Name: ______________________________ (please print)

Signature: ___________________________

ECE 2202 – Exam #2

April 9, 2022

Keep this exam closed until you are told to begin.

1. This exam is closed book, closed notes. You may use one 8.5" x 11" crib sheet, or its equivalent.
2. Show all work on these pages. Show all work necessary to complete the problem. A solution without the appropriate work shown will receive no credit. A solution that is not given in a reasonable order will lose credit. Clearly indicate your answer (for example by enclosing it in a box).
3. Show all units in solutions, intermediate results, and figures. Units in the exam will be included between square brackets.
4. Do not use red ink. Do not use red pencil.
5. You will have 90 minutes to work on this exam.

1. _______________/40

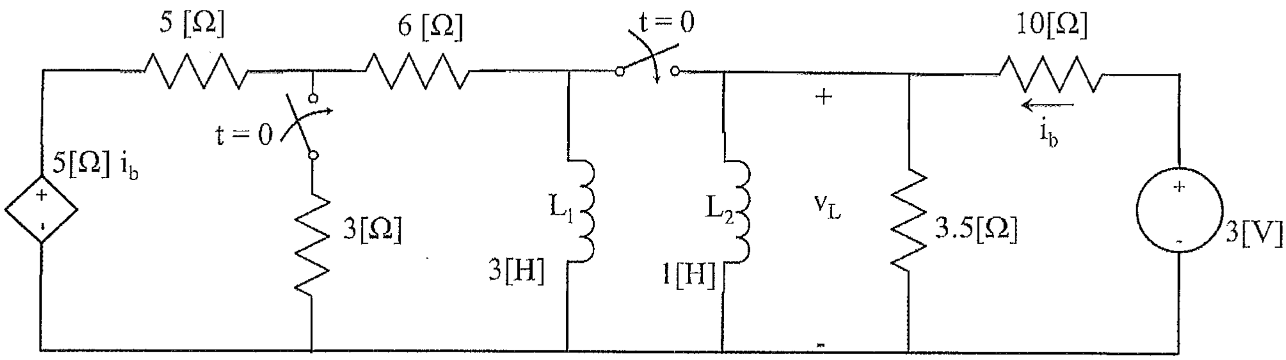
2. _______________/40

3. _______________/20

Total = 100

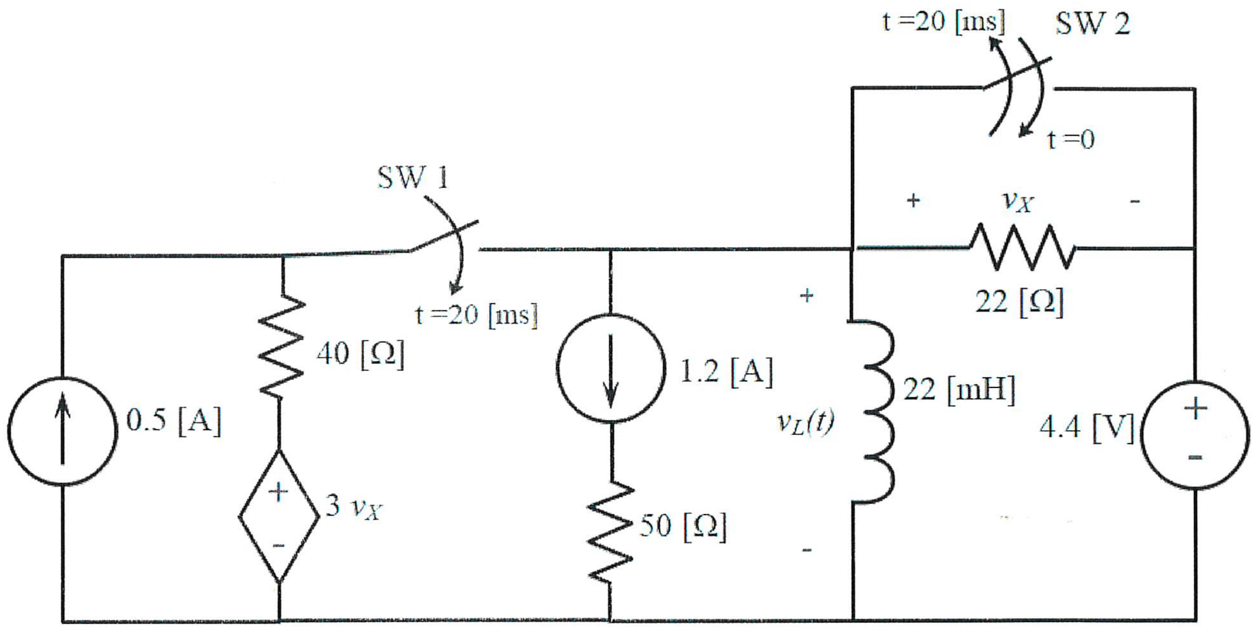
1. {40 Points} Both switches in the circuit below were open for a long time, before they closed at $t = 0$.

a) Find a numerical expression for $v_L(t)$ for $t > 0$.
b) Find the energy stored in inductor L_1 at $t = +\infty$.



Room for extra work

2. {40 Points} In the circuit below, switch SW1 was open for a long time and closed at t = 20 [ms]. Switch SW2 was open for a long time. It closed at t = 0 and opened again at t = 20 [ms]. Find $v_L(t)$ for t > 0.



Room for extra work

Room for extra work

3. {20 Points} The complex numbers $z_1 = -1.7e^{j\frac{70\pi}{180}}$ and $z_2 = 1.9e^{j\frac{200\pi}{180}}$ are given.

a) In what quadrant(s) do z_1 and z_2 lie?
b) What is the sum z_s of z_1 and z_2? Express z_s as a complex exponential (i.e., the same format as z_1 and z_2).

c) What are the real and imaginary components of $z_r = \frac{z_1}{z_2}$?

Room for extra work

1. {40 Points} Both switches in the circuit below were open for a long time, before they closed at $t = 0$.

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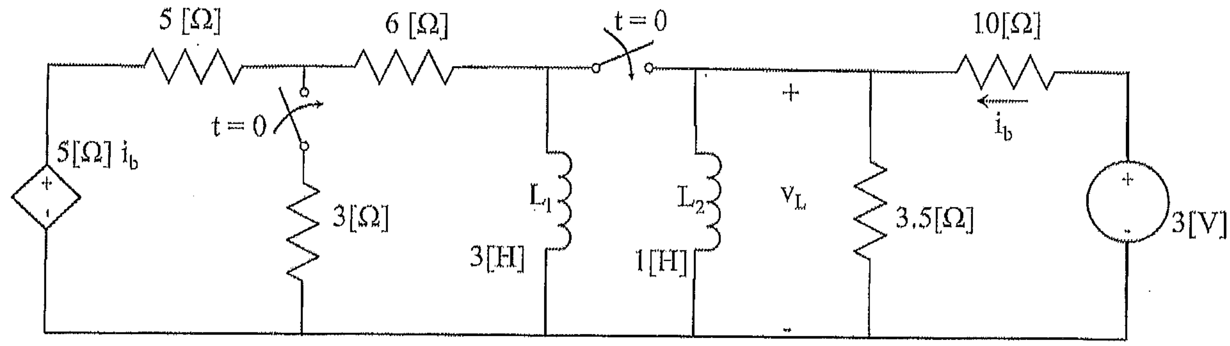
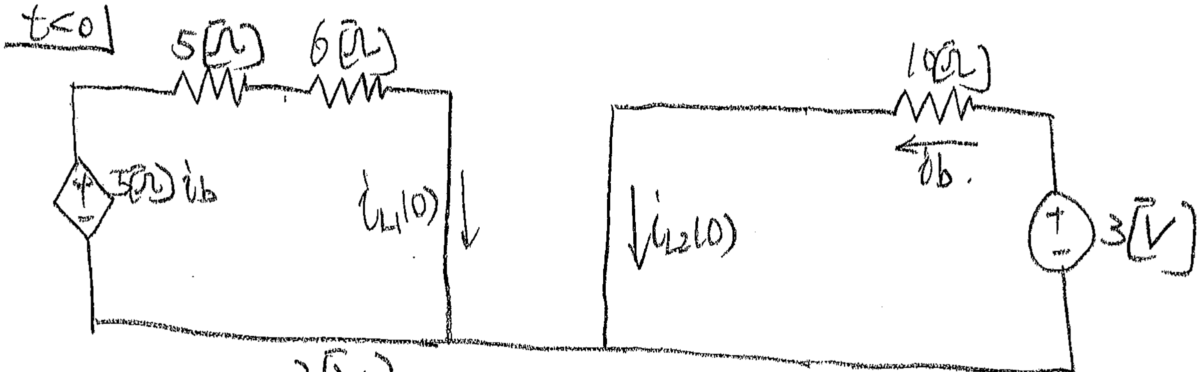
b) Find the energy stored in inductor L_1 at $t = +\infty$.



t<0 |

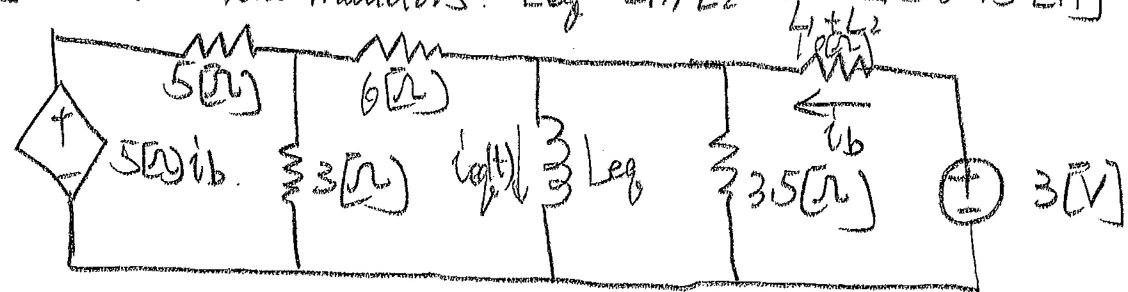


$$i_b = \frac{3[V]}{10[\Omega]} = 0.3[A]$$

$$i_{L2}(0) = i_b = 0.3[A]$$

$$i_{L1}(0) = \frac{5 \times i_b}{5+6} = \frac{1.5}{11} = 0.136[A]$$

t>0 | Combine two inductors: $L_{eq} = L_1 // L_2 = \frac{L_1 \cdot L_2}{L_1 + L_2} = 0.75[H]$

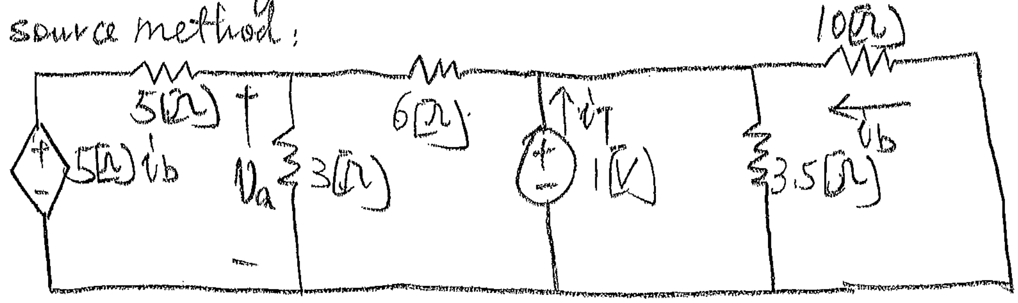


Initial condition: $i_{eq}(0) = i_{L1}(0) + i_{L2}(0) = 0.436\ [A]$

Room for extra work

To find out $\tau = \frac{L_{eq}}{R_{eq}}$ we need to know R_{eq}.

Test source method:



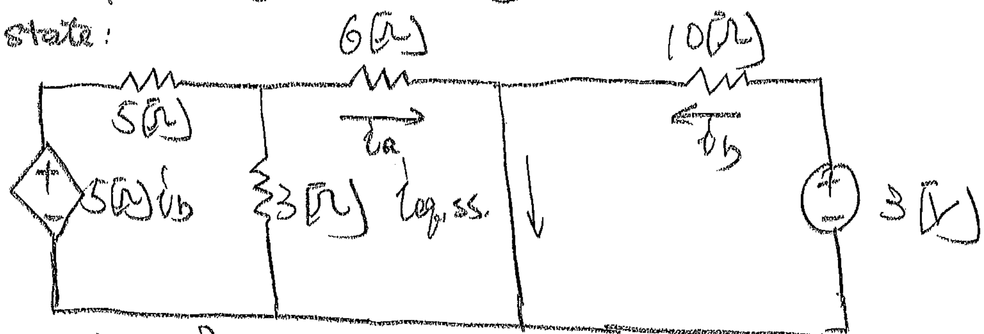
$$\begin{cases} i_b = -\frac{1(V)}{10(\Omega)} = -0.1\,[A] \\ \text{KCL: } -i_T + \frac{1}{3.5} + \frac{1}{10} + \frac{1-V_a}{6} = 0 \\ \text{KCL: } \frac{V_a - 5i_b}{5} + \frac{V_a}{3} + \frac{V_a - 1}{6} = 0 \end{cases}$$

solve: $i_T = 0.536\ [A]$

$R_{eq} = \frac{1(V)}{i_T} = \frac{1}{0.536} = 1.865\ (V)$

$\tau = \frac{L_{eq}}{R_{eq}} = \frac{0.75}{1.865} = 0.4\ [s]$

Steady. state:



$i_b = \frac{3}{10} = 0.3\,[A]$

$i_a = \frac{5 i_b}{5 + 3//6} \times \frac{3}{3+6} = 0.071\ [A]$

$i_{eq,ss} = i_a + i_b = 0.371\ [A]$

$i_{eq}(t) = 0.371 + (0.436 - 0.371) e^{-\frac{t}{0.4}}$

$= 0.371 + 0.065 e^{-2.5t} \quad (t \geq 0)$

Room for extra work

$$V_L(t) = L_{eq} \cdot \frac{d i_{eq}}{dt} = 1.85 \times 0.065 \times (-2.5) \times e^{-2.5t}$$

a). $\boxed{V_L(t) = -0.303 e^{-2.5t} \quad (t > 0)}$

$$i_{L1}(t) = \frac{1}{L_1} \int_0^t V_L(t) dt + i_{L1}(0)$$

$$i_{L1}(\infty) = \frac{1}{3} \int_0^\infty (-0.303 e^{-2.5t}) dt + 0.136$$

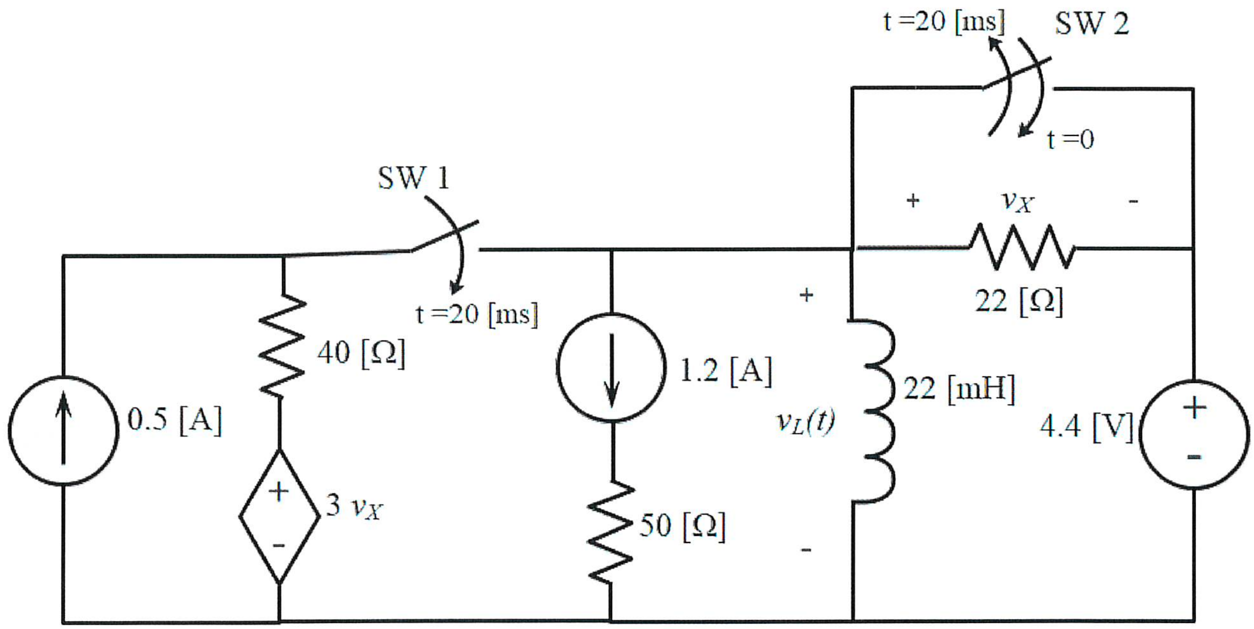
$$= \frac{-0.303}{3 \times (-2.5)} \int_0^\infty e^{-2.5t} d(-2.5t) + 0.136$$

$$= \frac{0.1}{2.5} e^{-2.5t} \Big|_0^\infty + 0.136$$

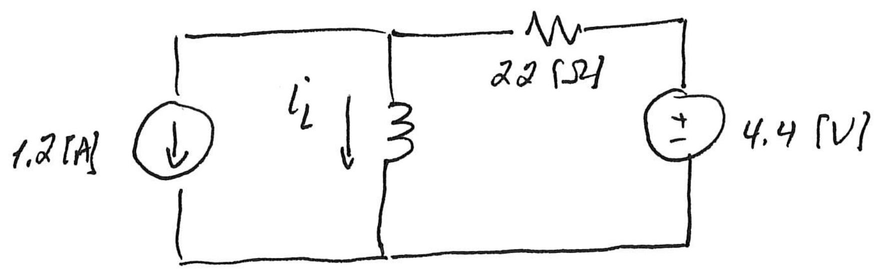
$$= 0.176 \text{ [A]}$$

$$\boxed{W_{sto, L_1} = \frac{1}{2} L_1 i_{L_1}^2(\infty) = \frac{1}{2} \times 3 \times (0.176)^2 = 0.046 \text{ [J]}}$$

2. {40 Points} In the circuit below, switch SW1 was open for a long time and closed at t = 20 [ms]. Switch SW2 was open for a long time. It closed at t = 0 and opened again at t = 20 [ms]. Find $v_L(t)$ for t > 0.



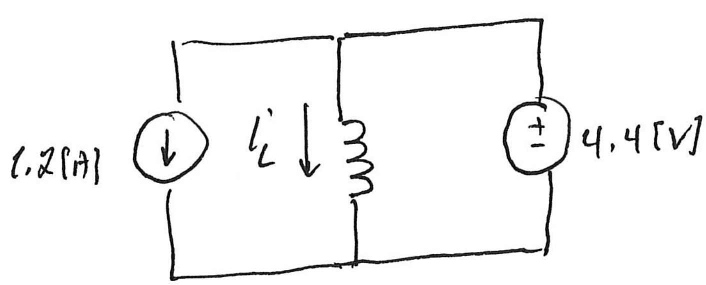
Draw for t < 0 ⇒ sw1 open, sw2 open. Define i_L'.



$$i_L' = -1.2 + \frac{4.4}{22} = -1\ [A] = i_L'(0).$$

Draw for 0 < t < 20 [ms]

$$i_L'(t) = \frac{1}{L}\int_0^t 4.4\,dt + i_L'(0)$$

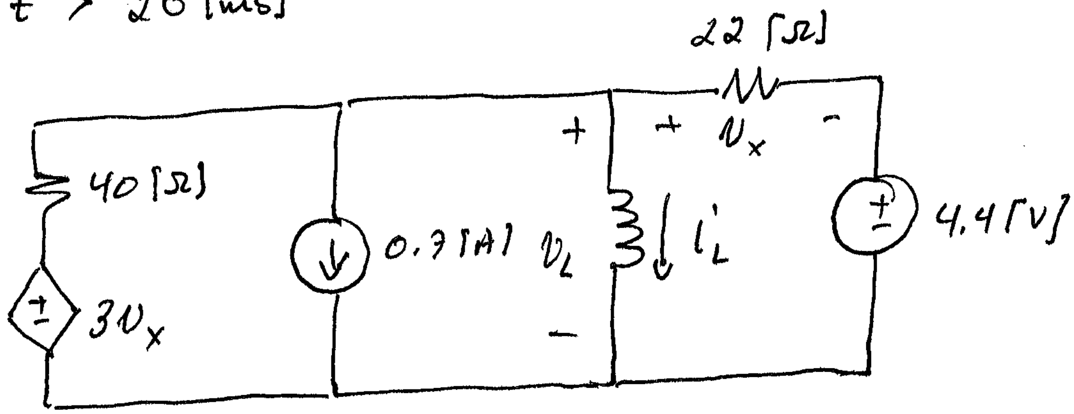


Room for extra work

$$i_L'(t) = \frac{1}{0.022}\int_0^t 4.4\,dt - 1 \quad [A]$$

$$= 200t - 1 \ [A] \qquad i_L'(20[ms]) = 3\,[A]$$

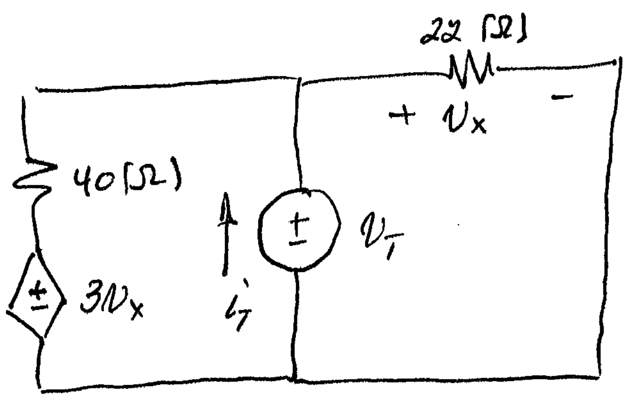
Draw for $t > 20\,[ms]$



$i_{L,f}'$: $L \rightarrow$ short so $v_x = -4.4\,[V]$

$$i_{L,f}' = \frac{4.4}{22} - 0.7 + \frac{3v_x}{40} = -0.83\,[A]$$

R_{TH} :



$v_T = v_x$

$$i_T = \frac{v_T}{22} + \frac{v_T - 3v_T}{40} \Rightarrow R_{TH} = \frac{v_T}{i_T} = -220\,[\Omega]$$

$$\tau_L = L/R_{TH} = \frac{0.022}{-220} = -100\,[\mu s]$$

Room for extra work

So...

$$i_L(t) = -0.83 + (3 - (-0.83))\, e^{+(t - 20\,[ms])/0.1\,[ms]} \quad [A]$$

$$t \geq 20\,[ms]$$

$$v_L(t) = L \frac{di_L}{dt} = (0.022)(3.83) \frac{1}{0.1 \times 10^{-3}}\, e^{(t - 20\,[ms])/0.1\,[ms]}$$

$$\boxed{v_L(t) = 842.6\, e^{(t - 20\,[ms])/0.1\,[ms]} \quad [V] \quad t > 20\,[ms]}$$

$$\boxed{v_L(t) = 4.4\,[V] \quad 0 < t < 20\,[ms]}$$

$z_1 = -1.7 e^{j\frac{70\pi}{180}}$ $z_2 = 1.9 e^{j\frac{200\pi}{180}}$

(a) $z_1 = -1.7 \cos\left(\frac{70\pi}{180}[rad]\right) + j(-1.7)\sin\left(\frac{70\pi}{180}[rad]\right)$

$= -.581 + -j\,1.60$

3 rd quadrant

$z_2 = 1.9\cos\left(\frac{200\pi}{180}[rad]\right) + j\,1.9\sin\left(\frac{200\pi}{180}[rad]\right)$

$= -1.79 - j\,0.65$

3 rd quadrant

(b) $z_s = z_1 + z_2 = (-0.581 - 1.79) + j(-1.60 - 0.65)$

$= -2.37 - j\,2.25$ (3rd quadrant)

In complex exponential form:

$z_s = \sqrt{(2.37)^2 + (2.25)^2}\, e^{j\tan^{-1}\left(\frac{-2.25}{-2.37}\right)}$

$= 3.26\, e^{j0.76} = \boxed{3.26\, e^{j\left(-\frac{136.5\pi}{180}\right)} = 3.26 \angle -136.5^\circ}$

Note: $\tan^{-1}\left(\frac{-2.25}{-2.37}\right) = 0.76\,[rad] = \frac{43.5^\circ \pi}{180^\circ}$.

But given that z_s lies in the third quadrant we must add 180°;

so $\frac{43.5^\circ\pi}{180^\circ} \Rightarrow \frac{(43.5^\circ + 180^\circ)\pi}{180} = \frac{223.5^\circ\pi}{180^\circ} = -\frac{136.5^\circ\pi}{180^\circ}$

$z_s = 3.26 \angle -136.5^\circ = 3.26 \angle 223.5^\circ$ Both are acceptable answers.

(c) $z_r = \frac{z_1}{z_2} = \frac{-1.7 e^{j\frac{70\pi}{180}}}{1.9 e^{j\frac{200\pi}{180}}} = \left(\frac{-1.7}{1.9}\right) e^{j\left(\frac{70\pi}{180} - \frac{200\pi}{180}\right)}$

$= -0.89\, e^{j\left(-\frac{130\pi}{180}\right)}$

$= -0.89\cos\left(-\frac{130\pi}{180}[rad]\right) + j(-0.89)\sin\left(-\frac{130\pi}{180}[rad]\right)$

$= \boxed{0.58} + j\,\boxed{0.69}$

real   imaginary