Name: ______________________________ (please print)

Signature: ___________________________

ECE 2202 – Exam 1

February 26, 2022

Keep this exam closed until you are told to begin.

1. This exam is closed book, closed notes. You may use one 8.5" x 11" crib sheet, or its equivalent.
2. Show all work on these pages. Show all work necessary to complete the problem. A solution without the appropriate work shown will receive no credit. A solution that is not given in a reasonable order will lose credit. Clearly indicate your answer (for example by enclosing it in a box).
3. Show all units in solutions, intermediate results, and figures. Units in the exam will be included between square brackets.
4. If the grader has difficulty following your work because it is messy or disorganized, you will lose credit.
5. Do not use red ink. Do not use red pencil.
6. You will have 90 minutes to work on this exam.

1. ________________/35

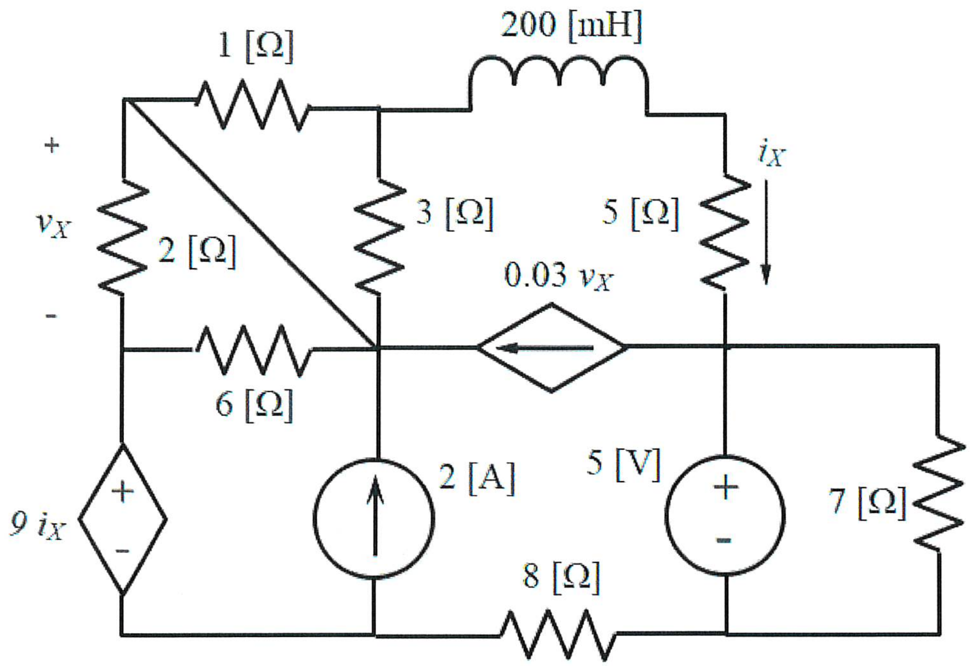
2. ________________/35

3. ________________/30

Total = 100

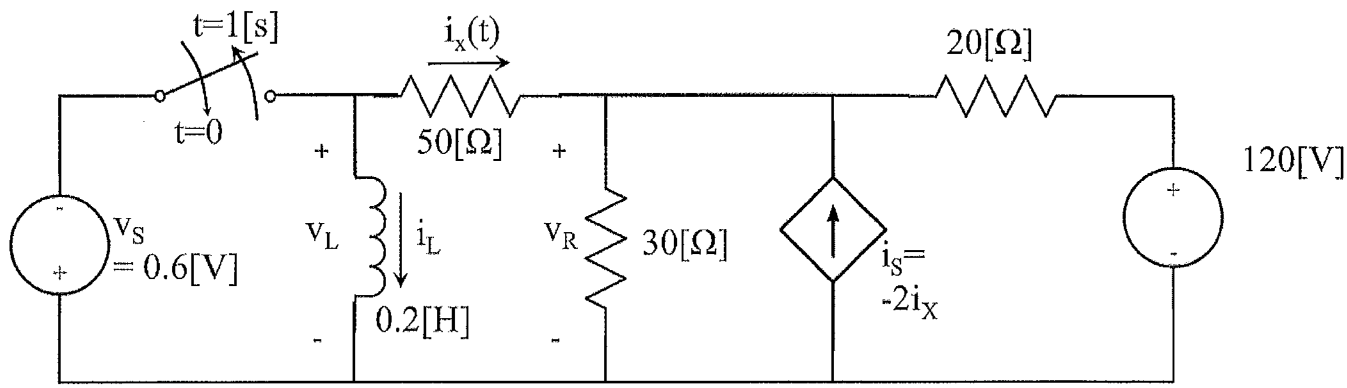
Room for extra work

1. {35 Points} For the circuit below, find the Thevenin equivalent seen by the inductor. Draw your Thevenin equivalent and label the terminals.



Room for extra work

2. {35 Points} The switch in the circuit below was open for a long time before it closed at t = 0, and then reopened at t = 1 [s]. Please find:
   a) $v_L(0^-)$ and $i_L(0^-)$;
   b) $v_L(0^+)$ and $i_L(0^+)$;
   c) $i_L(1[s])$;
   d) $v_R(1[s]^+)$;

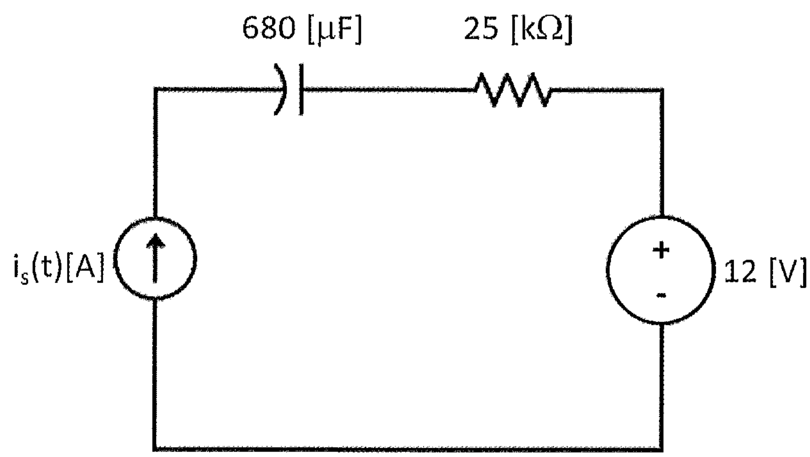


Room for extra work

3. {30 Points} A capacitor with a capacitance of 680 [μF] is connected into a circuit as shown below. The current source is specified as follows:

$$i_S(t) = 0 \quad t < 0$$
$$i_S(t) = 8\ [\text{A}] \quad t = 0$$
$$i_S(t) = B\cos(350[\text{rad/s}]t) + 0.5B\sin(350[\text{rad/s}]t)\ [\text{A}] \quad t \geq 0$$

What is the energy stored in the capacitor when t is equal to half the period?



3. {30 Points} A capacitor with a capacitance of 680 [μF] is connected into a circuit as shown below. The energy in the capacitor at t = 0 is 0. The current source is specified as follows:

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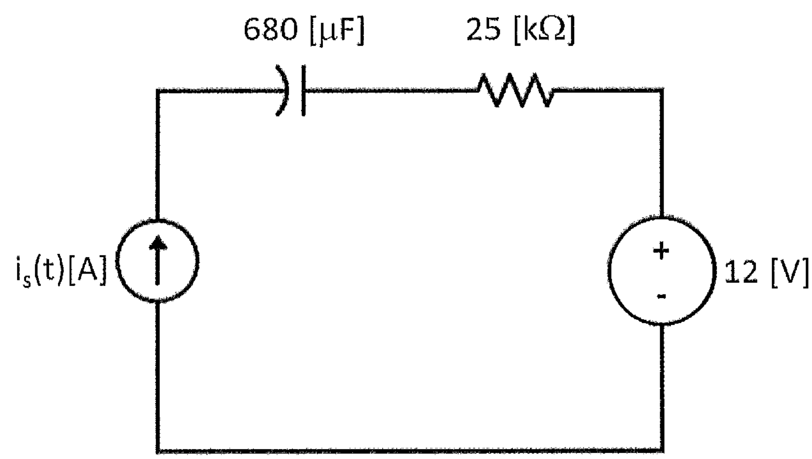
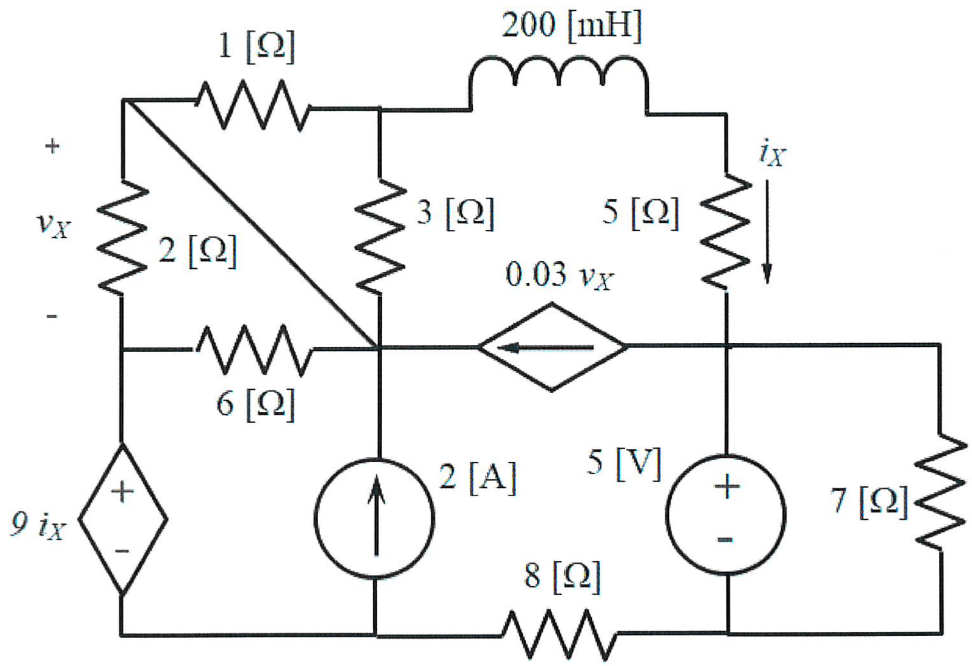
What is the energy stored in the capacitor when t is equal to half the period?



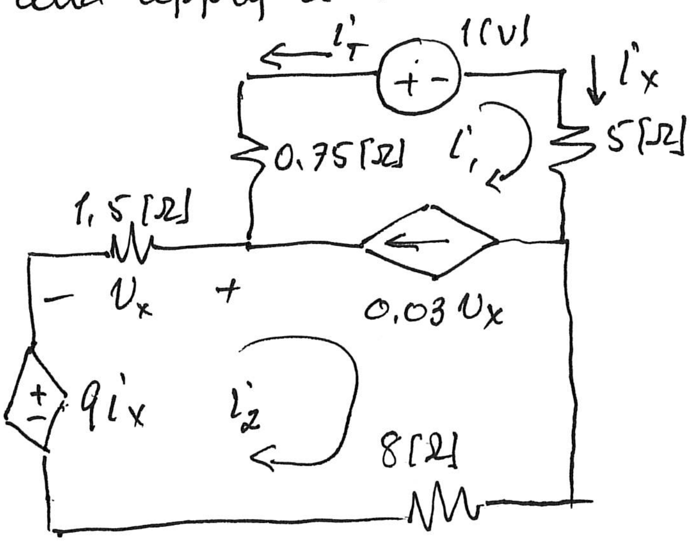
Room for extra work

1. {35 Points} For the circuit below, find the Thevenin equivalent seen by the inductor. Draw your Thevenin equivalent and label the terminals.



We can simplify a little bit: 1 [Ω] and 3 [Ω] are in parallel, as are 2 [Ω] and 6 [Ω].

We combine those resistances, remove the inductor, and apply a test source:



$$1 + 5.75 i_1' + 9.5 i_2' - 9 i_x' = 0$$

$$i_x' = i_1'$$

$$i_1' - i_2' = 0.03 v_x$$

$$v_x = -1.5 i_2'$$

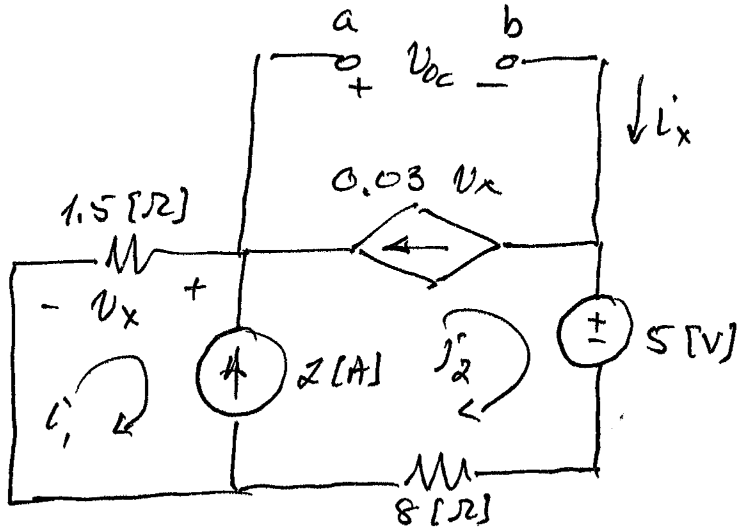
$$i_1' = -0.14931 \text{ [A]}$$

$$i_2' = -0.15634 \text{ [A]}$$

$$i_T' = -i_1' = 0.14931 \text{ [A]} \Rightarrow R_{TH} = 6.6975 \text{ [Ω]}$$

Room for extra work

Open-circuit voltage is simpler than short-circuit current:



$i_x = 0$

$i_2 = -0.03\, v_x$

$v_x = -1.5\, i_1$

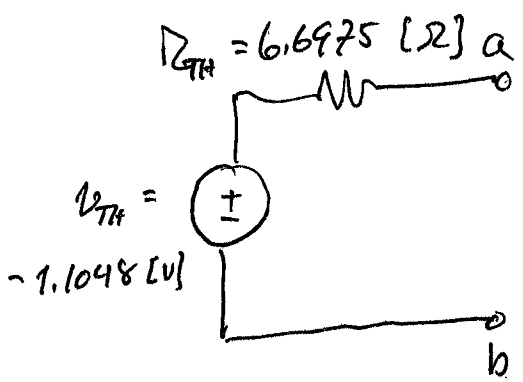
$i_2 - i_1 = 2$

$i_1 = -2.0942$ [A]

$i_2 = -0.09424$ [A]

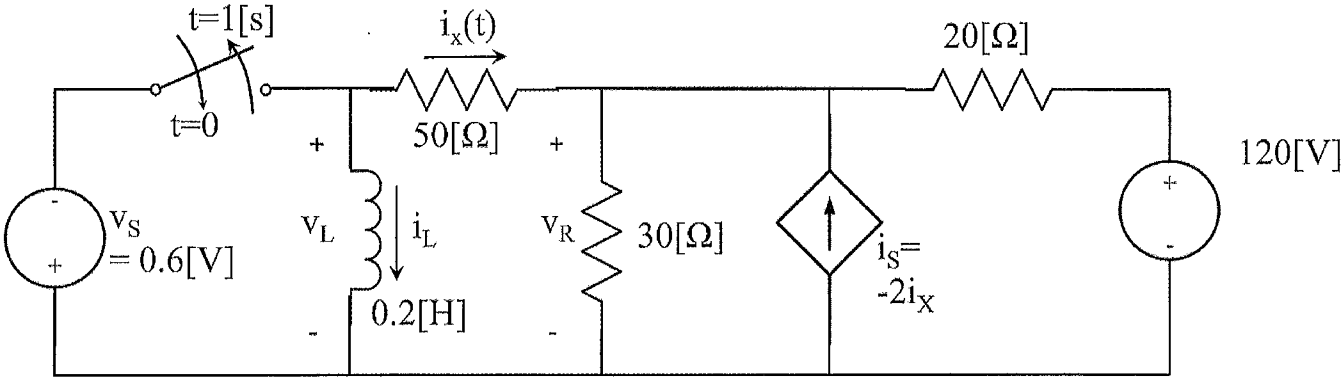
$v_{oc} + 5 + 8\, i_2 + 1.5\, i_1 = 0$

$\Rightarrow \boxed{v_{oc} = -1.1048 \text{ [V]}} = v_{TH}$

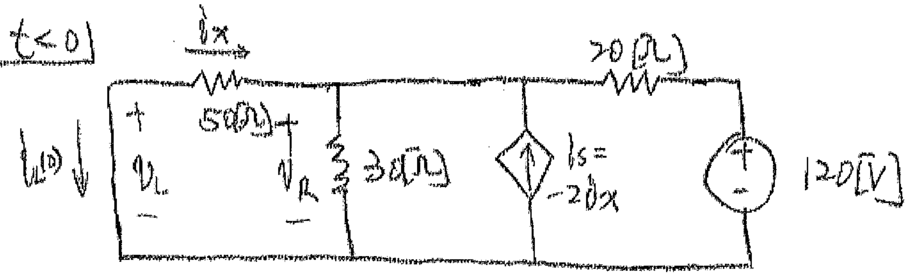


2. {35 Points} The switch in the circuit below was open for a long time before it closed at t = 0, and then reopened at t = 1 [s]. Please find:

a) $v_L(0^-)$ and $i_L(0^-)$;
b) $v_L(0^+)$ and $i_L(0^+)$;
c) $i_L(1[s])$;
d) $v_R(1[s]^+)$;



$t<0$ |

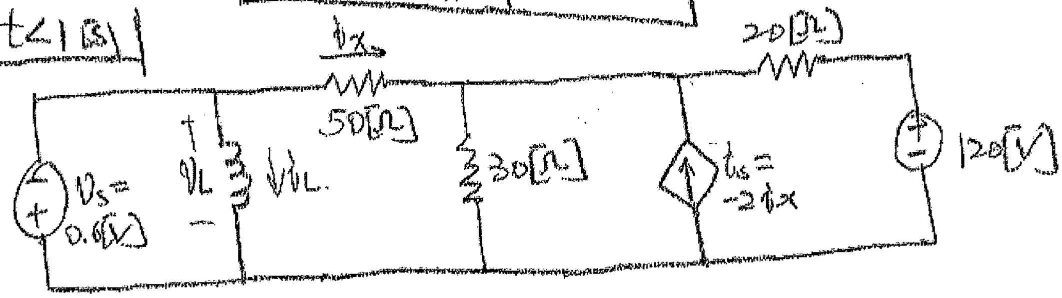


$$\begin{cases} i_L(0) = -i_x \\ i_L(0) + \dfrac{i_L(0) \times 50[\Omega]}{30[\Omega]} - (-2i_x) + \dfrac{i_L(0) \times 50[\Omega] - 120[V]}{20[\Omega]} = 0 \end{cases}$$

solve: we have: $i_L(0) = 1.895[A]$

Therefore: a): $\boxed{\begin{array}{l} i_L(0^-) = i_L(0) = 1.895[A] \\ v_L(0^-) = 0\,[V] \end{array}}$

$0 < t < 1[s]$ |



Room for extra work

Since the inductor is connected to a voltage source.

Therefore: $v_L(0^+) = -v_s = -0.6[V]$

$\boxed{b): \ v_L(0^+) = -0.6[V] \quad i_L(0^+) = i_L(0) = 1.895\,[A]}$

To obtain $i_L(1[s])$: We need to use intigration equation (Active sign Convention)

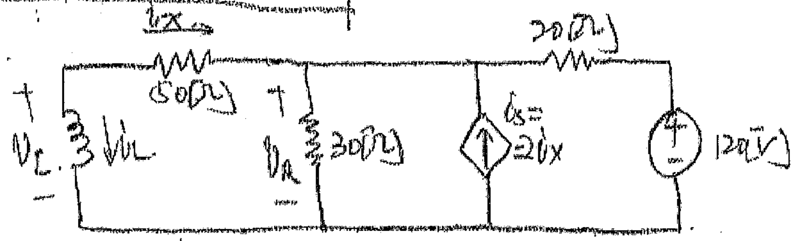
$$i_L(1[s]) = -\frac{1}{0.2[H]} \cdot \int_0^{1[s]} 0.6[V] \cdot dt + i_L(0)$$

$$= -\frac{1}{0.2} \int_0^1 0.6\,dt + 1.895$$

$$= -1.1\ [A]$$

$\boxed{c): \ i_L(1[s]) = -1.1\,[A]}$

$t > 1[s]$



Solve at $t = 1^+[s]$:
$$\begin{cases} i_x = -i_L(1[s]) = 1.1[A] \\ -1.1[A] + \frac{v_R}{30[\Omega]} - (-2i_x) + \frac{v_R - 120}{20[\Omega]} = 0 \end{cases}$$

solve, we get: $v_R(1[s]^+) = 58.5[V]$

$\boxed{d): \ v_R(1[s]^+) = 58.5\,[V]}$

3. {30 Points} A capacitor with a capacitance of 680 [μF] is connected into a circuit as shown below. The energy in the capacitor at t = 0 is 0. The current source is specified as follows:

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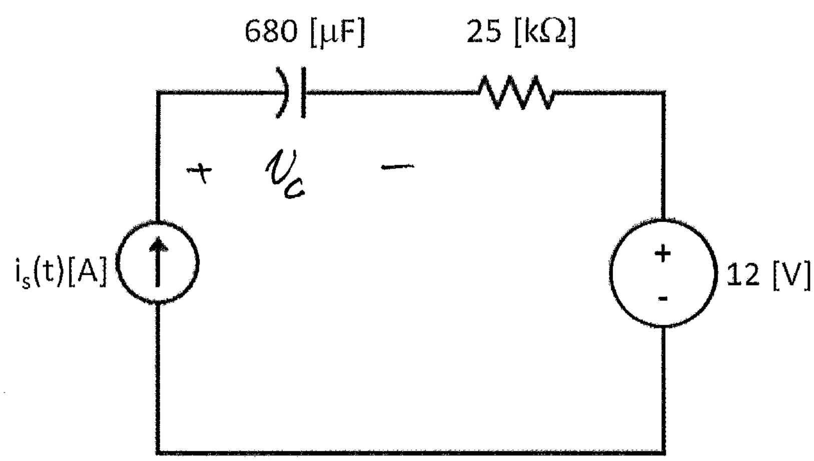
What is the energy stored in the capacitor when t is equal to half the period?



First we solve for B:

$$i_s(t=0) = B\cos(350[\tfrac{rad}{s}]\cdot 0) + 0.5B\sin(350[\tfrac{rad}{s}]\cdot 0)$$
$$= B + 0$$
$$= 8\,[A] \Rightarrow \boxed{B = 8\,[A]}$$

Energy: $w_c = \frac{1}{2} C v_c^2$ $\qquad i_s = C\frac{dv_c}{dt}$

$$v_c(t) = \frac{1}{C}\int i_s(t)\,dt + \cancel{v_c(0)}^{\,0}$$

Room for extra work

$$v_c(t) = -\frac{1}{680\times10^{-6}} \cdot \left[\int_0^t 8\cos(350t)\,dt + \int_0^t 4\sin(350t)\,dt \right]$$

$$= -\frac{1}{680\times10^{-6}} \left[\frac{8\,[A]}{350\,[\frac{rad}{s}]} \sin(350t)\Big|_0^t - \frac{4\,[A]}{350\,[\frac{rad}{s}]} \cos(350t)\Big|_0^t \right]$$

Find $\frac{1}{2}$ period: $\omega = 350 = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi}{350\,[\frac{rad}{s}]}$

$$\therefore \frac{1}{2}T = \frac{\pi}{350}\,[s]$$

$$v_c(\tfrac{1}{2}T) = \frac{-8}{(680\times10^{-6})(350)}\left[\cancel{\sin\pi}^{\,0} - \cancel{\sin 0}^{\,0}\right] + \frac{4}{(680\times10^{-6})(350)}\left[\cos\pi - \cos 0\right]$$

$$= \frac{4}{(680\times10^{-6})(350)}\left[-1 - 1\right]$$

$$v_c(\tfrac{1}{2}T) = -33.6\,[V]$$

$$\therefore \boxed{w_c = \frac{1}{2}(680\times10^{-6})(33.6)^2 = 384\,[mJ]}$$