Name: ______________________________ (please print)

Signature: ___________________________

ECE 2202 – Final Exam

December 6, 2023

Keep this exam closed until you are told to begin.

1. This exam is closed book, closed notes. You may use one 8.5" x 11" crib sheet, or its equivalent.
2. Show all work on these pages. Show all work necessary to complete the problem. A solution without the appropriate work shown will receive no credit. A solution that is not given in a reasonable order will lose credit. Clearly indicate your answer (for example by enclosing it in a box).
3. Show all units in solutions, intermediate results, and figures. Units in the exam will be included between square brackets.
4. If the grader has difficulty following your work because it is messy or disorganized, you will lose credit.
5. Do not use red ink. Do not use red pencil.
6. You will have 160 minutes to work on this exam.

1. _______________/50

2. _______________/40

3. _______________/50

4. _______________/40

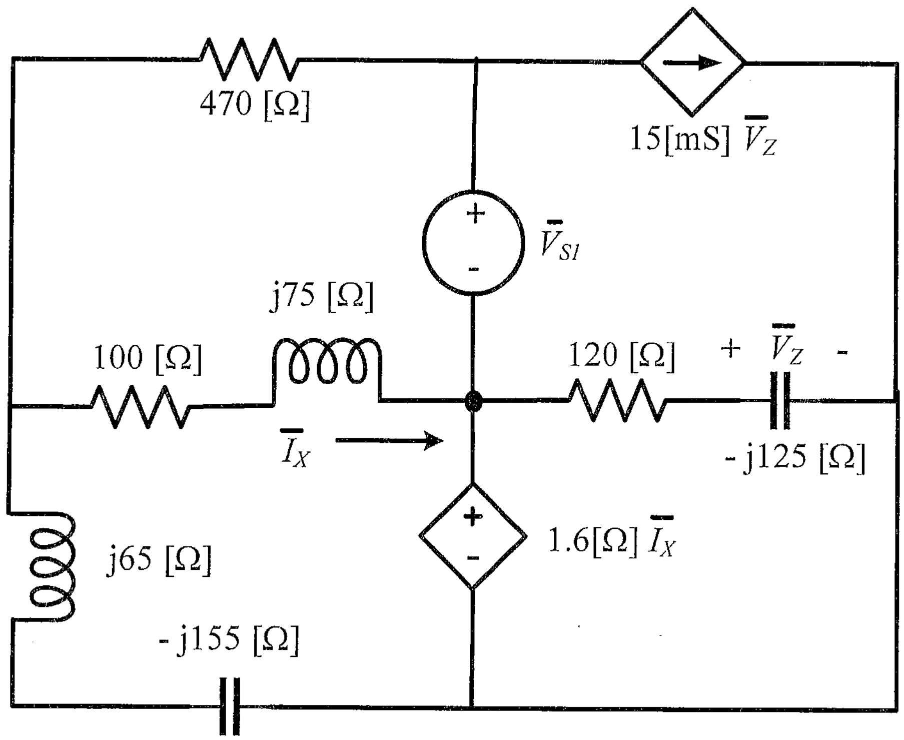
5. _______________/40

Total = 220

Room for extra work

1. {50 Points} For the circuit below, do the following.

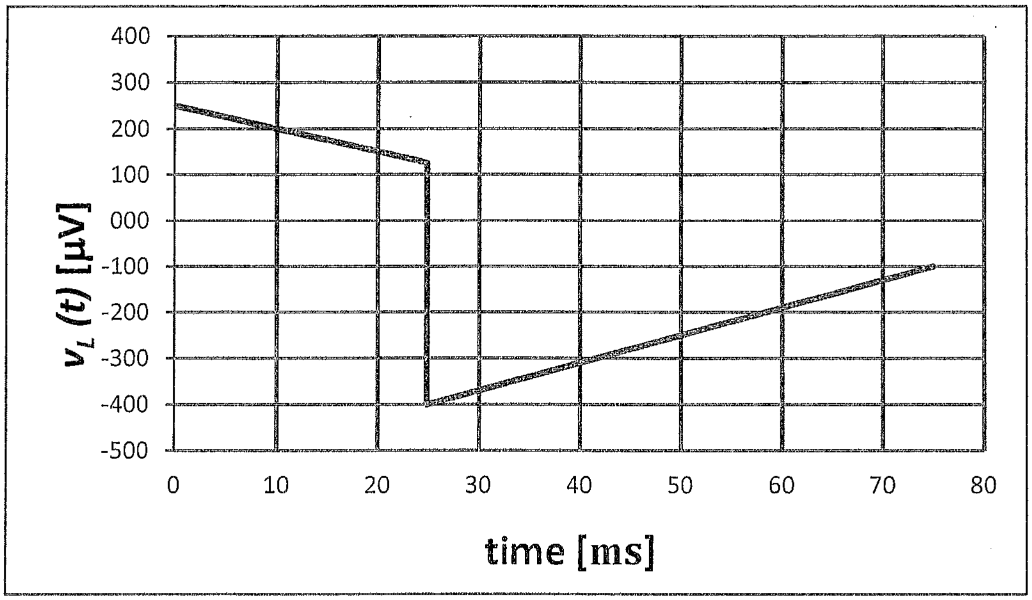
   a) Find the Thevenin equivalent impedance seen by $\bar{V}_{S1}$.
   b) Express the Thevenin equivalent impedance in terms of resistors, capacitors, and inductors, assuming the operating frequency is 350 [Hz].

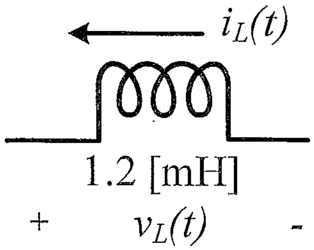


Room for extra work

2. {40 Points} The voltage $v_L(t)$ across a 1.2 [mH] inductor is given in the graph below. The inductor voltage polarity and current direction are shown in the figure. It is known that $i_L(50[mS]) = 2.25$ [mA].

Find i_L at 10 [ms].

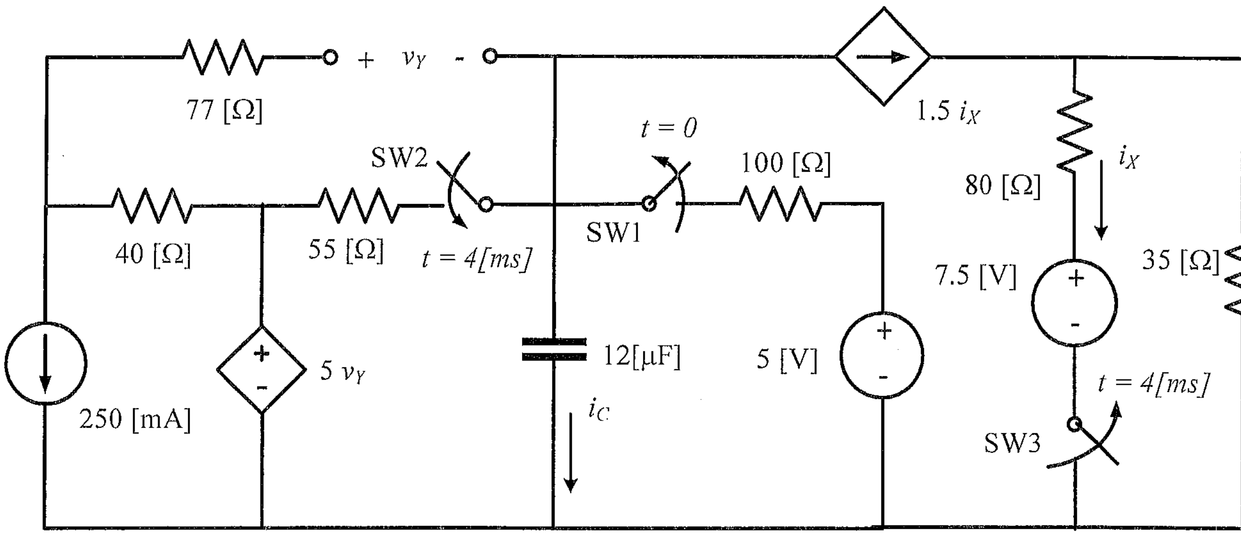




Room for extra work

3. {50 Points} In the circuit below, switch SW1 was closed for a long time, SW2 was open for a long time, and SW3 was closed for a long time. At t = 0, SW1 opened. At t = 4 [ms], SW2 closed, and SW3 opened.

Find an expression for the current i_C as a function of time for t > 4 [ms].



Room for extra work

Room for extra work

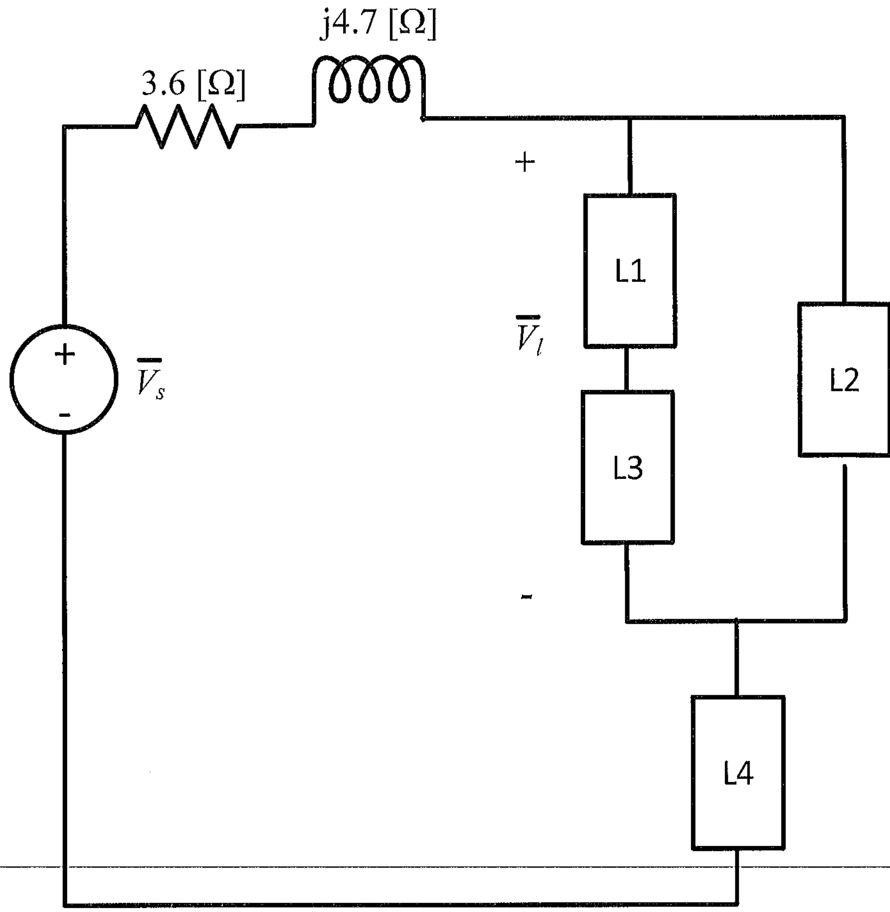
4. {40 points} In the circuit below, the following is known.

Load L1 absorbs 15 [kW] at 0.84 power factor leading.
Load L2 absorbs 22 [kVA] at 0.66 power factor lagging.
Load L3 absorbs 12 [kW] and delivers 16 [kVAR].
Load L4 is a 2.7 [Ω] resistor in parallel with a reactance of -j41 [Ω].

In the time domain, the voltage indicated by $\bar{V}_l$ is

$$v_L(t) = 678.82[\text{V}]\cos\left(377\left[\frac{rad}{s}\right]t + 20°\right).$$

Find $\bar{V}_s$.

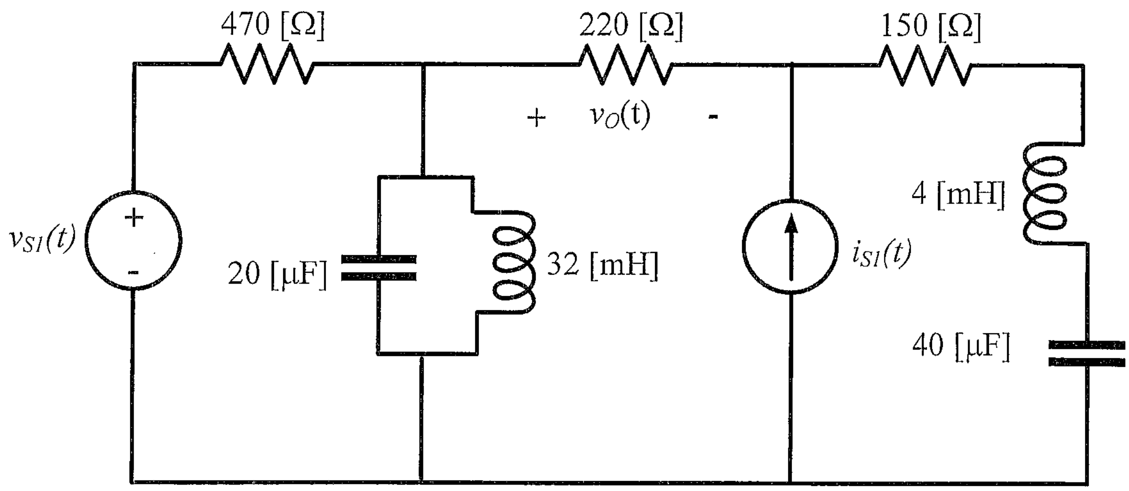


Room for extra work

5. {40 points} For the following circuit, find $v_O(t)$. It is given that

$$v_{S1}(t) = 17.5\ [mV]\cos\left(1250\left[\frac{rad}{s}\right]t - 22°\right), \text{ and}$$

$$i_{S1}(t) = 32.7\ [mA]\sin\left(2500\left[\frac{rad}{s}\right]t + 13°\right).$$



Room for extra work

1. {50 Points} For the circuit below, do the following.

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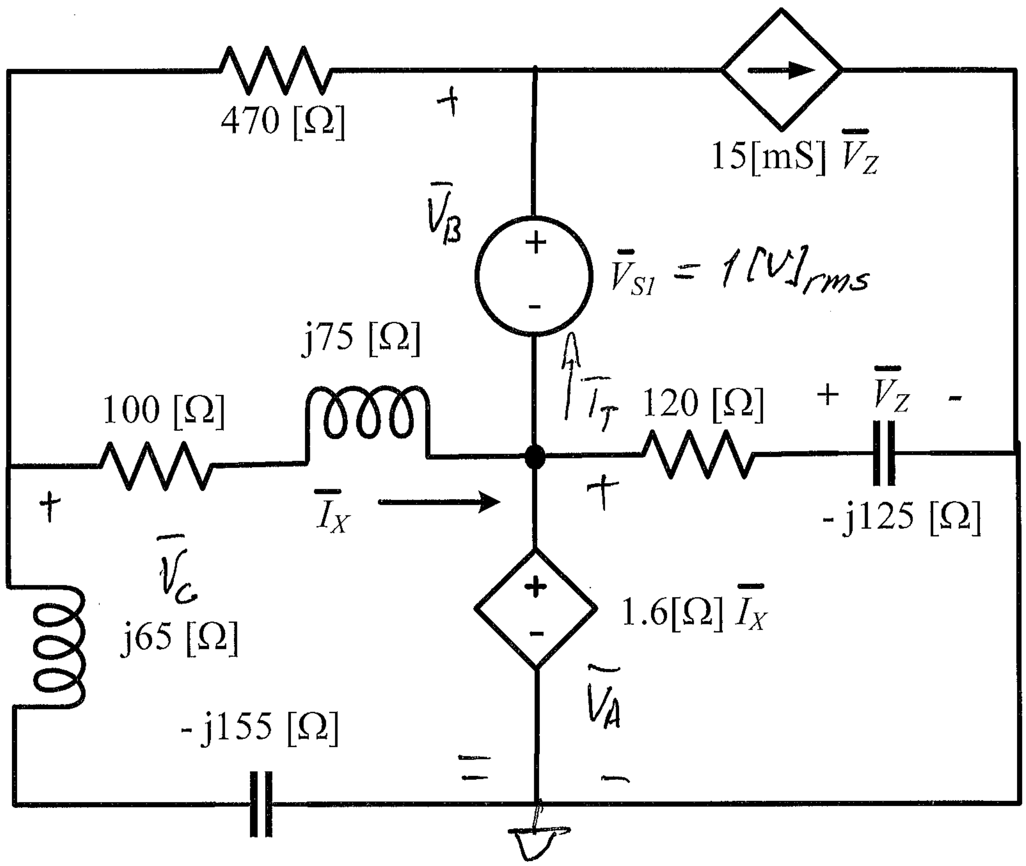
   b) Express the Thevenin equivalent impedance in terms of resistors, capacitors, and inductors, assuming the operating frequency is 350 [Hz].



We can make $\bar{V}_{S1}$ a test source of 1 [V]rms.

+4 $\bar{V}_A = 1.6\,\bar{I}_X$ +5 $\bar{I}_X = \dfrac{\bar{V}_C - \bar{V}_A}{100 + j75}$ +4 $\bar{V}_B = 1 + \bar{V}_A$

+10 $\dfrac{\bar{V}_C}{j65 - j155} + \dfrac{\bar{V}_C - \bar{V}_A}{100 + j75} + \dfrac{\bar{V}_C - \bar{V}_B}{470} = 0$

+6 $\bar{I}_T = \dfrac{\bar{V}_B - \bar{V}_C}{470} + 0.015\,\bar{V}_Z$ +6 $\bar{V}_Z = \bar{V}_A \cdot \dfrac{-j125}{120 - j125}$

+35

Room for extra work

+4

$\bar{V}_A = 0.716 - j2.43 = 2.534 \angle -73.6^\circ$ [mV]

$\bar{V}_B = 1000 - j0.2431 = 1000 \angle -0.14^\circ$ [mV]

$\bar{V}_C = 159.4 - j120.8 = 200.0 \angle -37.2^\circ$ [mV]

$\bar{I}_X = 0.447 - j1.519 = 1.584 \angle -73.6^\circ$ [mV]

$\bar{V}_Z = -0.842 - j1.623 = 1.828 \angle -117.4^\circ$ [mV]

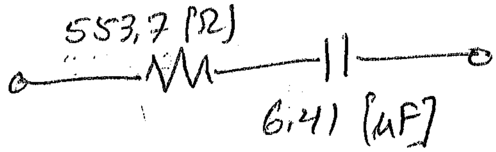
$\bar{I}_T = 1.777 + j0.2276 = 1.792 \angle 7.296^\circ$ [mA]

+3 $Z_{TH} = \frac{1}{\bar{I}_T} = 553.7 - j70.91$ [Ω] $= 538.4 \angle -7.57^\circ$ $= R_{TH} + jX_{TH}$

The real and imaginary parts of Z_{TH} give R_{TH} and X_{TH}

$R_{TH} = 553.7$ [Ω] $\quad X_{TH} = -70.91$ [Ω]

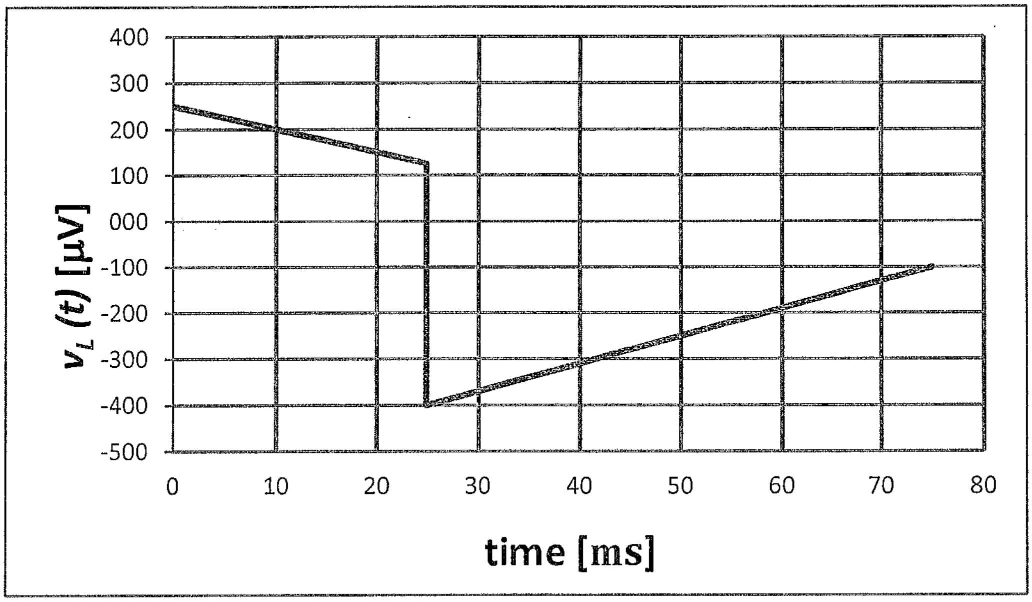
+4

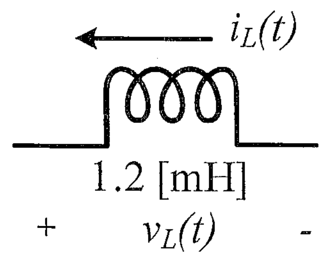


+4 $\frac{1}{\omega C} = 70.91 \therefore C = \frac{1}{70.91\,\omega} = \frac{1}{2\pi(350)(70.91)} = 6.41$ [μF]

2. {40 Points} The voltage $v_L(t)$ across a 1.2 [mH] inductor is given in the graph below. The inductor voltage polarity and current direction are shown in the figure. It is known that $i_L(50[mS]) = 2.25$ [mA].

   Find i_L at 10 [ms].





We need equations for the lines:

$0 < t < 25$ [ms] $\quad m = \dfrac{(125-250)\times10^{-6}}{25\times10^{-3}} = -5\times10^{-3}\,[\tfrac{V}{s}]$

+8 $\quad b = 250\times10^{-6}$ [V]

$\therefore\ V_L(t) = -5\times10^{-3}\,[\tfrac{V}{s}]\,t + 250\times10^{-6}$ [V] $\quad 0 < t < 25$ [ms]

$t > 25$ [ms] $\quad m = \dfrac{(-100-(-400))\times10^{-6}}{(75-25)\times10^{-3}} = 6\times10^{-3}\,[\tfrac{V}{s}]$

$V_L(t) = 6\times10^{-3}\,[\tfrac{V}{s}]\,t + b \qquad t = 25\times10^{-3}\,[s] \Rightarrow V_L = -400\times10^{-6}$ [V]

$\therefore\ b = -550\times10^{-6}$ [V]

$\therefore\ V_L(t) = 6\times10^{-3}\,[\tfrac{V}{s}]\,t - 550\times10^{-6}$ [V] $\quad t > 25$ [ms]

+8

28

Room for extra work

$$i_L(t) = \frac{-1}{L}\int_{t_0}^{t} v_L(t)\,dt + i_L(t_0)$$

$$i_L(50[ms]) = 2.25\times10^{-3}[A] = \frac{-1}{0.0012[H]}\int_{25\times10^{-3}[s]}^{50\times10^{-3}[s]} \left(6\times10^{-3}[\tfrac{V}{s}]t - 550\times10^{-6}[V]\right)dt + i_L(25[ms])$$

+ 12

$$2.25\times10^{-3} = 6.771\times10^{-3} + i_L(25[ms])$$

$$i_L(25[ms]) = -4.520\times10^{-3}[A]$$

$$i_L(25[ms]) = -4.520\times10^{-3}[A] = \frac{-1}{0.0012[H]}\int_{10\times10^{-3}[s]}^{25\times10^{-3}[s]} \left(-5\times10^{-3}[\tfrac{V}{s}]t + 250\times10^{-6}[V]\right)dt + i_L(10[ms])$$

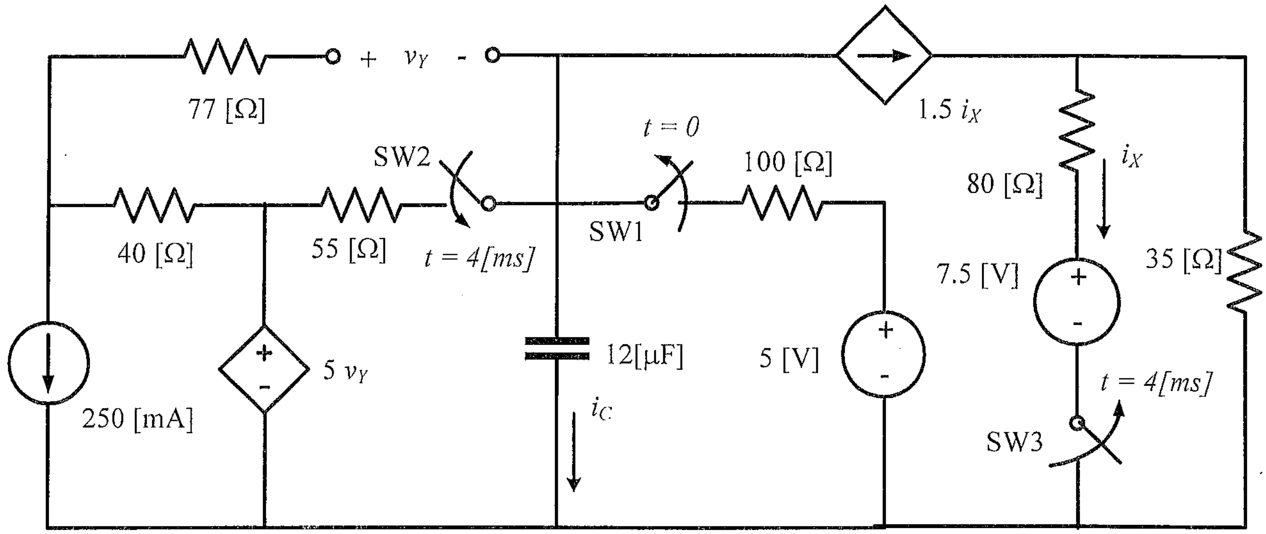
$$-4.52\times10^{-3} = -2.031\times10^{-3} + i_L(10[ms])$$

+ 12

$$\Rightarrow \boxed{i_L(10[ms])} = -2.489\times10^{-3}[A]$$

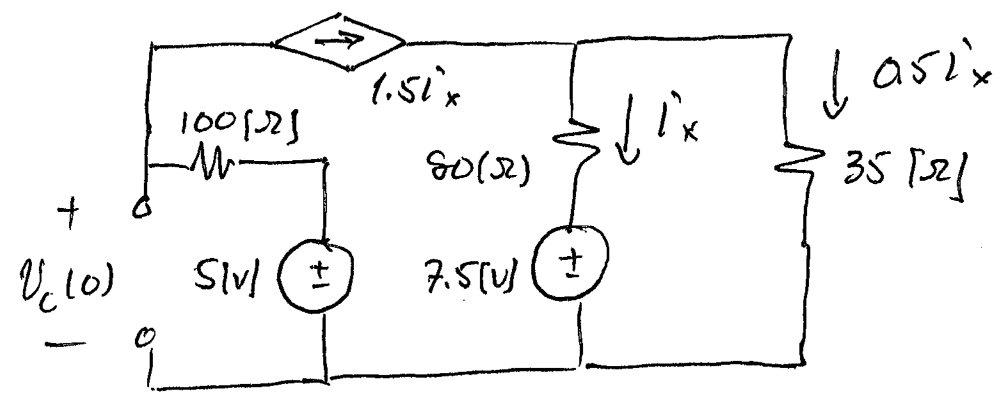
3. {50 Points} In the circuit below, switch SW1 was closed for a long time, SW2 was open for a long time, and SW3 was closed for a long time. At t = 0, SW1 opened. At t = 4 [ms], SW2 closed, and SW3 opened.

Find an expression for the current i_C as a function of time for t > 4 [ms].



Draw for t<0 : SW1 closed, SW2 open, SW3 closed

Find $V_c(0)$ ⇒ C → open circuit.



+4

$V_c(0) = -1.5\, i_x + 5$

$-7.5 - 80\, i_x + 35(0.5 i_x) = 0$

$\Rightarrow i_x = -0.120$ [A]

$V_c(0) = -1.5(-0.120) + 5$

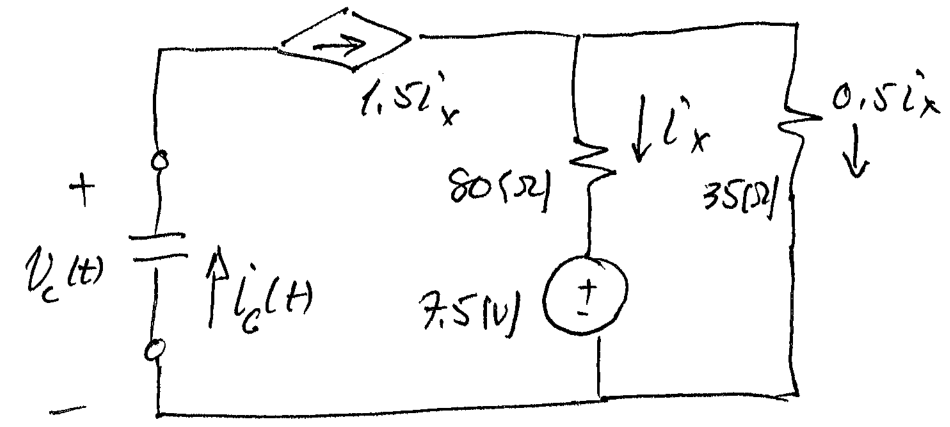
$V_c(0) = 23$ [V]

+6

Room for extra work

Draw for $0 < t < 4\,[ms]$ sw1 open, sw2 open, sw3 closed

+4



+2 $\quad -7.5 - 80 i_x + 35(0.5\, i_x) = 0 \quad \Rightarrow \quad i_x = -0.120\ [A]$

+10 $\quad v_c(t) = \dfrac{-1}{C} \displaystyle\int_0^t i_c(t)\,dt + v_c(0)$

$\quad = \dfrac{-1}{12 \times 10^{-6}} \displaystyle\int_0^t 1.5(-0.12)\,dt + 23\ [V]$

+2

$v_c(4\,[ms]) = 60 + 23 = 83\ [V]$

Draw for $t > 4\,[ms]$ sw1 open sw2 closed sw3 open

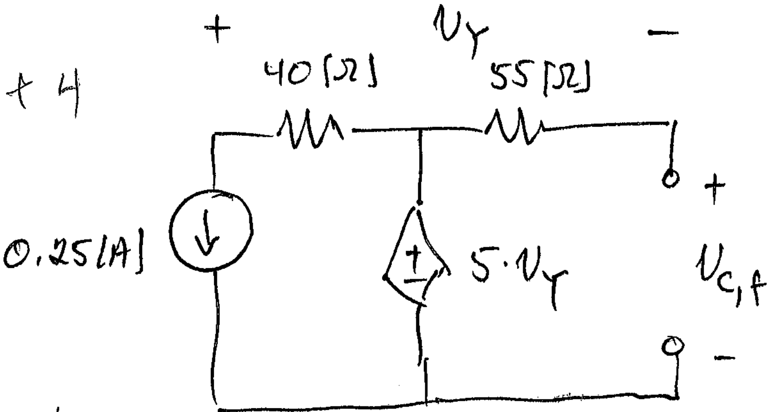
sw3 open $\Rightarrow i_x = 0$

Find $v_{c,f}$ $\Rightarrow$ C → open circuit

Note that 77 [Ω] can be ignored.

Room for extra work

+4



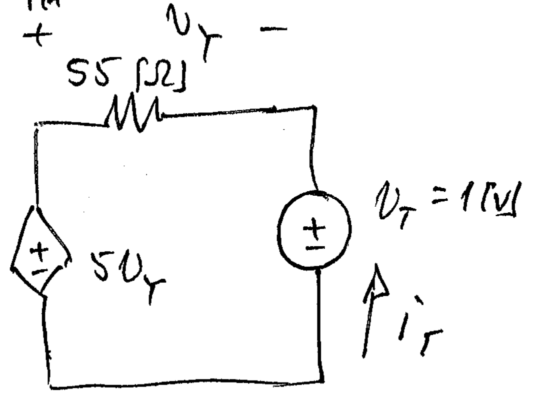
$$v_{C,f} = 5\, v_Y$$

$$-v_Y - 40(0.25) = 0$$

$$\Rightarrow v_Y = -10\ [V]$$

+4 $v_{C,f} = -50\ [V]$

Find R_{TH}:



$$-1 + 55\, i_T + 5\, v_Y = 0$$

$$v_Y = -55\, i_T$$

$$\therefore i_T = -4.546\ [mA]$$

+4 $\Rightarrow R_{TH} = -220\ [\Omega]$ $\qquad \tau_C = R_{TH} C = -2.64\ [ms]$

+2 $\therefore v_C(t) = v_{C,f} + (v_C(4\,[ms]) - v_{C,f})\, e^{-\frac{(t - 4\,[ms])}{\tau_C}}$

+4 $v_C(t) = -50 + (83 - (-50))\, e^{-\frac{(t-4\,[ms])}{-2.64\,[ms]}}\ [V]$

+2 $i_C(t) = C \dfrac{dv_C(t)}{dt} = 12 \times 10^{-6} \cdot 133 \cdot \dfrac{1}{2.64 \times 10^{-3}}\, e^{-\frac{(t-4\,[ms])}{-2.64\,[ms]}}$

+2 $i_C(t) = 0.6046\ [A]\, e^{+\frac{(t-4\,[ms])}{2.64\,[ms]}}$

4. {40 points} In the circuit below, the following is known.

Load L1 absorbs 15 [kW] at 0.84 power factor leading.
Load L2 absorbs 22 [kVA] at 0.66 power factor lagging.
Load L3 absorbs 12 [kW] and delivers 16 [kVAR].
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In the time domain, the voltage indicated by $\bar{V}_l$ is

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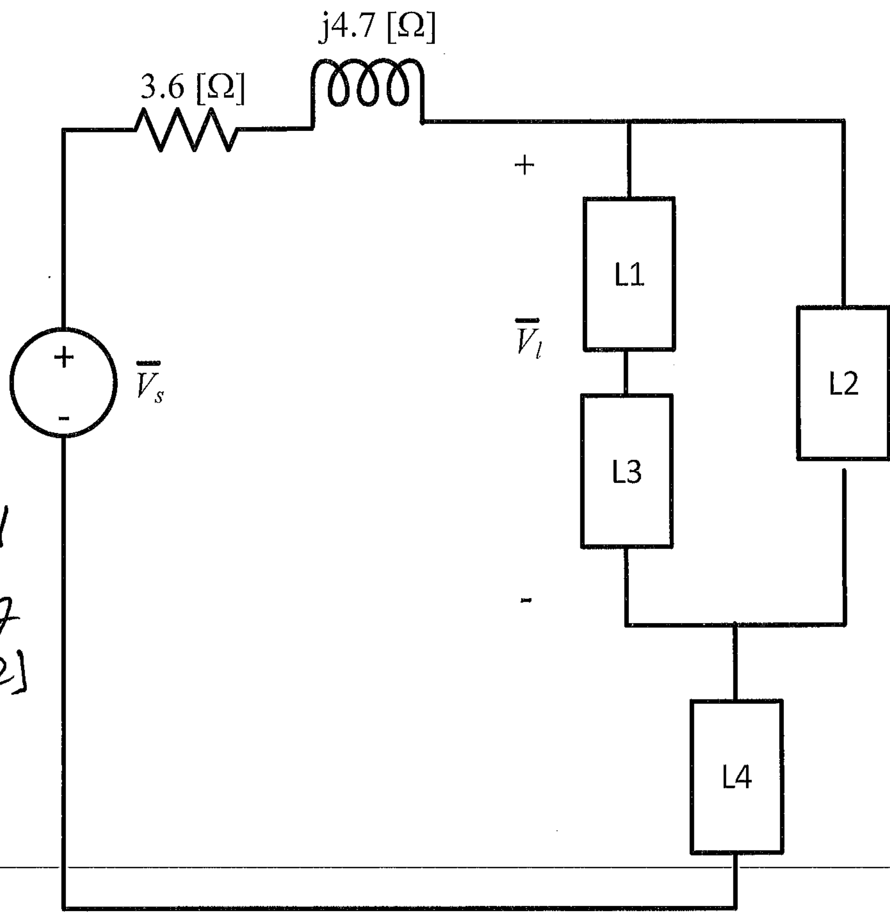
Find $\bar{V}_s$.



For later use:

$2.7[\Omega] \,//\, -j41[\Omega]$

$= 2.688 - j0.1777 \; [\Omega]$

$2.69 \angle -3.78°$

We will use rms amplitudes, since on the diagram, $\bar{V}_l$ and $\bar{V}_s$ are indicated, and not $\bar{V}_{l,m}$, $\bar{V}_{s,m}$. So...

$$\bar{V}_l = \frac{678.82}{\sqrt{2}} \angle 20° \; [\text{V}]_{rms} = 480 \angle 20° \; [\text{V}]_{rms}$$

↗

Room for extra work

L1: $P_{abs,1} = 15000$ [W] $pf = 0.84 \Rightarrow rf = \sqrt{1-0.84^2} = 0.543$

$|S_{abs,1}| = \frac{15000}{0.84} = 17{,}857$ [VA] leading $\Rightarrow Q < 0$

$S_{abs,1} = 15000 - j\,17{,}857(0.543) = 15000 - j\,9696.4$ [VA] $17861\angle -32.9°$

L2: $|S_{abs,2}| = 22000$ [VA] $pf = 0.66 \Rightarrow rf = \sqrt{1-0.66^2} = 0.751$

$S_{abs,2} = 22000(0.66) + j\,22000(0.751)$ lagging $\Rightarrow Q > 0$

$S_{abs,2} = 14520. + j\,16522$ [VA] $21996\angle 48.69°$

L3: $S_{abs,3} = 12000 - j\,16{,}000$ [VA] $20000\angle -53.13°$

$S_{abs,123} = S_{abs,1} + S_{abs,2} + S_{abs,3} = 41520 - j\,9174.4$ [VA]

$\therefore \; I_s = \left(\frac{S_{abs,123}}{\bar{V}_L}\right)^* = 74.75 + j\,47.54$ [A]$_{rms}$

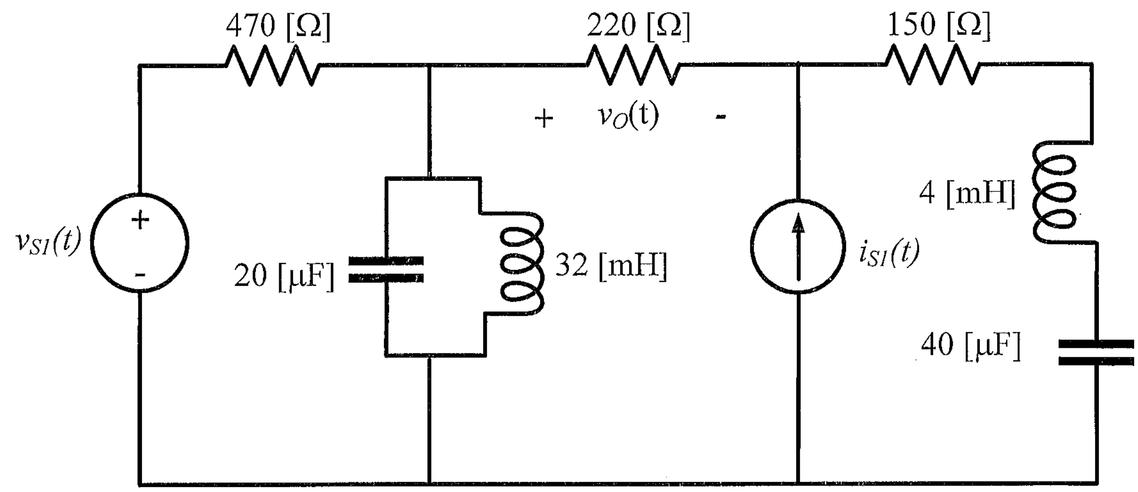
$\bar{V}_s = I_s(3.6 + j4.7) + 480\angle 20° + I_s(2.688 - j0.1777)$

$\bar{V}_s = 706.09 + 801.14$ [V]$_{rms}$

5. {40 points} For the following circuit, find $v_O(t)$. It is given that

$$v_{S1}(t) = 17.5\,[mV]\cos\left(1250\left[\frac{rad}{s}\right]t - 22°\right), \text{ and}$$

$$i_{S1}(t) = 32.7\,[mA]\sin\left(2500\left[\frac{rad}{s}\right]t + 13°\right).$$



This requires superposition. We'll begin with $v_{S1}(t)$.

x2 Solve with $v_{S1}(t) \Rightarrow i_{S1} \to$ open $\quad \bar{V}_{S1,m} = 17.5\angle -22°$ [mV]
$16.23 - j6.556$ [mV]

x2 $\quad 4\,[mH] \to j(1250)(0.004) = j5\,[\Omega]$

x2 $\quad 40\,[\mu F] \to -j/(1250)(40\times10^{-6}) = -j20\,[\Omega]$

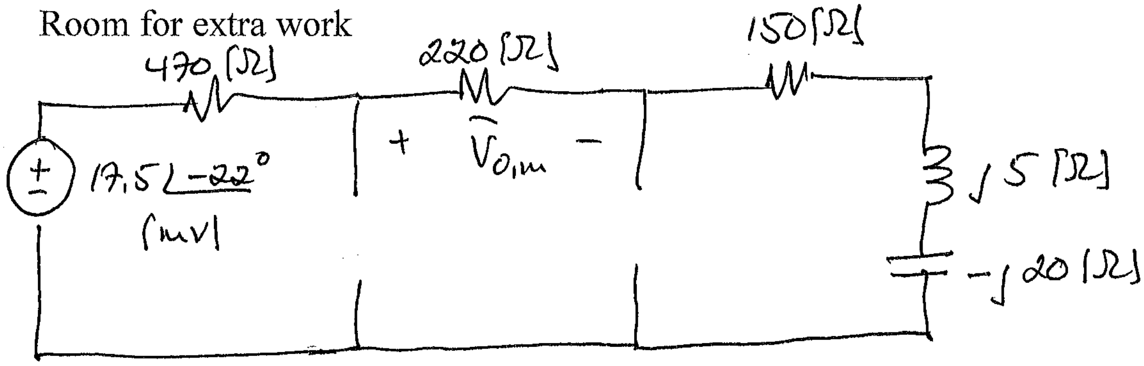
x2 $\quad 32\,[mH] \to j(1250)(0.032) = j40\,[\Omega]$

x2 $\quad 20\,[\mu F] \to -j/(1250)(20\times10^{-6}) = -j40\,[\Omega]$

Note that $j40\,[\Omega]\,//\,-j40\,[\Omega] = \infty$ so the 20 [μF] capacitor and 32 [mH] inductor $\Rightarrow$ open circuit.

↗

Room for extra work



$$\bar{V}_{0,m} = (17.5\times10^{-3}\angle -22^\circ)\,\frac{220}{470+220+150+j5-j20}$$

+5 $= 4.279 - j1.640$ [mV] $= 4.583\angle -20.98^\circ$ [mV]

+3 Solve with $i_{s_1}(t) \Rightarrow v_{s_1} \rightarrow$ short $\bar{I}_{s1,m} = 32.7\angle -77^\circ$ [mA]

$(13^\circ - 90^\circ)$ $7.356 - j31.86$ [mA]

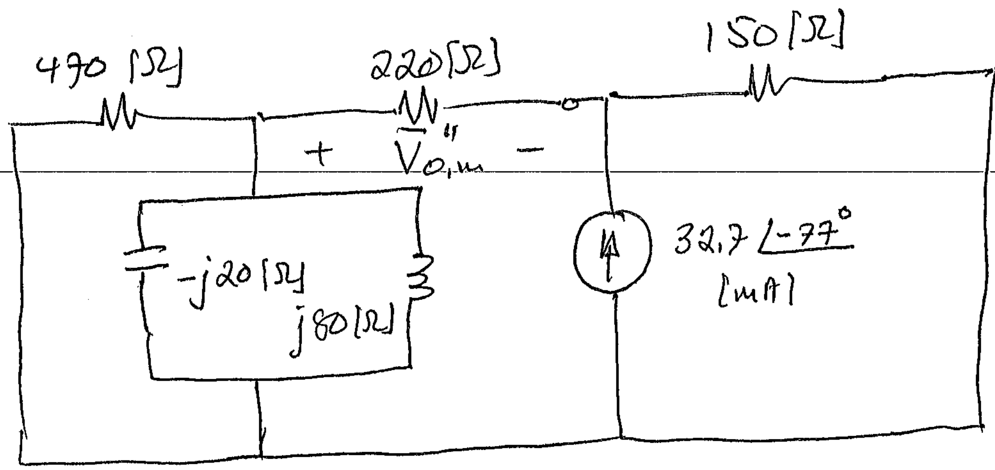
+2 4 [mH] → $j(2500)(0.004) = j10$ [Ω]

+2 40 [μF] → $-j/(2500)(40\times10^{-6}) = -j10$ [Ω]

These components in series are a short.

+2 32 [mH] → $j(2500)(0.032) = j80$ [Ω]

+2 20 [μF] → $-j/(2500)(20\times10^{-6}) = -j20$ [Ω]



pg. 2

Room for extra work

$$\frac{1}{Z_{eq}} = \frac{1}{470} + \frac{1}{-j220} + \frac{1}{j80} \Rightarrow Z_{eq} = 1.508 - j26.58 \, [\Omega]$$

$$26.62 \angle -86.75$$

$$\bar{V}_{o,m}'' = -(32.7\times10^{-3} \angle -77^\circ) \cdot \frac{150}{Z_{eq} + 220 + 150} \cdot 220$$

+6

$$= -0.8515 + j2.769 \, [V] = -2.897 \angle -72.9^\circ \, [V]$$

So

$$v_o(t) = 0.00458 \cos(1250 [\tfrac{rad}{s}] t - 20.98^\circ) \, [V]$$

$$-2.897 \cos(2500 [\tfrac{rad}{s}] t - 72.9^\circ) \, [V]$$

+8