left(\sqrt{1-0.7^2}\right)$$

$$= 2500 + j\,2550.5\ [VA]$$

x 5

(+5)

+3

Room for extra work

$S_{abs,L1} = 2500 + j2550.5$ [VA]
$3571 \angle 45.6°$ [VA]

+ Load 2: $S_{abs,L2} = 3000 + j1500$ [VA]
$3354 \angle 26.56°$ [VA]

Load 3: $S_{abs,L3} = 5500(0.8) - j5500(\sqrt{1-0.8^2})$
$4400 - j3300$

+4 $= \cancel{2580 - j1530.9}$ [VA]
$5500 \angle -36.87°$

$S_{abs,L1} = \bar{V}_{L1} \cdot \bar{I}^*_{L12} \Rightarrow \bar{I}^*_{L12} = \dfrac{2500 + j2550.5}{120 \angle 60°}$

$= 28.82 - j7.415$ [A]$_{rms}$

+5 $\bar{I}_{L12} = 28.82 + j7.415$ [A]$_{rms}$
$29.76 \angle 14.43°$

$S_{abs,L2} = \bar{V}_{L2} \bar{I}^*_{L12} \Rightarrow \bar{V}_{L2} = \dfrac{3000 + j1500}{28.82 - j7.415}$

+5

$= 85.06 + j73.92$ [V]$_{rms}$
$112.7 \angle 40.99$

+5 $\bar{I}_s = \dfrac{\bar{V}_s - \bar{V}_{L1} - \bar{V}_{L2}}{2 - j4} = 45.06 + j1.202$ [A]$_{rms}$
$= 45.08 \angle 1.528°$ [A]$_{rms}$

+3 $\therefore \bar{I}_{L3} = \bar{I}_s - \bar{I}_{L12} = 16.24 - j6.213$ [A]$_{rms}$
$= 17.39 \angle -20.94°$ [A]$_{rms}$

a) $S_{del\ by\ \bar{V}_s} = \overbrace{\underbrace{S_{L1} + S_{L2} + S_{L3}}_{9898.5 + j751.0}}^{9927 \angle 4.339} + \underbrace{|\bar{I}_s|^2(2 - j4)}_{4064.4 - j8128.8}^{9088.3 \angle -63.43} + \underbrace{|\bar{I}_{L3}|^2(j7.5)}_{j2268.1}$?

+2 +3 +3

$= \cancel{12.144 - j3.341}$ [kVA] $= \cancel{12.60 \angle -15.38°}$ [kVA]
$13.96 - j5.110$ [kVA] $\quad 14.87 \angle -20.1°$ [kVA]

b) apparent power is $|S_{del\ by\ \bar{V}_s}| = \cancel{12.60}$ [kVA]
14.87

+2

$S_{del\ by\ \bar{V}_s} = 240 \cdot \bar{I}^*_s = 10.815 - j0.288$ [kVA] $= 10.819 \angle -1.525°$ [kVA]

Room for extra work

c) Total power absorbed by Load 3 and $j7.5\,[\Omega]$ is

$$(\bar{V}_{L1} + \bar{V}_{L2})\,\bar{I}_{L3}^{\,*} = 1250.7 + j\,3790.0\ [VA]$$
$$= 3991.0\ \angle 71.737^\circ\ [VA]$$

+5 $\boxed{\text{So } pf = \cos(71.74^\circ) = 0.3133}$

$$3991.0\ \angle 71.737^\circ - |\bar{I}_{L3}|^2 (j\,7.5) = 1274.9 + j\,1513.8\ [VA]$$
$$\neq S_{abs,L3}$$

$$Z_{LOAD\,3} + j\,7.5 = 14.94\ \angle -13.20^\circ\ [\Omega]$$

$$\Rightarrow pf = 0.9736^\circ$$