Name: ______________________________ (please print)

Signature: ___________________________

ECE 2202 – Quiz 5

November 2, 2023

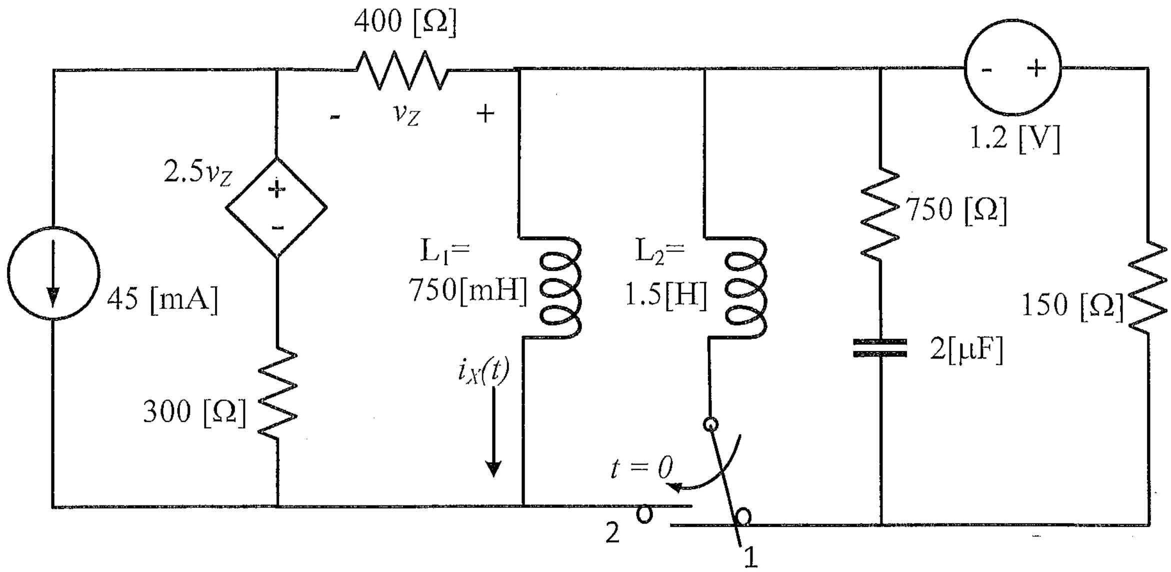
1. This quiz is closed book, closed notes. You may have one 8.5 x 11" crib sheet.
2. Show all work necessary to complete the problem. A solution without the appropriate work shown will receive no credit. A solution which is not given in a reasonable order will lose credit.
3. Show all units in solutions, intermediate results, and figures. Units in the quiz will be included between square brackets.
4. If the grader has difficulty following your work because it is messy or disorganized, you will lose credit.
5. Do not use red ink. Do not use red pencil.
6. You will have 40 minutes to work on this quiz.

_________________/20

Room for extra work

In the circuit below, the switch was in position '1' for a long time. At t = 0 it moved to position '2'.

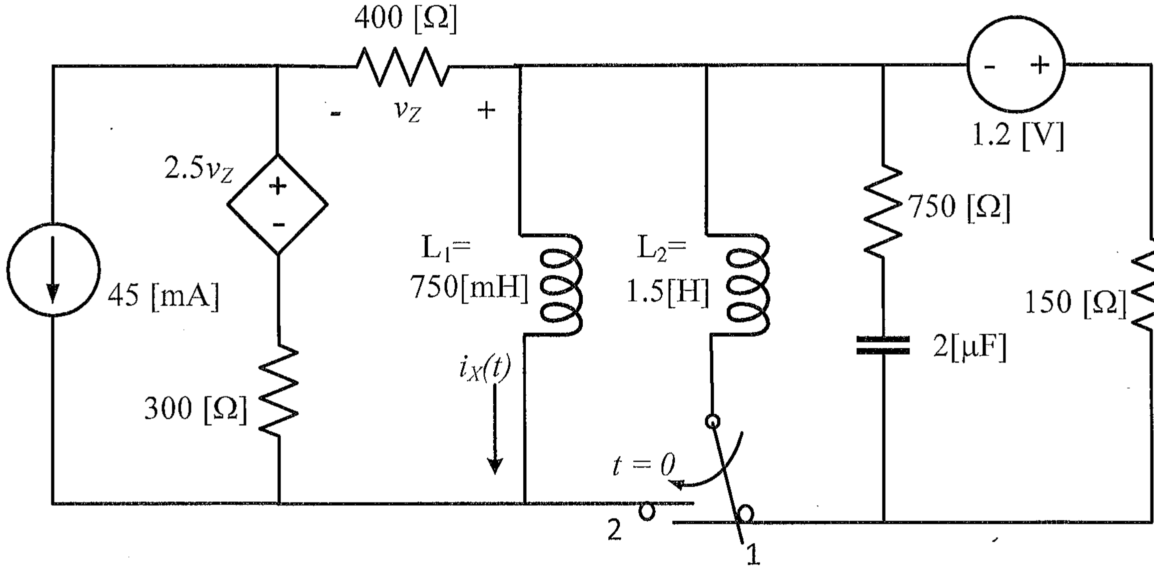
- Find $i_X(t)$, the current in inductor L_1 as a function of time, for t > 0.
- Find the value of v_Z after two time constants.



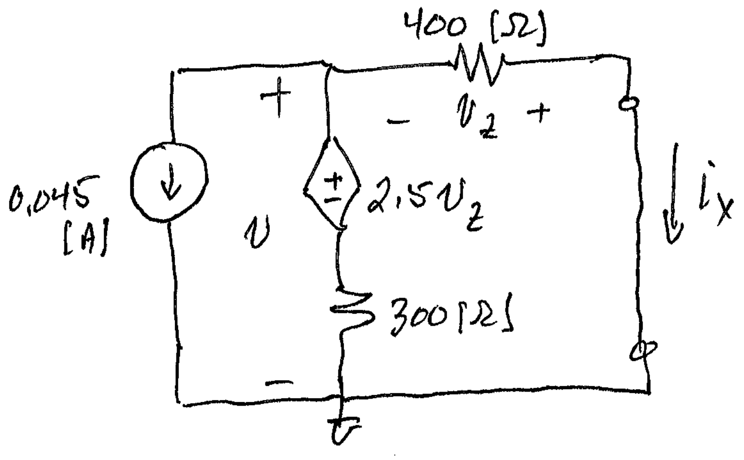
Room for extra work

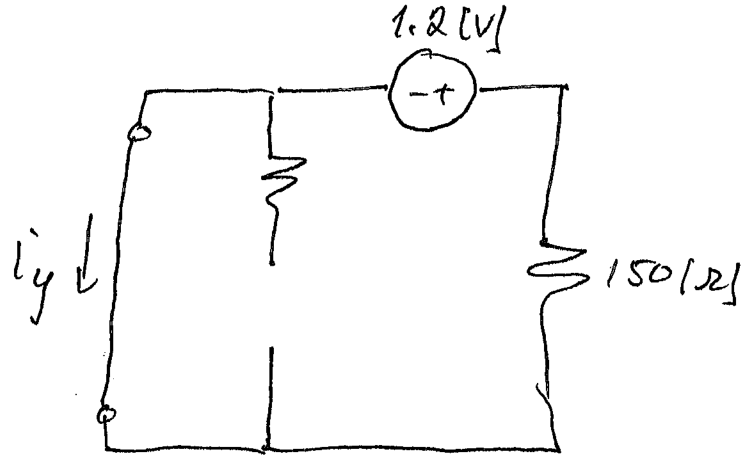
In the circuit below, the switch was in position '1' for a long time. At t = 0 it moved to position '2'.

- Find $i_X(t)$, the current in inductor L_1 as a function of time, for t > 0.
- Find the value of v_Z after two time constants.



For t<0, the inductors are shorts, and the capacitor is an open circuit.





$$\frac{v - 2.5v_z}{300} + 0.045 + \frac{v}{400} = 0$$

$$v_z = -v$$

$\Rightarrow$ $v = -3.176$ [V]

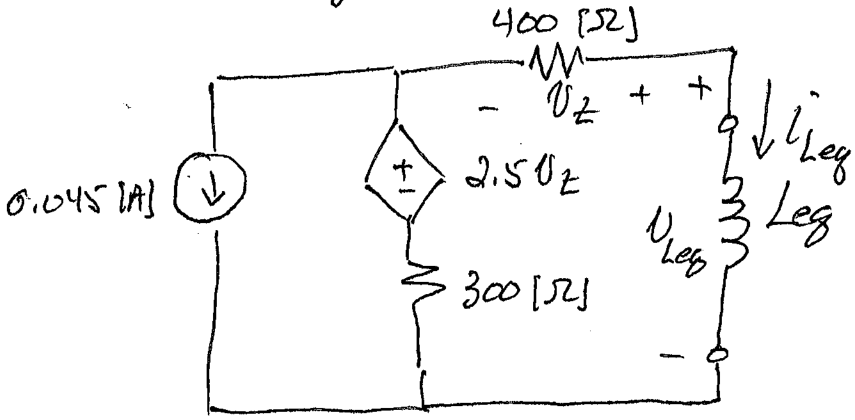
$\Rightarrow$ $i_x(0) = v/400 = -7.941$ [mA]

$$150 i_y + 1.2 = 0$$

$$i_y(0) = -8.00 \text{ [mA]}$$

Room for extra work

$t>0$ To find $i_x(t)$, we will combine the inductors into an equivalent L_{eq} and find the current in L_{eq}. Then we'll differentiate to get the inductor voltage, and integrate to get $i_x(t)$.

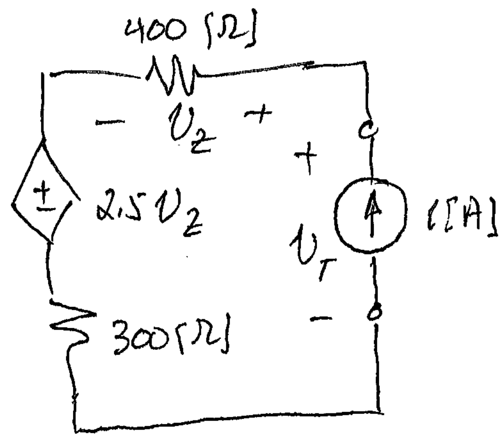


$$L_{eq} = \left(\frac{1}{L_1} + \frac{1}{L_2}\right)^{-1} = 0.5\,[H]$$

$$i_{Leq}(t) = i_{Leq,f} + \left(i_{Leq}(0) - i_{Leq,f}\right)e^{-t/\tau_L}$$

$$i_{Leq}(0) = i_x(0) + i_y(0) = -15.941\,[A]$$

R_{TH}:



$$v_T = 3.5\,v_Z + 300$$

$$v_Z = 400\,[V]$$

$$v_T = 3.5(400) + 300 = 1700\,[V]$$

$$\therefore R_{TH} = 1700\,[\Omega]$$

$$\tau_L = L_{eq}/R_{TH} = 0.2941\,[ms]$$

Room for extra work

$i_{Leq,f}$: Note that in steady state, with $L_{eq} \rightarrow$ short, we have the same circuit as for finding $i_x(0)$.
So $i_{Leq,f} = -7.941$ [mA].

$$i_{Leq}(t) = -7.941\,[\text{mA}] + \left(-15.941\,[\text{mA}] - (-7.941\,[\text{mA}])\right) e^{-t/0.294\,[\text{ms}]} \quad t \geq 0$$

Now we can find $v_{Leq}(t)$:

$$v_{Leq}(t) = L_{eq} \frac{d}{dt} i_{Leq}(t) = 13.601\,[\text{V}]\, e^{-t/0.294\,[\text{ms}]} \quad t > 0$$

Then
$$i_x(t) = \frac{1}{L_1} \int v_{Leq}(t)\, dt + i_x(0)$$
$$\underline{i_x(t) = -5.333\,[\text{mA}]\, e^{-t/0.294\,[\text{ms}]}} \Big|_0^t - 7.941\,[\text{mA}] \quad t \geq 0$$

Note that $i_x(t=0) \neq 0$ so we need to evaluate the result of the integration at t and at $\emptyset$:

$$i_x(t) = -5.333\,[\text{mA}]\, e^{-t/0.294\,[\text{ms}]} - (-5.333\,[\text{mA}]) - 7.941\,[\text{mA}]$$

Finally $\boxed{i_x(t) = -5.333\,[\text{mA}]\, e^{-t/0.294\,[\text{ms}]} - 2.608\,[\text{mA}]}$.

Note that at $t=0$, $i_x(0) = -7.941$ [mA], as it must be.

After two time constants, $e^{-t/\tau_L} \rightarrow e^{-2}$. So...

$$\boxed{v_t = -i_{Leq}(t = 2\tau_L) \times 400 = \left(+7.941\,[\text{mA}] + 8.00\,[\text{mA}]\, e^{-2}\right) \times 400 = 3.609\,[\text{V}]}$$

Room for extra work

We have our answer, but let's look at a more elegant solution to find $i_x(t)$ from simple calculus ideas:

$$v_{Leq}(t) = L_{eq} \frac{d}{dt} i_{Leq}(t)$$

$$i_x(t) = \frac{1}{L_1} \int_0^t v_{Leq}(t)\,dt + i_x(0)$$

$$= \frac{L_{eq}}{L_1} \int_0^t \frac{d}{dt} i_{Leq}(t)\,dt + i_x(0)$$

$$= \frac{L_{eq}}{L_1} \left(i_{Leq}(t) - i_{Leq}(0) \right) + i_x(0)$$

Plugging in the appropriate values gives us the same $i_x(t)$ we found above.

Another note: We may be tempted to use the current divider rule here:

$$\text{CDR} \Rightarrow \quad i_x(t) = \frac{L_2}{L_1 + L_2} i_{Leq}(t)$$

But looking at the last equation above for $i_x(t)$, we see that $\frac{L_{eq}}{L_1} = \frac{L_2}{L_1 + L_2}$, but CDR does not include $i_{Leq}(0)$ or $i_x(0)$, so it doesn't work here. But CDR does work <u>if</u> there are no initial conditions, i.e., $i_{Leq}(0) = i_x(0) = 0$.