Name: $\qquad$ (Print)

Signature
Date:

ECE 2300 -- Quiz \#1 S.R. Brankovic Section - TuTh 8:30 AM February 7, 2006

## KEEP THIS QUIZ CLOSED AND FACE UP UNTIL YOU ARE TOLD TO BEGIN.

1. This quiz is closed book, closed notes. You may use one 8.5 " x 11 " crib sheet, or its equivalent.
2. Show all work on these pages. Show all work necessary to complete the problem including appropriate referent current and voltage sign convention. A solution without the appropriate work shown will receive no credit. A solution which is not given in a reasonable order will lose credit.
3. It is assumed that your work will begin on the same page as the problem statement. If you choose to begin your work on another page, you must indicate this on the page with the problem statement, with a clear indication of where the work can be found. If your work continues on to another page, indicate clearly where your work can be found. Failure to indicate this clearly will result in a loss of credit.
4. Show all units in solutions, intermediate results, and figures. Units in the quiz will be included between square brackets.
5. Do not use red ink. Do not use red pencil.
6. You will have 25 minutes to work on this quiz.

Problem \#1. The electrical circuit defined in Figure 1 is the part of more complex electronic device circuitry. Find the value of $R_{X}$ for which the ratio $v_{A B} / v_{0}$ is equal to 1 . If the power delivered to the system by $5[\Omega]$ resistor is $-10[\mathrm{~W}]$, find the value of $v_{A B}$.. Find the energy absorbed by the same resistor in the period of 100 sec .


Figure 1

## Solution:

In order to solve this problem, the first thing we need to do is properly label the circuit in Figure 1. If we have done this, the circuit should look like the one presented below.


Figure 2
Now we can start. Let's first consider the loop labeled as KVL\#3:

$$
\begin{equation*}
-v_{A B}-i_{x} R_{x}-i_{5 x} 5 R_{x}=0 \tag{1}
\end{equation*}
$$

Now we have to define the current in this equation, let's look the closed surfaces KCL\#1, KCL\#3, KCL\#4 and KCL\#5 and write the appropriate equations:

KCL\#1:
KCL\#3:

$$
\begin{align*}
& i_{11}=0  \tag{2}\\
& -i_{2}+i_{x}+i_{22}=0  \tag{3}\\
& -i_{22}=0  \tag{4}\\
& -i_{22}-i_{5 x}+i_{3}=0 \tag{5}
\end{align*}
$$

Combining (3) and (4), and (4) and (5) we get:

$$
\begin{align*}
& i_{2}=i_{x}  \tag{6}\\
& i_{3}=i_{5 x} \tag{7}
\end{align*}
$$

Using (6) and (7), now we can write equations for loops labeled as KVL\#1 and KVL\#2 as:

KVL\#1: $\quad v_{0}+i_{x} \cdot 1[\Omega]+i_{x} \cdot R_{x}=0$
KVL\#2: $\quad 5 v_{0}+i_{5 x} \cdot 25[\Omega]+i_{5 x} \cdot 5 R_{x}=0$
Expressing the $i_{x}$ and $i_{5 x}$ from (8) and (9) and substituting them in (1) we get:

$$
\begin{equation*}
-v_{A B}+v_{0} \cdot \frac{R_{x}}{1[\Omega]+R_{x}}+5 v_{0} \cdot \frac{5 R_{x}}{25[\Omega]+5 R_{x}}=0 \tag{10}
\end{equation*}
$$

After rearrangement we get:

$$
\begin{equation*}
\frac{v_{A B}}{v_{0}}=\frac{R_{x}}{1[\Omega]+R_{x}}+\frac{5 R_{x}}{5[\Omega]+R_{x}} \tag{11}
\end{equation*}
$$

Knowing that problem is asking for value of $R_{X}$ for which the $v_{A B} / v_{0}=1$, equation (11) reduces to quadratic equation expressed below;

$$
\begin{equation*}
5 R_{x}^{2}+4 R_{x}-5=0 \tag{12}
\end{equation*}
$$

There are two solutions for this equation; however, only one has the physical meaning related to this problem;

$$
R_{x}=0.677[\Omega]
$$

Now, we can go back to the $5[\Omega]$ resistor and write the appropriate Ohm's law equation:

$$
\begin{equation*}
v_{5[\Omega]}=i_{10} \cdot 5[\Omega] \tag{13}
\end{equation*}
$$

For the closed surface KCL\#2, the appropriate equation is:

$$
\begin{equation*}
-i_{1}+i_{11}+i_{10}=0 \tag{14}
\end{equation*}
$$

Combining (2) and (14) we get:

$$
\begin{equation*}
i_{10}=i_{1} \tag{15}
\end{equation*}
$$

And from the Figure 2 knowing that $i_{1}=5[S] v_{A B}$ (13) becomes:

$$
\begin{equation*}
v_{5[\Omega]}=5[S] \cdot v_{A B} \cdot 5[\Omega] \tag{16}
\end{equation*}
$$

Power delivered to the system by $5[\Omega]$ resistors is:
$p_{\text {DEL.BY. } 5[\Omega] \text { resistor }}=-v_{5[\Omega]} \cdot i_{1}=-i_{1}^{2} \cdot 5[\Omega]=-\left(5[S] v_{A B}\right)^{2} \cdot 5[\Omega]=-10[W]$
and

$$
v_{A B}=\sqrt{\frac{10}{125}[V]^{2}}=0.283[V]
$$

Finally the energy absorbed by $5[\Omega]$ resistors is:
$w_{A B S . B Y .5[\Omega] \text { resistor }}=-\int_{0}^{100 s} p_{\text {DEL.BY.5[ת]resistor }} \cdot d t=+10 \cdot(100-0)[W \cdot s]=1000[\mathrm{~J}]$
or
$w_{A B S . B Y .5[\Omega] \text { resistor }}=\int_{0}^{100 s} p_{A B S . B Y .5[\Omega] \text { resistor }} d t=v_{5[\Omega]} i_{1} \int_{0}^{100 s} d t=(5[S] \cdot 0.283[V])^{2} \cdot 5[\Omega] \cdot(100-0)[s]$
$w_{A B S . B Y .5[\Omega] \text { resistor }}=1000[J]$.

