Name: ______________________________ (please print)

Signature: ___________________________

ECE 2202 – Exam 2

November 4, 2023

Keep this exam closed until you are told to begin.

1. This exam is closed book, closed notes. You may use one 8.5" x 11" crib sheet, or its equivalent.
2. Show all work on these pages. Show all work necessary to complete the problem. A solution without the appropriate work shown will receive no credit. A solution that is not given in a reasonable order will lose credit. Clearly indicate your answer (for example by enclosing it in a box).
3. Show all units in solutions, intermediate results, and figures. Units in the exam will be included between square brackets.
4. If the grader has difficulty following your work because it is messy or disorganized, you will lose credit.
5. Do not use red ink. Do not use red pencil.
6. You will have 90 minutes to work on this exam.

1. ________________/35

2. ________________/25

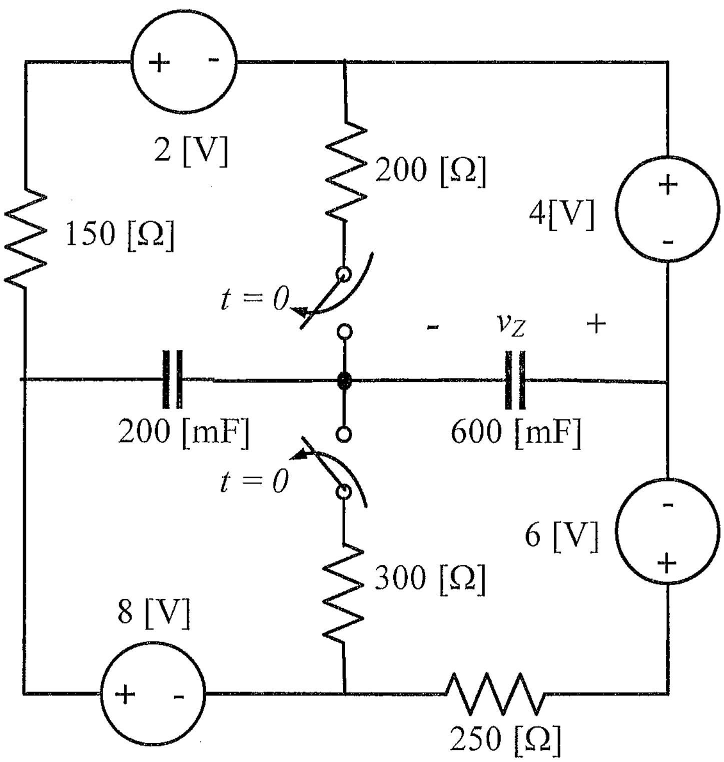
3. ________________/40

Total = 100

Room for extra work

1. {35 Points} In the circuit below, both switches were closed for a long time, and then opened at t = 0.

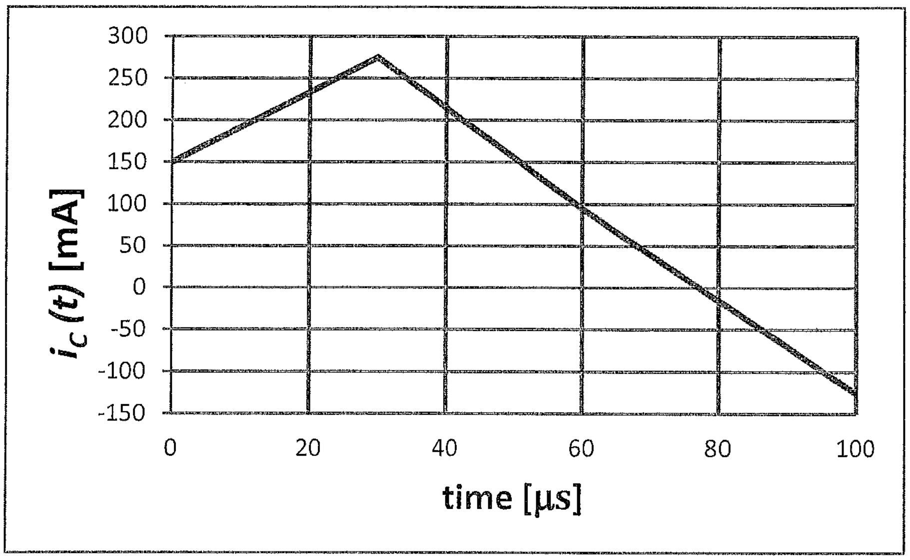
   a) Find an expression for the voltage v_Z as a function of time for $t \geq 0$.
   b) Find the value of v_Z after two time constants.

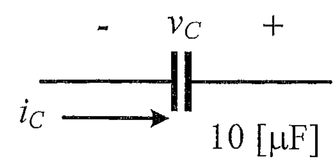


Room for extra work

2. {25 Points} The current $i_C(t)$ through a 10 [μF] capacitor is given in the graph below. It is known that $v_C(60[\mu S]) = 225$ [mV]. The capacitor voltage polarity and current direction are shown in the figure.

Find v_C at 0 [μs].

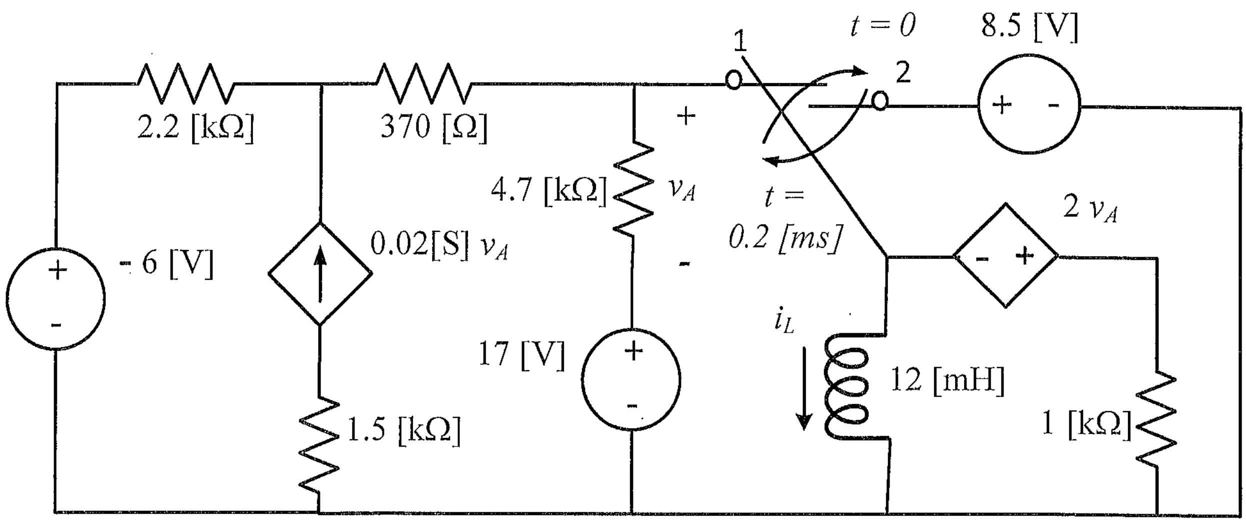




Room for extra work

3. {40 Points} In the circuit below, the switch was in position 1 for a long time, and moved to position 2 at t = 0. At t = 0.2 [ms], the switch moved back to position 1.

   a) Find an expression for the current i_L as a function of time for t ≥ 0.
   b) Find v_A at 0.3 [ms].

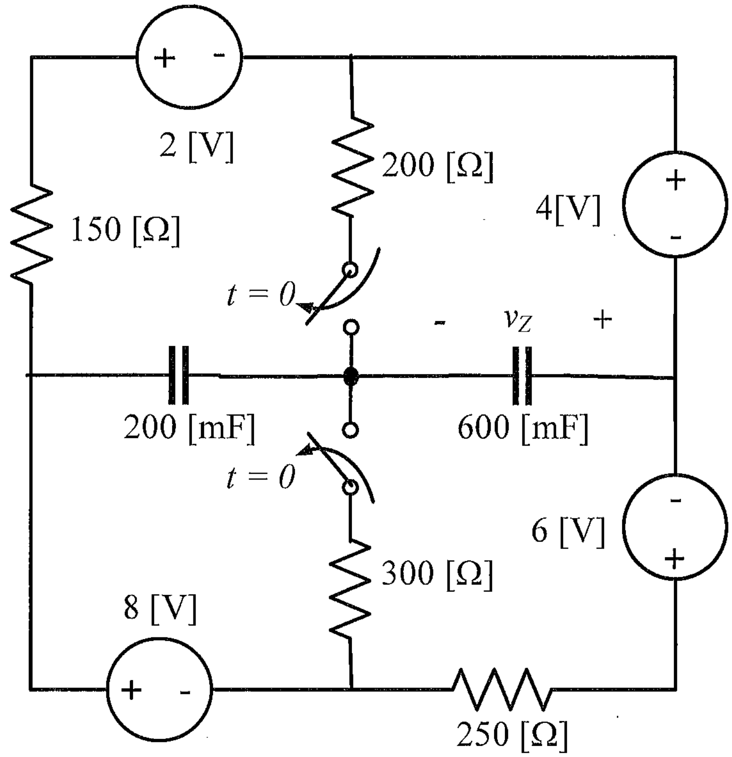


Room for extra work

Room for extra work

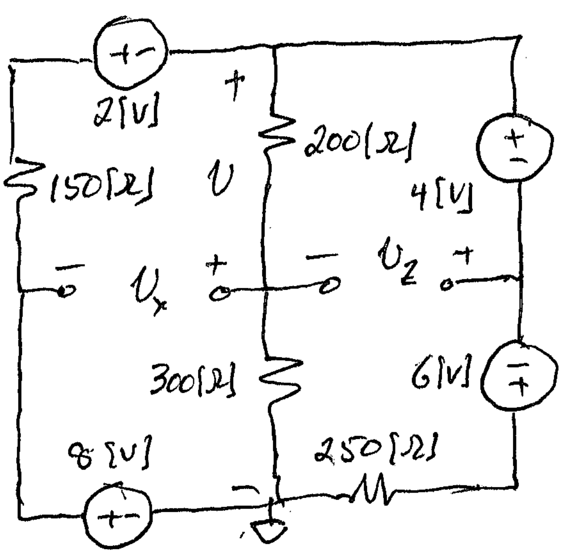
1. {35 Points} In the circuit below, both switches were closed for a long time, and then opened at t = 0.

   a) Find an expression for the voltage v_Z as a function of time for $t \geq 0$.
   b) Find the value of v_Z after two time constants.



t<0
with switches closed, and after a long time so that C → open, we have:

a)



$$\frac{v}{500} + \frac{v-8+2}{150} + \frac{v+6-4}{250} = 0$$

$$\Rightarrow v = 2.526 \text{ [V]}$$

$$v_x + 8 + \frac{-v}{500} \cdot 300 = 0$$

$$\Rightarrow v_x = -6.484 \text{ [V]}$$

$$v_z - \frac{v}{500} \cdot 200 + 4 = 0$$

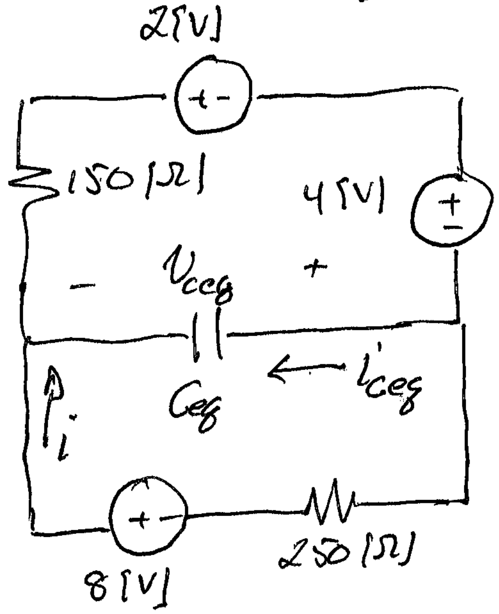
$$\Rightarrow v_z = -2.990 \text{ [V]}$$

These are the initial capacitor values $v_x(0)$, $v_z(0$

Room for extra work

$t>0$ switches are open. Combine capacitors into C_{eq}:

$$C_{eq} = \left(\frac{1}{0.2} + \frac{1}{0.6}\right)^{-1} = 0.15\,[H]$$



$R_{TH} = 150 \| 250 = 93.75\,[\Omega]$

$\tau_c = C_{eq} R_{TH} = 14.06\,[s]$

$V_{c_{eq}}(0) = V_x(0) + V_z(0) = -9.474\,[V]$

$V_{c,f}$: with $C_{eq} \to$ open :

$2 + 4 - 6 - 8 + 400\,i = 0 \Rightarrow i = 20\,[mA]$

So $V_{ceq,f} + 150\,i + 2 + 4 = 0 \Rightarrow V_{ceq,f} = -9{,}000\,[V]$

$$V_{ceq}(t) = V_{ceq,f} + \left(V_{ceq}(0) - V_{ceq,f}\right) e^{-t/\tau_c}$$

$$V_{ceq}(t) = -9.000 + \left(-9.474 - (-9.000)\right) e^{-t/14.06[s]}\ [V] \quad t \geq 0$$

$$V_{ceq}(t) = -9 - 0.474\, e^{-t/14.06[s]}\ [V] \quad t \geq 0$$

We now differentiate to find $i_{ceq}(t)$ and then integrate that to find $V_z(t)$ ...

↗ p.2

Room for extra work

$$i_{ceq}(t) = C_{eq}\frac{d}{dt}(v_{ceq}(t))$$

$$v_z(t) = \frac{1}{0.6}\int_0^t i_{ceq}(t)\,dt + v_z(0)$$

$$= \frac{0.15}{0.6} v_{eq}(t)\Big|_0^t + v_z(0)$$

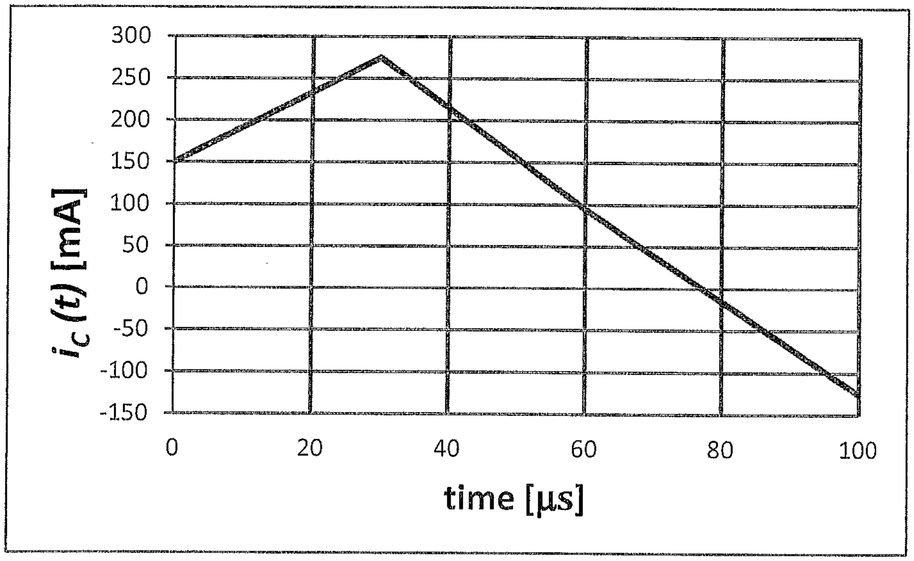
$$= 0.25(-9 - 0.474e^{-t/14.06[s]}) + (-2.990)$$

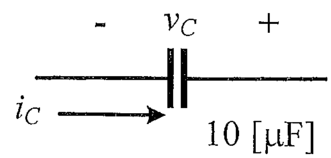
$$v_z(t) = (-5.240 - 0.1185e^{-t/14.06[s]})[V] \quad t \geq 0$$

b) $v_z(t = 2\tau_c) = -5.240 - 0.1185\,e^{-2} = -5.256\,[V]$

2. {25 Points} The current $i_C(t)$ through a 10 [μF] capacitor is given in the graph below. It is known that $v_C(60[\mu S]) = 225$ [mV]. The capacitor voltage polarity and current direction are shown in the figure.

Find v_C at 0 [μs].





If we have equations for the two lines, we can integrate from 30[μs] to 60[μs] to find $v_C(30[\mu s])$, and then from 0 to 30[μs] to find $v_C(0)$.

$0 < t < 30[\mu s]$

$$i_C(t) = \frac{0.275 - 0.150}{30\times10^{-6}}\,t + 0.150$$

$$i_C(t) = 4166.7\left[\tfrac{A}{s}\right]t + 0.150\,[A]$$

$t > 30\,[\mu s]$

$$i_C(t) = mt + b$$

$$0.275 = m(30\times10^{-6}) + b$$
$$-0.125 = m(100\times10^{-6}) + b$$
$$\Rightarrow \quad m = -5714.3\left[\tfrac{A}{s}\right], \quad b = 0.4464\,[A]$$

$$i_C(t) = -5714.3\left[\tfrac{A}{s}\right]t + 0.4464\,[A]$$

Room for extra work

$$v_c(60[\mu s]) = \frac{-1}{C}\int_{30\times10^{-6}[s]}^{60\times10^{-6}[s]} (-5714.3t + 0.4464)\,dt + v_c(30[\mu s]) = 0.225[A]$$

$$= -10^5 \cdot 5.6778\times10^{-6} + v_c(30[\mu s]) = 0.225$$

$$\Rightarrow \quad v_c(30[\mu s]) = 0.7928\ [V]$$

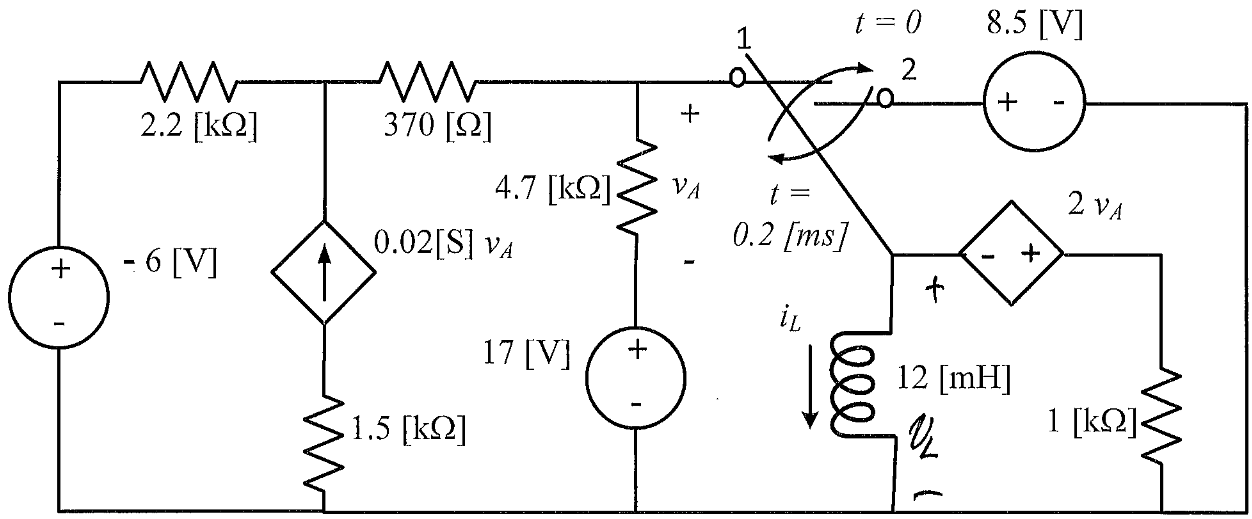
$$v_c(30[\mu s]) = \frac{-1}{C}\int_{0}^{30\times10^{-6}[s]} (4166.7t + 0.150)\,dt + v_c(0) = 0.7928\ [V]$$

$$= -10^5 \cdot 6.375\times10^{-6} + v_c(0) = 0.7928\ [V]$$

$$\Rightarrow \quad v_c(0) = 1.430\ [V]$$

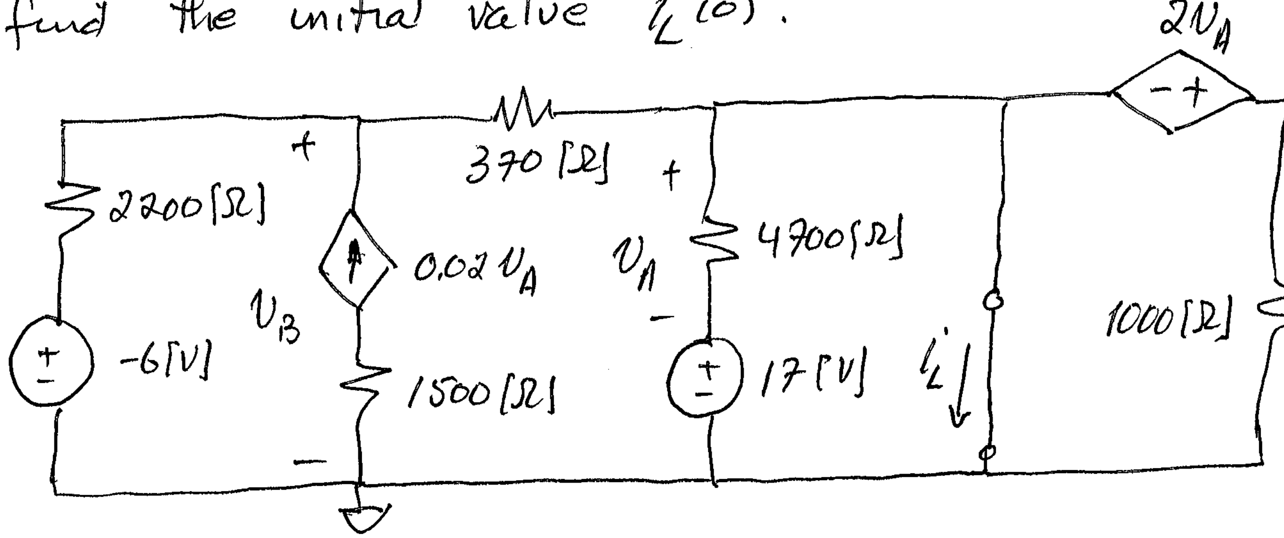
3. {40 Points} In the circuit below, the switch was in position 1 for a long time, and moved to position 2 at t = 0. At t = 0.2 [ms], the switch moved back to position 1.

   a) Find an expression for the current i_L as a function of time for $t \geq 0$.
   b) Find v_A at 0.3 [ms].



a)

For $t<0$ we model the inductor as a short to find the initial value $i_L(0)$.



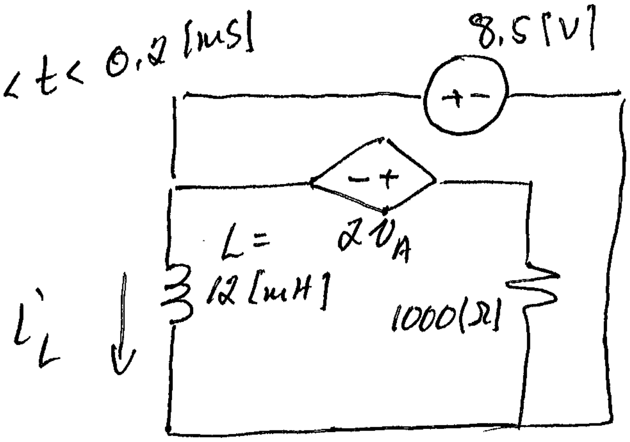
$$-0.02\,V_A + \frac{V_B+6}{2200} + \frac{V_B}{370} = 0 \quad \Rightarrow \quad V_B = -108.55\ [V]$$

$$V_A = -17\ [V]$$

Room for extra work

KCL $i_L(0) = \frac{-2v_A}{1000} - \frac{v_A}{4700} + \frac{v_B}{370} = -255.76\,[mA]$

$0 < t < 0.2\,[ms]$



The inductor is in parallel with a voltage source, so...

$$i_L(t) = \frac{1}{L}\int_0^t 8.5\,dt + i_L(0)$$

$0 < t < 0.2\,[ms]$

$$i_L(t) = \frac{1}{0.012}\int_0^t 8.5\,dt - 255.76\,[mA] = 708.33t - 255.76\,[mA]$$

$t = 0.2\,[ms] \Rightarrow i_L(0.2\,[ms]) = 708.33\left[\frac{A}{s}\right](2\times10^{-4}\,[s]) - 0.25576\,[A]$

$$i_L(0.2\,[ms]) = -114.09\,[mA]$$

$t > 0.2\,[ms]$

When the switch returns to position 1, we have the same configuration as for $t<0$. So the final value for i_L is the same as the initial value:

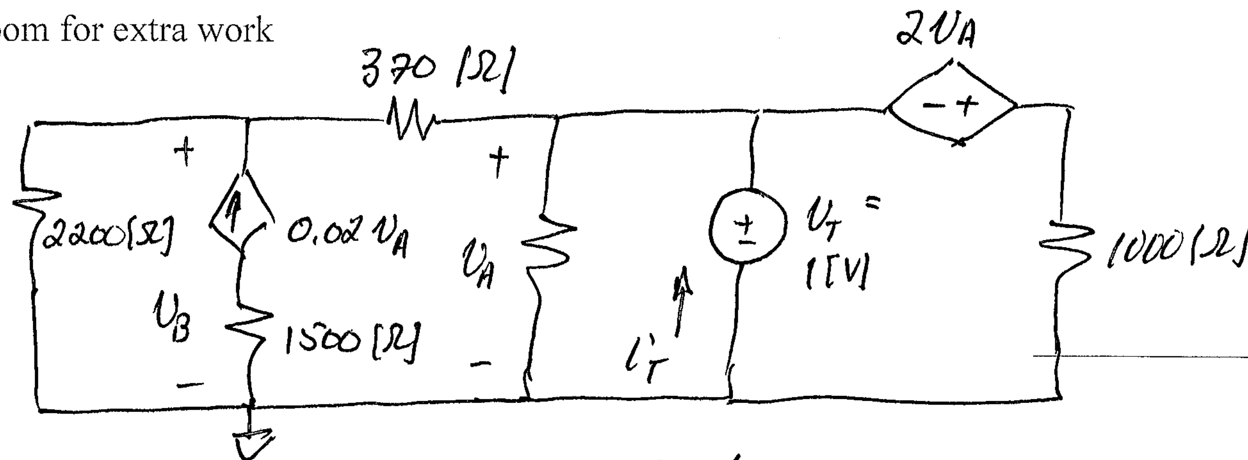
$$i_{L,f} = i_L(0) = -255.76\,[mA]$$

We also need a time constant:

→ p. 9

Room for extra work

R_{TH} :



$$v_A = 1\ [V] \qquad -0.02(1) + \frac{v_B}{2200} + \frac{v_B - 1}{370} = 0 \Rightarrow v_B = 7.1907\ [V]$$

$$i_T = \frac{1+2}{1000} + \frac{1}{4700} + \frac{1 - 7.1907}{370} = -13.52\ [mA]$$

$$R_{TH} = -73.97\ [\Omega] \Rightarrow \tau_L = L/R_{TH} = \frac{0.012}{-73.97} = -0.1622\ [ms]$$

$$i_L(t) = i_{L,f} + (i_L(.2\,[ms]) - i_{L,f})\, e^{-\frac{(t-0.2[ms])}{-0.1622[ms]}}$$

$$= -255.76\ [mA] + (-114.09\ [mA] - (-255.76\ [mA]))\, e^{-\frac{(t-0.2[ms])}{-0.1622[ms]}}$$

$$\boxed{i_L(t) = -255.76\ [mA] + 141.67\ [mA]\, e^{\frac{t-0.2[ms]}{0.1622[ms]}}} \qquad t \geq 0.2\ [ms]$$

b) with v_L defined in the original circuit diagram:

$$v_A(t) + 17 - v_L(t) = 0 \qquad v_A(t) = v_L(t) - 17\ [V]$$

$$v_L(t) = L\frac{di_L(t)}{dt} = (0.012)\frac{1}{0.1622\,[ms]}\, e^{\frac{t-0.2[ms]}{0.1622[ms]}}$$

$$v_L(0.3\,[ms]) = 73.98\ [V]\, e^{\frac{(0.3-0.2)[ms]}{0.1622[ms]}} = 137.05\ [V]$$

$$\boxed{v_A(0.3\,[ms]) = 120.05\ [V]}$$