Name: ______________________________ (please print)

Signature: ___________________________

ECE 2202 – Quiz #2

March 6, 2018

Keep this quiz closed and face up until you are told to begin.

1. This quiz is closed book, closed notes. You may use one 8.5" x 11" crib sheet, or its equivalent.

2. Show all work on these pages. Show all work necessary to complete the problem. A solution without the appropriate work shown will receive no credit. A solution which is not given in a reasonable order will lose credit.

3. It is assumed that your work will begin on the same page as the problem statement. If you choose to begin your work on another page, you must indicate this on the page with the problem statement, with a clear indication of where the work can be found. **If your work continues on to another page, indicate clearly where your work can be found. Failure to indicate this clearly will result in a loss of credit.**

4. Show all units in solutions, intermediate results, and figures. Units in the quiz will be included between square brackets.

5. Do not use red ink. Do not use red pencil.

6. You will have 30 minutes to work on this quiz.

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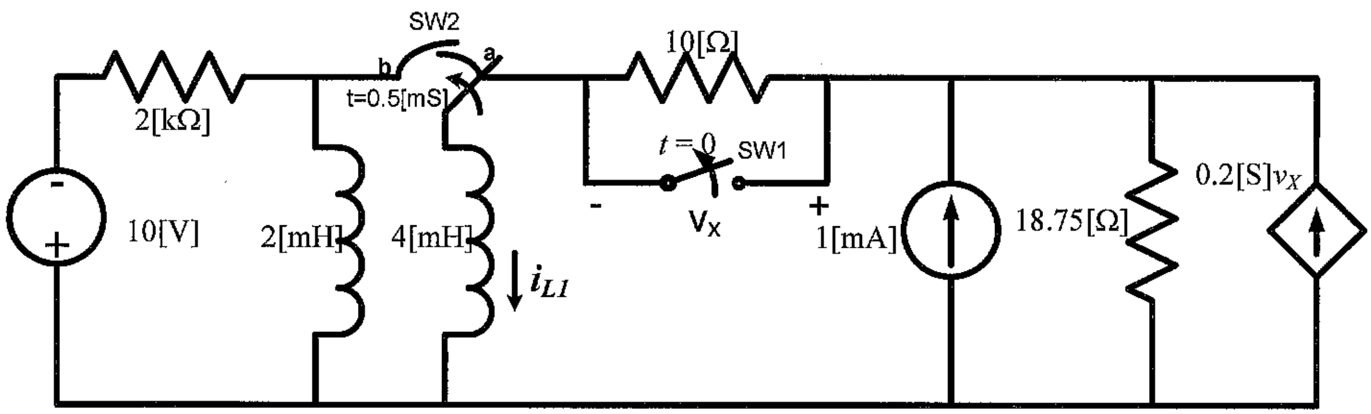
Room for extra work

The circuit below has two switches, SW1 and SW2. SW1 has been closed for a long time and is opened at t=0, while SW2 has been at position a for a long time before switch from position a to position b at t = 0.5 [mS].

a) Find the i_{L1}(0.5 [mS]).
b) Find total energy stored in the inductors at t = ∞.



The circuit below has two switches, SW1 and SW2. SW1 has been closed for a long time and is opened at t=0, while SW2 has been at position a for a long time before switch from position a to position b at t = 0.5 [mS].

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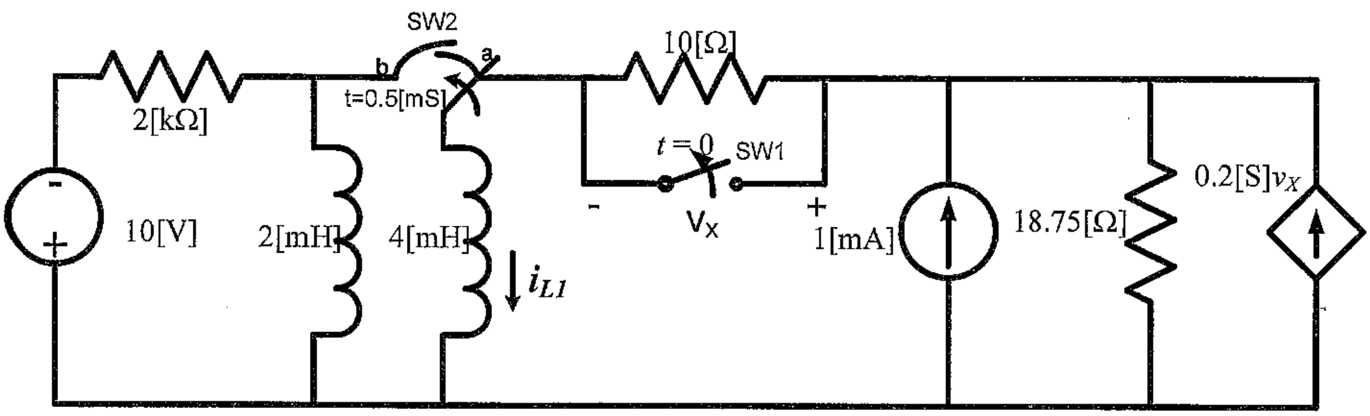
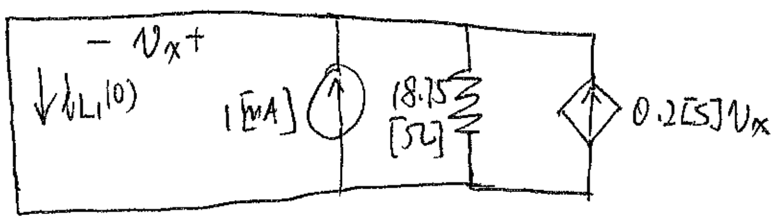


a). Redraw for t<0



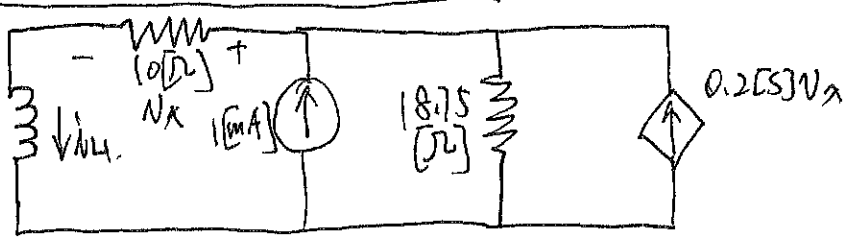
Since the SW1 has been closed for long time, inductor ⇒ wire.

$v_x = 0$ so. $0.2[S]\, v_x = 0$

18.75[Ω] resistor will be shorted by the wire

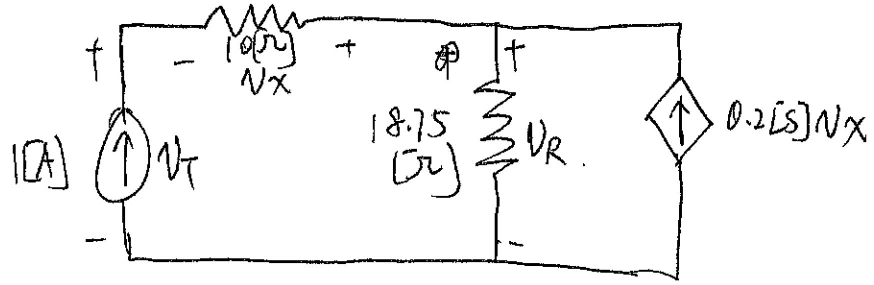
so. $i_{L1}(0) = 1[mA]$

Redraw for 0<t<0.5 ms



To obtain R_{eq}: set indep. source = 0 & use test source.

Room for extra work



$v_x = -1[A] \times 10[\Omega] = -10[V]$

KCL: $-1[A] + \frac{v_R}{18.75[\Omega]} - 0.2[S] \times (-10[V]) = 0$

$v_R = -18.75 [V]$

KVL: $-v_T - v_x + v_R = 0$

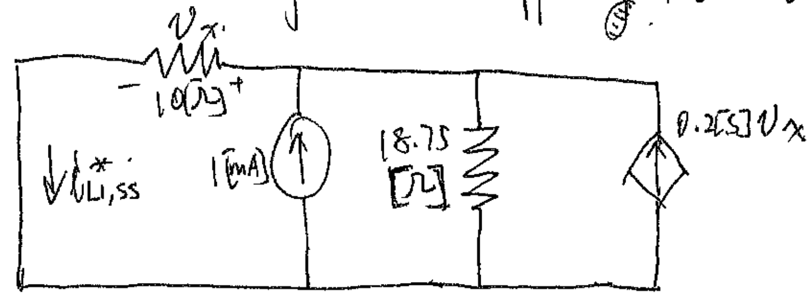
$v_T = v_R - v_x = -18.75 + 10 = -8.75[V]$

$R_{eq} = \frac{V_T}{1[A]} = -8.75[\Omega]$

$\tau = \frac{L}{R_{eq}} = \frac{4[mH]}{-8.75[\Omega]} = -0.46[ms]$

Redraw for $0 < t < 0.5ms$, steady, state

Since $\tau < 0$. steady-state is happened at $-\infty$.



KCL: $i^*_{L1,ss} - 1[mA] + \frac{v_x}{18.75[\Omega]} - 0.2 \times v_x = 0$

$v_x = 10[\Omega] \times i^*_{L1,ss}$

solve: $i^*_{L1,ss} = -2.14 [mA]$

Room for extra work

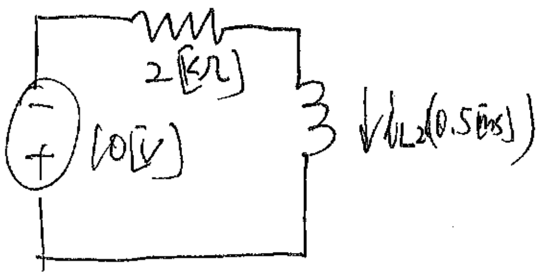
Therefore: $i_L(t) = -2.14[mA] + (1[mA] + 2.14[mA])\, e^{+\frac{t}{0.46[ms]}}$ $0 \le t \le 0.5[ms]$

$i_L(0.5[ms]) = -2.14 + 3.14 \cdot e^{+\frac{0.5[ms]}{0.46[ms]}}$

$\boxed{= 7.17\,[mA]}$

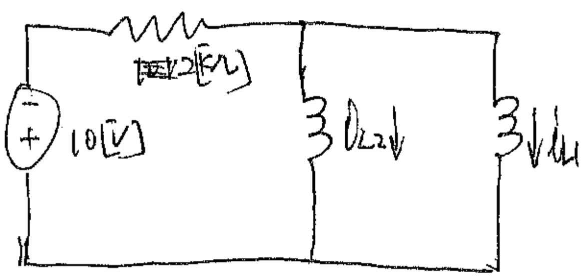
b) We need to find $i_{L1,ss}$, $i_{L2,ss}$.

Redraw for $t < 0.5[ms]$

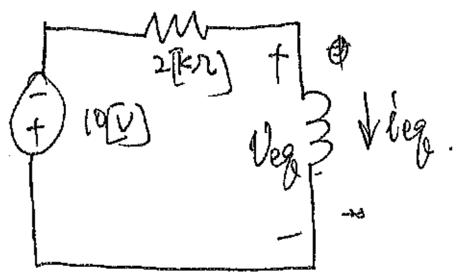


$i_{L2}(0.5[ms]) = -\frac{10[V]}{2[k\Omega]} = -5[mA]$

Redraw for $t > 0.5[ms]$



$\Rightarrow$



For: equivalent circuit.

initial condition: $i_{eq}(0.5[ms]) = i_{L1}(0.5\,ms) + i_{L2}(0.5\,ms) = (7.17 - 5)[mA] = 2.17[mA]$

$L_{eq} = 2[mH] // 4[mH] = 1.33\,[mH]$

$\tau = \frac{L_{eq}}{2[k\Omega]} = \frac{1.33[mH]}{2[k\Omega]} = 0.66\,[\mu s]$

$i_{eq,ss} = -5\,[mA]$

so: $i_{eq}(t) = -5 + (2.17 + 5)\, e^{-\frac{t - 0.5[ms]}{\tau}}$ for $t \ge 0.5[ms]$

$= -5 + 7.17\, e^{-\frac{t-0.5[ms]}{0.66[\mu s]}}$ for $t \ge 0.5[ms]$

Room for extra work

$$V_{eq} = L_{eq}\frac{di_{eq}}{dt} = L_{eq} \times 7.17\,[mA] \cdot \frac{d\, e^{-\frac{t-0.5[ms]}{\tau}}}{dt}$$

$$= L_{eq} \times 7.17\,[mA] \times e^{-\frac{t-0.5[ms]}{\tau}} \cdot \left(-\frac{1}{\tau}\right)$$

$$= -\frac{L_{eq} \times 7.17[mA]}{\tau} \cdot e^{-\frac{t-0.5[ms]}{\tau}}$$

Go back to original circuit:

$$i_{L1,ss.} = \frac{1}{L_1} \cdot \int_{0.5}^{+\infty} V_{eq}(t)\,dt + i_{L1}(0.5[ms])$$

$$= \frac{1}{L_1} \times \left(-\frac{L_{eq} \times 7.17[mA]}{\tau}\right) \int_{0.5}^{\infty} e^{-\frac{t-0.5[ms]}{\tau}} dt + 7.17[mA]$$

$$= \frac{\frac{L_1 \times L_2}{L_1+L_2} \times 7.17[mA]}{L_1} \cdot \int_{0.5}^{\infty} e^{-\frac{t-0.5[ms]}{\tau}} d\left(-\frac{t-0.5[ms]}{\tau}\right) + 7.17[mA]$$

$$= \frac{L_2}{L_1+L_2} \times 7.17[mA] \left. \underbrace{e^{-\frac{t-0.5[ms]}{\tau}}}_{=(-1)} \right|_{0.5[ms]}^{\infty} + 7.17[mA]$$

$$= -\frac{L_2}{L_1+L_2} \times 7.17[mA] + 7.17[mA] = \boxed{4.78\,[mA]}$$

$$i_{L2,ss} = \frac{1}{L_2} \int_{0.5ms}^{+\infty} V_{eq}(t)\,dt + i_{L2}(0.5[ms])$$

$$= \frac{\frac{L_1 \times L_2}{L_1+L_2} \times 7.17[mA]}{L_2} \int_0^{\infty} e^{-\frac{t-0.5[ms]}{\tau}} d\left(-\frac{t-0.5[ms]}{\tau}\right) - 5[mA]$$

$$= -\frac{L_1}{L_1+L_2} \times 7.17[mA] - 5[mA] = \boxed{-9.78[mA]}$$

$$W_{STO.L1,ss} = \frac{1}{2} L_1 i_{L1,ss}^2 = \frac{1}{2} \times 4[mH] \times (4.78[mA])^2 = 4.57 \times 10^{-8} (J)$$

$$W_{STO.L2,ss} = \frac{1}{2} L_2 i_{L2,ss}^2 = \frac{1}{2} \times 2[mH] \times (-9.78[mA])^2 = 9.56 \times 10^{-8} (J)$$

$$\boxed{W_{STO.} = (4.57 + 9.56) \times 10^{-8} = 1.41 \times 10^{-7} (J)}$$

To verify: $i_{L1,ss} + i_{L2,ss} = (4.78 - 9.78)[mA] = -5[mA] = i_{eq,ss}$