Name: ______________________________ (please print)

Signature: ___________________________

ECE 2202 – Final Exam

May 4, 2022

Keep this exam closed until you are told to begin.

1. This exam is closed book, closed notes. You may use one 8.5" x 11" crib sheet, or its equivalent.
2. Show all work on these pages. Show all work necessary to complete the problem. A solution without the appropriate work shown will receive no credit. A solution that is not given in a reasonable order will lose credit. Clearly indicate your answer (for example by enclosing it in a box).
3. Show all units in solutions, intermediate results, and figures. Units in the exam will be included between square brackets.
4. Do not use red ink. Do not use red pencil.
5. You will have 160 minutes to work on this exam.

1. _______________/40

2. _______________/40

3. _______________/40

4. _______________/40

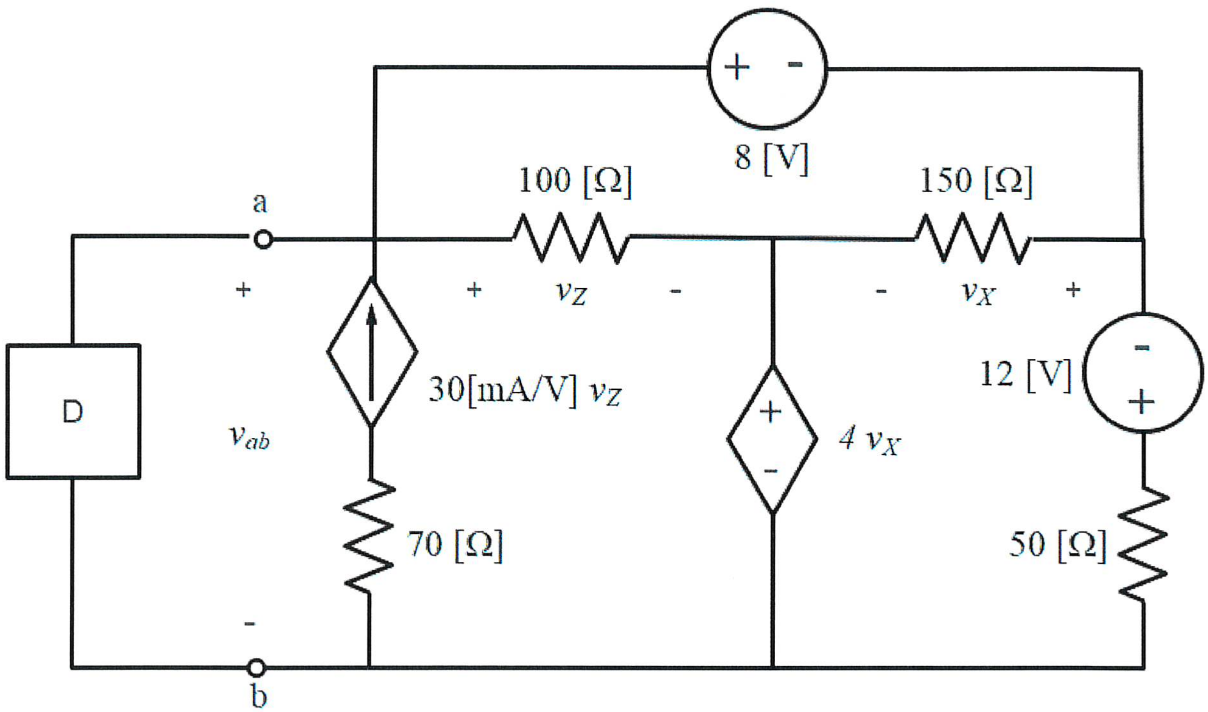
5. _______________/40

Total = 200

Room for extra work

1. {40 Points} Device D is connected to a circuit at terminals a, b, as shown below. Device D can be represented by a Norton equivalent with a Norton current of 100 [mA]. When it is connected to the circuit, a voltage v_{ab} = -1.7919 [V] is developed across terminals a, b, as shown in the figure.

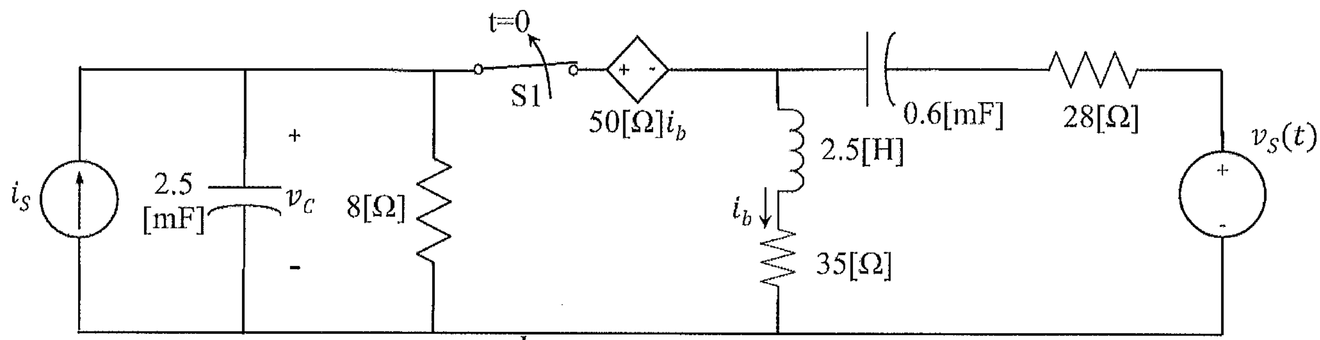
   a) Find the Thevenin Equivalent seen by device D.
   b) Find the Norton resistance for device D.



Room for extra work

Room for extra work

2. {40 Points} Circuit below has been in steady state for a long time before t = 0.
   At t = 0, switch S1 opens. Please find:
   1) $v_C(0)$ (Hint: It is a phasor problem for t<0).
   2) $v_C(10[ms])$.



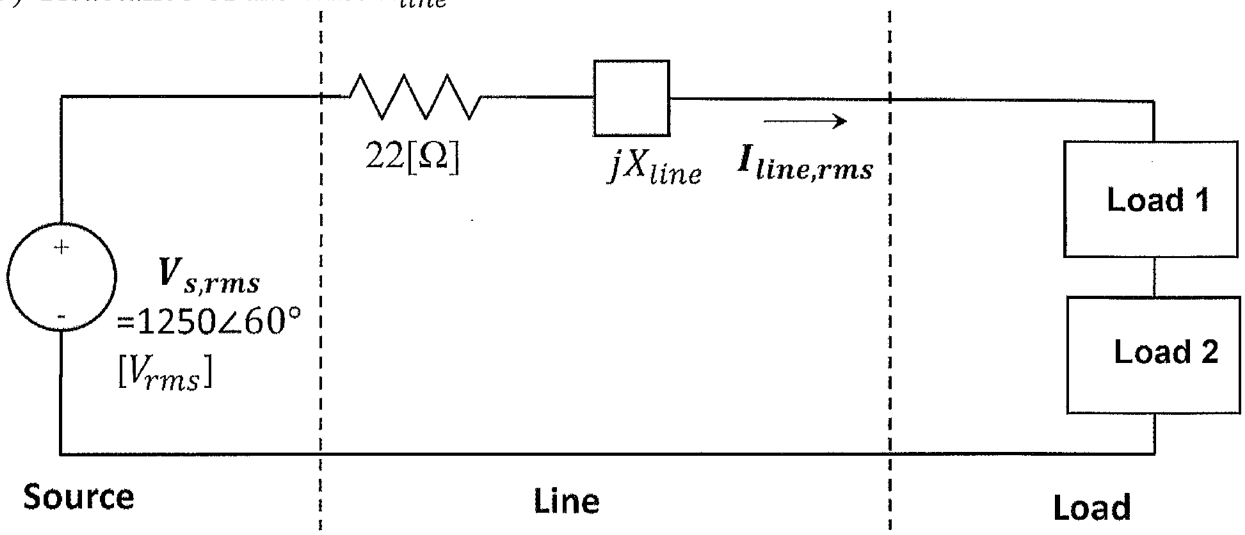
$$v_S(t) = 10 \cdot \sin\left(30\left[\frac{rad}{s}\right]t + 45°\right)[V]$$

$$i_S = 2[A]$$

Room for extra work

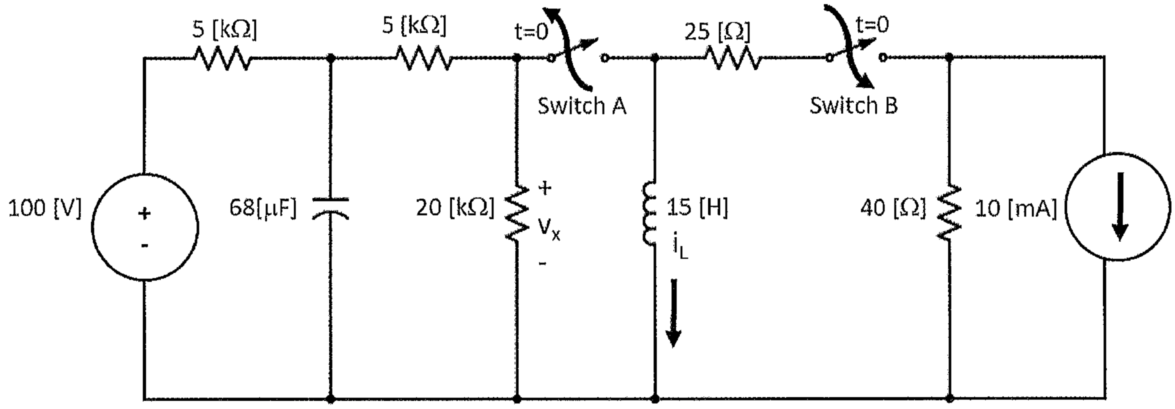
Room for extra work

3. {40 Points} Circuit below has been in steady state. Load 1 absorbs 500 [VA] with leading power factor of 0.55, Load 2 absorbs 550 [W] with power factor angle of 50°, and source delivers real power of 1000 [W].
Please find:
   1) Amplitude of RMS phasor current through line: $|\boldsymbol{I}_{line,rms}|$
   2) Total impedance of the Load: $Z_{Load} = Z_{Load\ 1} + Z_{Load\ 2}$
   3) Reactance of the line: X_{line}



Room for extra work

4. {40 Points} For the circuit shown below, Switch A has been closed and Switch B has been open for a long time. At time t=0, Switch A opens and Switch B closes.
   Find the current iL(100 [ms]) and voltage vx(100 [ms]).



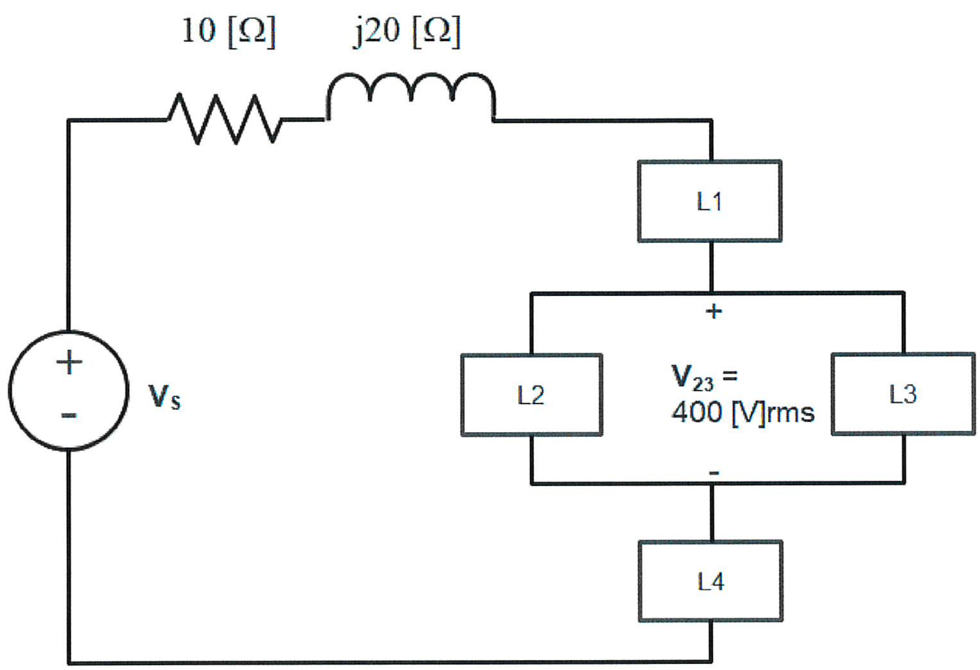
Room for extra work

5. {40 Points} In the circuit below, **Vs** is the source voltage, L1, L2, L3, and L4 are loads, and the impedance (10 + j20) [Ω] represents the line. The following is known.

- L1 absorbs $36000\angle 17°$ [VA].
- L2 is an impedance of 40 - j60 [Ω].
- L3 absorbs 45000 [W] at 0.6 power factor leading.
- L4 absorbs 15000 [W] and delivers 25000 [VAR].

a) What source voltage **Vs** is needed to provide 400 $[V]_{rms}$ across loads L2 and L3?

b) What is the power factor angle for the four loads combined?

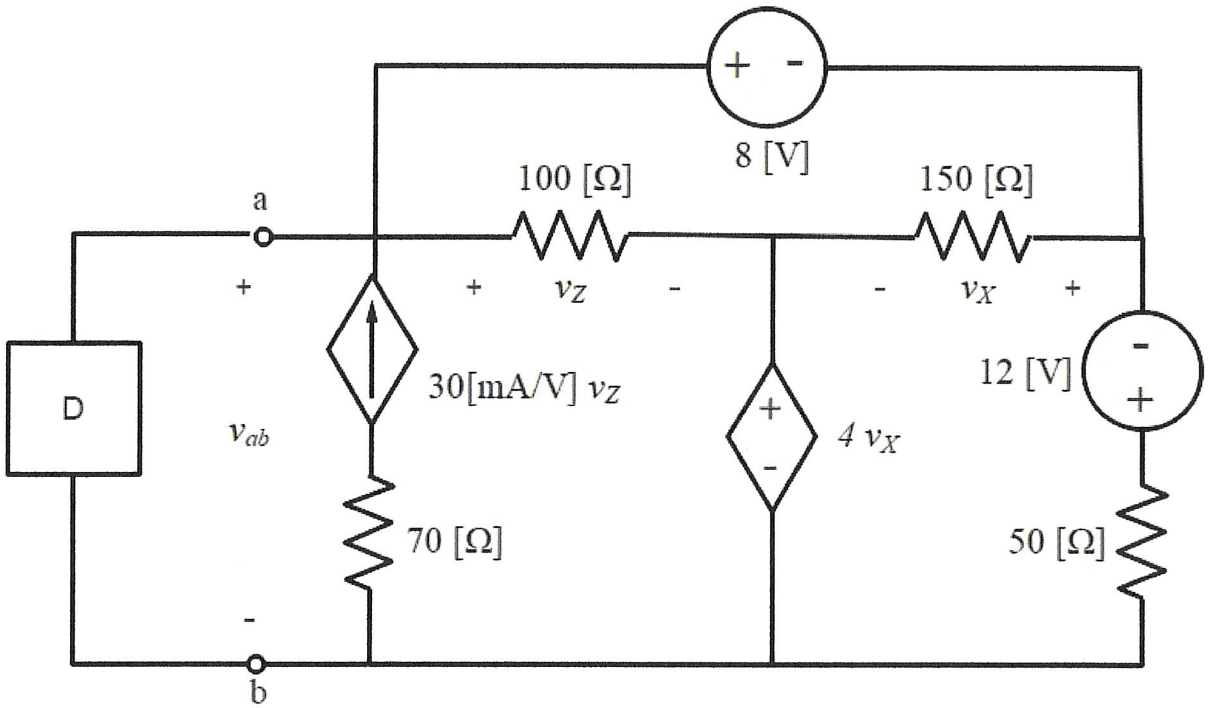


Room for extra work

Room for extra work

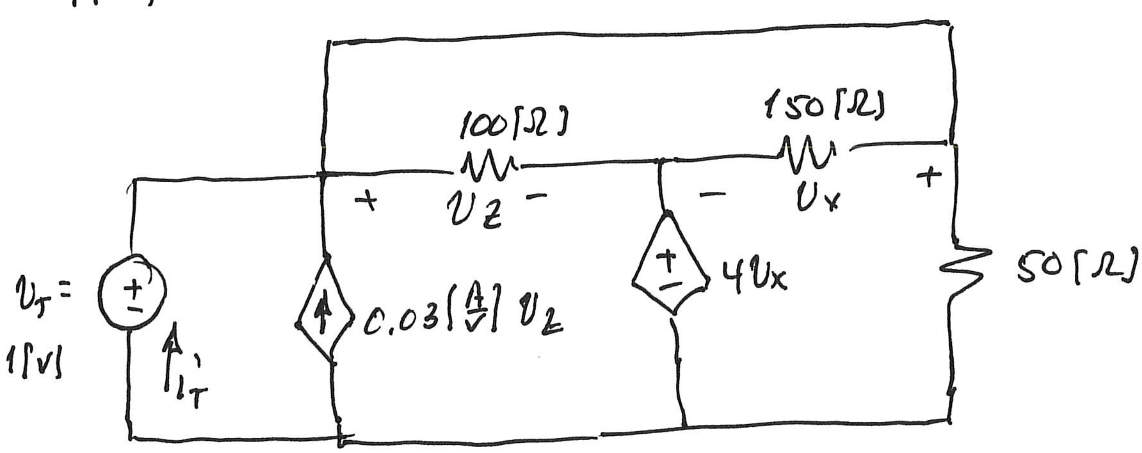
1. {40 Points} Device D is connected to a circuit at terminals a, b, as shown below. Device D can be represented by a Norton equivalent with a Norton current of 100 [mA]. When it is connected to the circuit, a voltage v_{ab} = -1.7919 [V] is developed across terminals a, b, as shown in the figure.

   a) Find the Thevenin Equivalent seen by device D.
   b) Find the Norton resistance for device D.



a) apply a test source; ignore 70 [Ω]

+4



no short for 8 [V] -4

remove .03v_Z for i_{sc} -7

Thev. drawing but no D attached -2

Room for extra work

$$i_T' = -0.03\, v_z + \frac{v_z}{100} + \frac{v_x}{150} + \frac{v_x + 4v_x}{50}$$

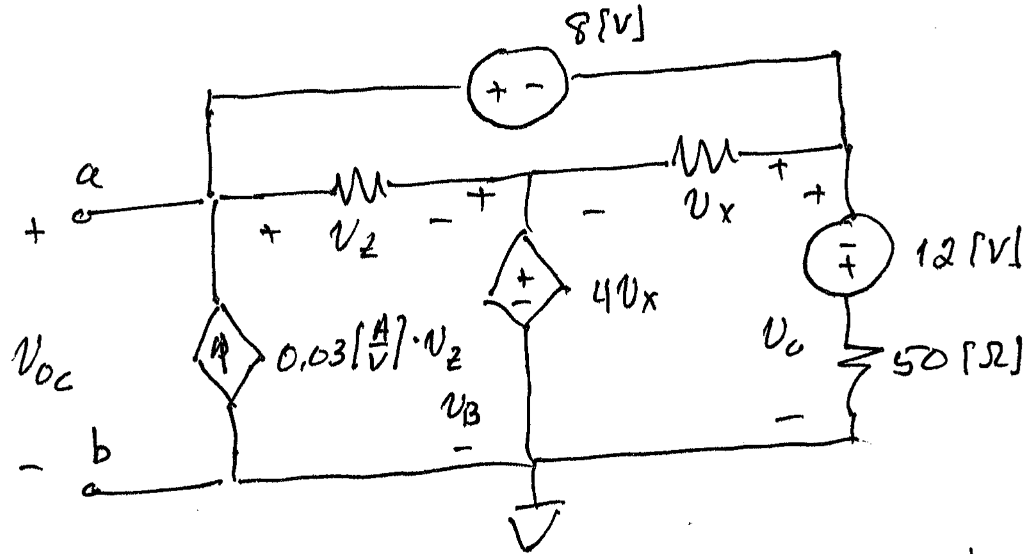
+9 $v_z + 4v_x = 1$ $\quad v_x = v_z$

$\Rightarrow$ $v_x = v_z = 0.2$ [V]

+2 $i_T' = 17.333$ [mA] $\Rightarrow$ $R_{TH} = 57.692$ [Ω]

v_{oc}:

+4



$$-0.03\, v_z + \frac{v_{oc} - v_B}{100} + \frac{v_c + 12}{50} + \frac{v_c - v_B}{150} = 0$$

+9 $v_B = 4v_x$ $\quad v_{oc} - v_c = 8$

$v_x = v_c - v_B$ $\quad v_z = v_{oc} - v_B$

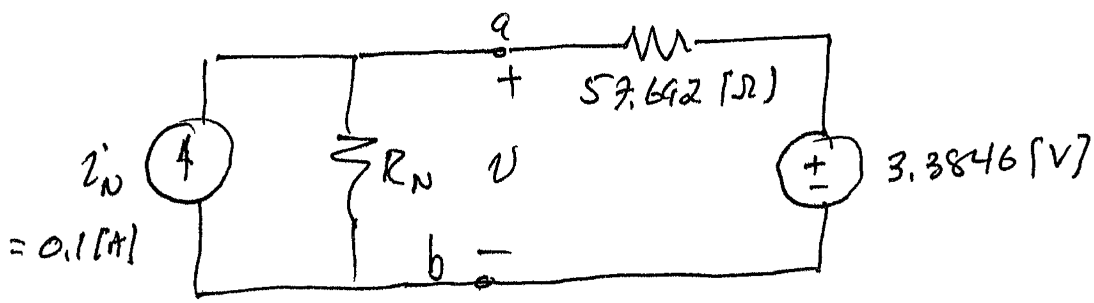
+2 $\Rightarrow$ $v_{oc} = v_{TH} = 3.3846$ [Ω]

$v_B = -3.6923$ [V]
$v_c = -4.6154$ [V]
$v_x = -0.92308$ [V]
$v_z = 7.0769$ [V]

Room for extra work

b) The direction of i_N was not specified, so we'll do this for both directions.

+4



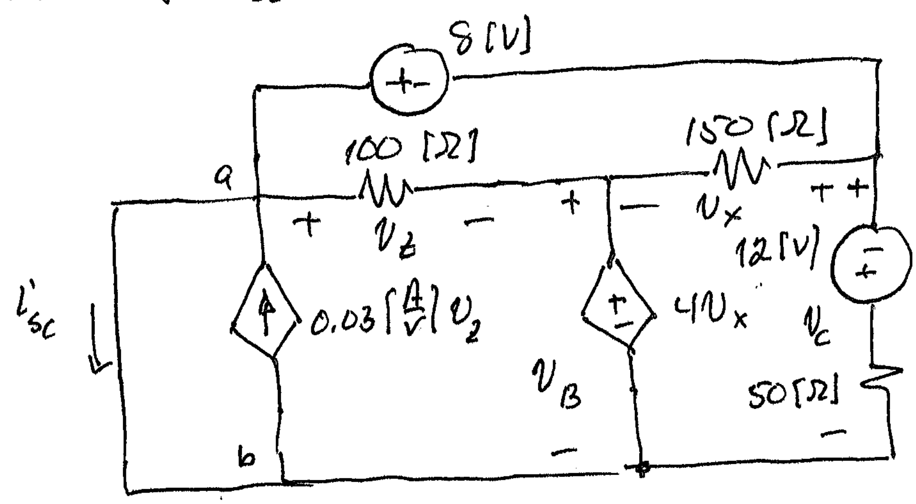
+4 GIVEN $v = -1.7919$ [V]

$$\frac{v}{R_N} - 0.1 + \frac{v - 3.3846}{57.642} = 0$$

+2 $\Rightarrow$ $R_N = -9.445$ [Ω]

If i_N is reversed, $R_N = 174.42$ [Ω]

Finding i_{sc} is also acceptable:



$v_B = 4v_x = -6.4$ [V]

$v_c = -8$

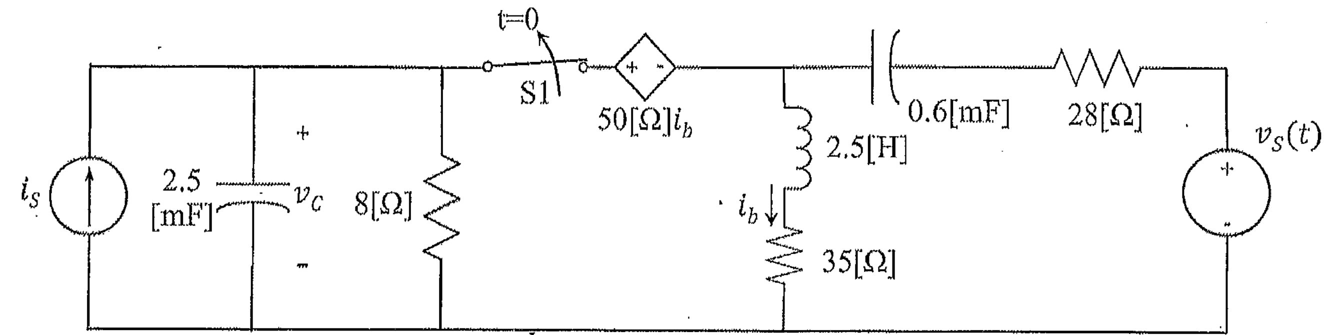
$v_x = v_c - v_B = -1.6$ [V]

$v_z = -v_B = 6.4$ [V]

$$i_{sc} = 0.03\, v_z - \frac{v_z}{100} - \frac{v_x}{150} - \frac{v_c + 12}{50}$$

$i_{sc} = 58.667$ [mA] $\Rightarrow$ $R_{TH} = \frac{v_{oc}}{i_{sc}} = \frac{3.3846}{0.058667} = 57.69$ [Ω]

2. {40 Points} Circuit below has been in steady state for a long time before t = 0. At t = 0, switch S1 opens. Please find:
   1) $v_C(0)$ (Hint: It is a phasor problem for t<0).
   2) $v_C(10[ms])$.



$$v_S(t) = 10 \cdot \sin\left(30\left[\frac{rad}{s}\right]t + 45°\right)[V]$$

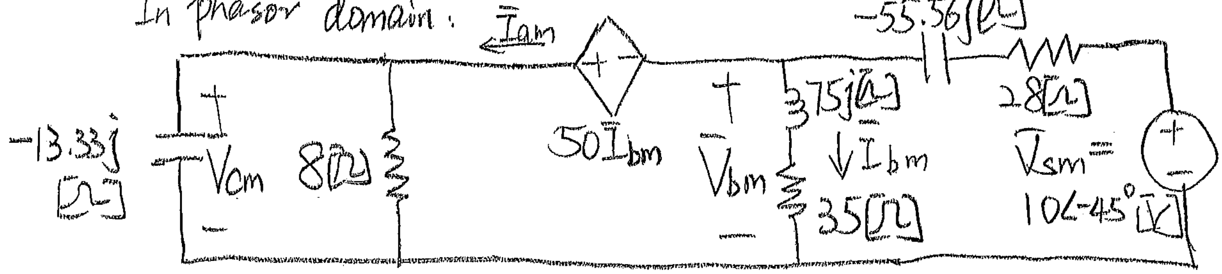
$$i_S = 2[A]$$

t<0 | There are two sources with different frequencies.

Use super position.

△ Take V_s, and set $i_s = 0$

In phasor domain:



$$\begin{cases} \dfrac{\bar{V}_{bm} - 10\angle{-45°}}{28 - 55.56j} + \bar{I}_{bm} + \bar{I}_{am} = 0 \\ \dfrac{\bar{V}_{bm} + 50\bar{I}_{bm}}{8} + \dfrac{\bar{V}_{bm} + 50\bar{I}_{bm}}{-13.33j} - \bar{I}_{am} = 0 \\ \bar{I}_{bm} = \dfrac{\bar{V}_{bm}}{35 + 75j} \end{cases}$$

Solve: we have:

$$\begin{cases} \bar{I}_{am} = 0.144 + 0.046j \\ \bar{I}_{bm} = 0.0053 - 0.0075j \\ \bar{V}_{bm} = 0.744 + 0.136j \end{cases}$$

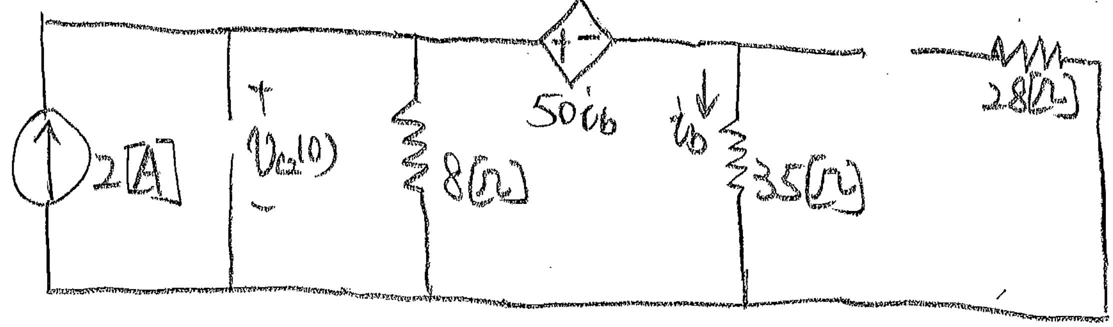
Room for extra work

$\bar{V}_{cm} = \bar{V}_{bm} + 50\bar{I}_{bm} = 1.037\angle -13.32^\circ$

$V_{c1}(t) = 1.037\cos(30t - 13.32^\circ)$ [V]

$V_{c1}(0) = 0.971$ [V]

△ Take i_s, set $v_s = 0$, it is a D.C. source.



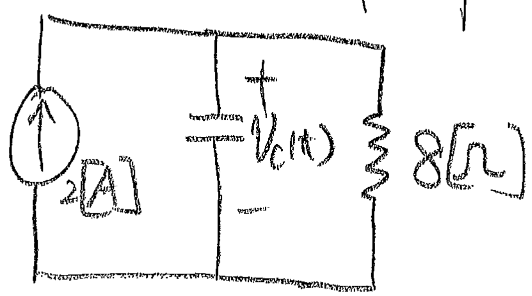
KCL: $\frac{35i_b + 50i_b}{8} + i_b - 2 = 0$

$i_b = 0.172$ [A]

$V_{c2}(0) = (50+35)i_b = 14.6$ [V]

$V_c(0) = V_{c1}(0) + V_{c2}(0) = 0.971 + 14.6 = 15.57$ [V]

t>0 | It is a step response:



$R_{eq} = 8$[Ω]

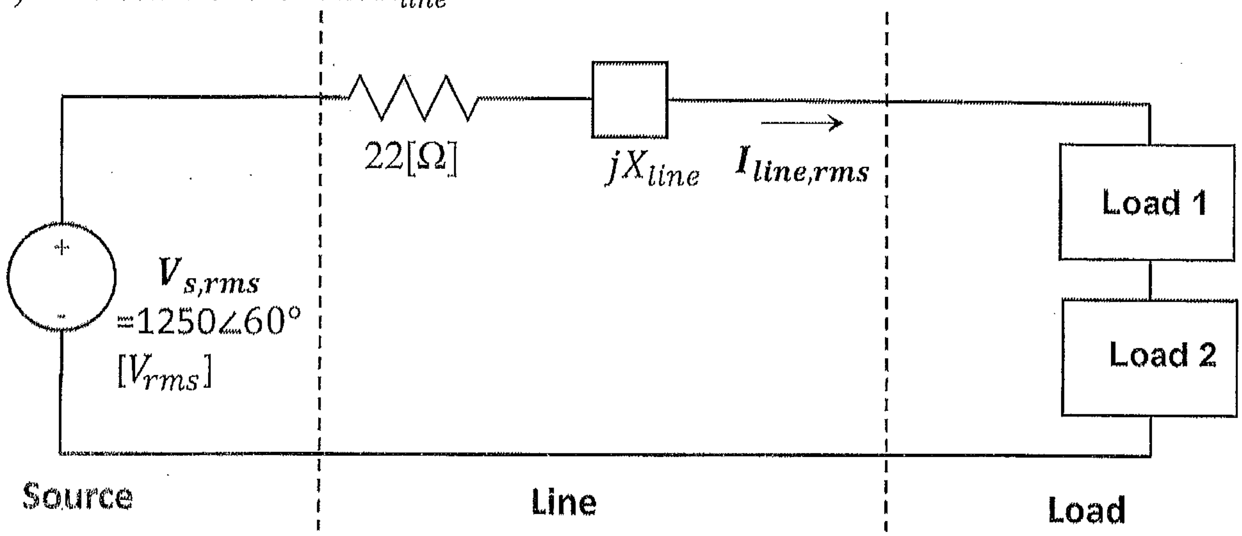
$\tau = R_{eq} \cdot C = 8 \times 2.5\text{mF} = 0.02$[s]

$V_{c,ss} = 8 \times 2 = 16$[V]

$V_c(t) = 16 + (15.57 - 16)e^{-\frac{t}{0.02}} = 16 - 0.43e^{-\frac{t}{0.02}}$ $(t \geq 0)$

$V_c(10\text{ms}) = 16 - 0.43e^{-\frac{0.01}{0.02}} = 15.73$ [V]

3. {40 Points} Circuit below has been in steady state. Load 1 absorbs 500 [VA] with leading power factor of 0.55, Load 2 absorbs 550 [W] with power factor angle of 50°, and source delivers real power of 1000 [W].
Please find:
   1) Amplitude of RMS phasor current through line: $|\boldsymbol{I}_{line,rms}|$
   2) Total impedance of the Load: $Z_{Load} = Z_{Load\,1} + Z_{Load\,2}$
   3) Reactance of the line: X_{line}



$\cos\theta = 0.55$. ∵ It is a leading power factor, $\theta < 0$

$\theta = \arccos(0.55) = -56.63°$

$P_{abs.\,by.\,Load\,1} = 500 \times \cos\theta = 500 \times 0.55 = 275\ [W]$

$Q_{abs.\,by.\,Load\,1} = 500 \times \sin\theta = -417.66\ [VAR]$

$P_{abs.\,by.\,Load\,2} = 550\ [W]$

$Q_{abs.\,by.\,Load\,2} = 550 \times \tan(50°) = 655.5\ [VAR]$

$P_{abs.\,by.\,Line} = 1000 - 550 - 275 = 175\ [W] = |I_{Line,rms}|^2 \times 22$

$\therefore |I_{Line,rms}| = \sqrt{\frac{175}{22}} = 2.82\ [A_{rms}]$

$P_{abs.\,by.\,Load\,1} = 275 = |I_{Line,rms}|^2 \cdot R_{Load\,1} \Rightarrow R_{Load\,1} = \frac{275}{2.82^2} = 34.58\ [\Omega]$

$Q_{abs.\,by.\,Load\,1} = -417.66 = |I_{Line,rms}|^2 \cdot X_{Load\,1} \Rightarrow X_{Load\,1} = -52.5\ [\Omega]$

Room for extra work

$P_{abs.\ by.\ Load2} = 550 = |\bar{I}_{line,rms}|^2 \cdot R_{Load2} \Rightarrow R_{Load2} = 69.14\ [\Omega]$

$Q_{abs.\ by.\ Load2} = 655.5 = |\bar{I}_{line,rms}|^2 \cdot X_{Load2} \Rightarrow X_{Load2} = 82.40\ (\Omega)$

$Z_{Load} = (34.58 - 52.5j) + (69.14 + 82.40j)$

$\boxed{Z_{Load} = 103.72 + 29.9j\ [\Omega]}$

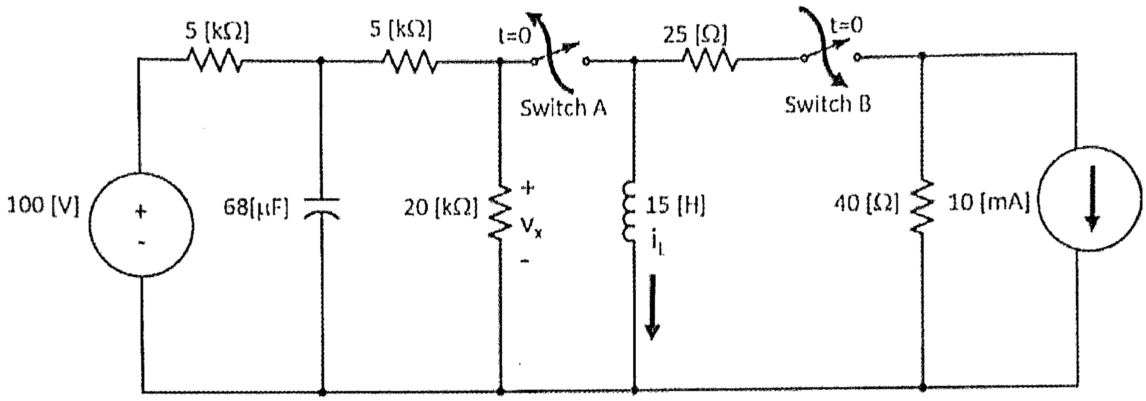
$\bar{I}_{line,rms} = |\bar{I}_{line,rms}| \angle \theta_{line} = 2.82 \angle \theta_{line}.$

$Z_{Load} \times \bar{I}_{line,rms} = \bar{V}_{s,rms}$

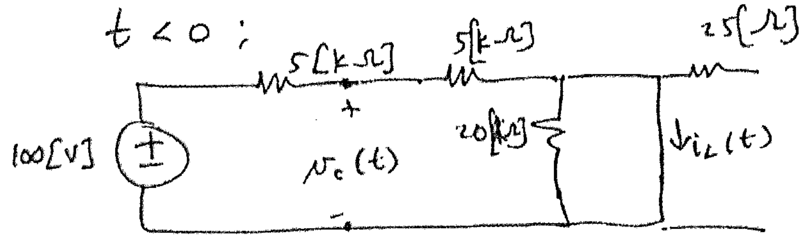
$[(103.72 + 29.9j) + (22 + X_{line}j)]\, 2.82 \angle \theta_{line} = 1250 \angle 60°.$

Solve: We have: $\boxed{X_j = 395.1\ [\Omega] \text{ or. } -454.9\ (\Omega)}$

4. {40 Points} For the circuit shown below, Switch A has been closed and Switch B has been open for a long time. At time t=0, Switch A opens and Switch B closes.
   Find the current iL(100 [ms]) and voltage vx(100 [ms]).



$t < 0$;



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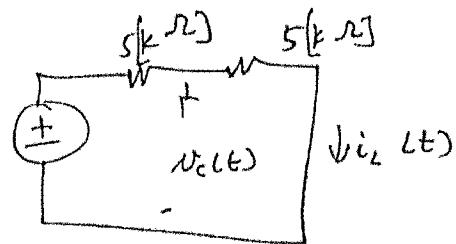


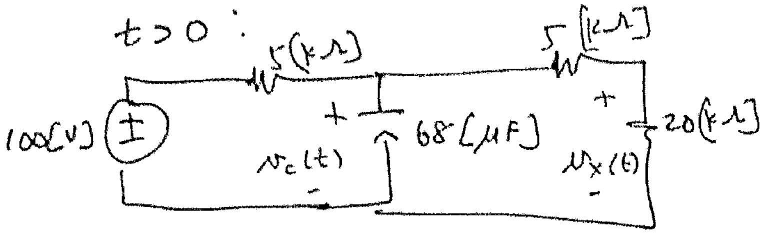
$$v_c(0^-) = v_c(0^+) = 100[V]\left(\frac{5[k\Omega]}{5[k\Omega] + 5[k\Omega]}\right) = 50[V]$$

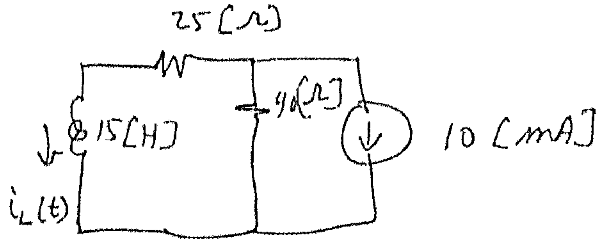
$$i_L(0^-) = i_L(0^+) = \frac{100[V]}{5[k\Omega] + 5[k\Omega]} = 0.01[A] = 10[mA]$$

next page →

Room for extra work

$t > 0$:





$i_L(t)$:

$$\tau_L = \frac{15[H]}{(25+40)[\Omega]} = 0.231\ [s]$$

$$i_L(0^+) = 0.01\ [A]$$

$$i_L(\infty) = (-.010[A])\left(\frac{40\ [\Omega]}{(25+40)\ [\Omega]}\right) = -.00615\ [A] = -6.15\ [mA]$$

$$i_L(t) = -.00615 + (.010 + .00615)\, e^{-t/.231[s]} \quad [A]$$

$$= -.00615 + 0.1615\, e^{-4.33t} \quad [A]$$

$$\text{(a)} \quad i_L(100\,[ms]) = -.00615 + 0.1615\, e^{-4.33(.1)}$$

$$= 0.004319\ [A] = 4.3\ [mA]$$

$v_c(t)$:

$$\tau_c = (68[\mu F])\left(5k[\Omega] \,\|\, (5k+20k)[\Omega]\right) = 0.283\ [s]$$

$$v_c(0^+) = 50\ [V]$$

$$v_c(\infty) = 100[V]\left(\frac{25\ [k\Omega]}{(25k + 5k)[\Omega]}\right) = 83.3\ [V]$$

$$v_c(t) = 83.3 + (50 - 83.3)\, e^{-t/0.283[s]} \quad [V]$$

$$v_x(t) = v_c(t)\left(\frac{20[k\Omega]}{5[k\Omega] + 20[k\Omega]}\right) = 0.8\, v_c(t)$$

$$v_x(100\,[ms]) = 66.67 - 26.67\, e^{-3.53\,(0.1)} \quad [V]$$

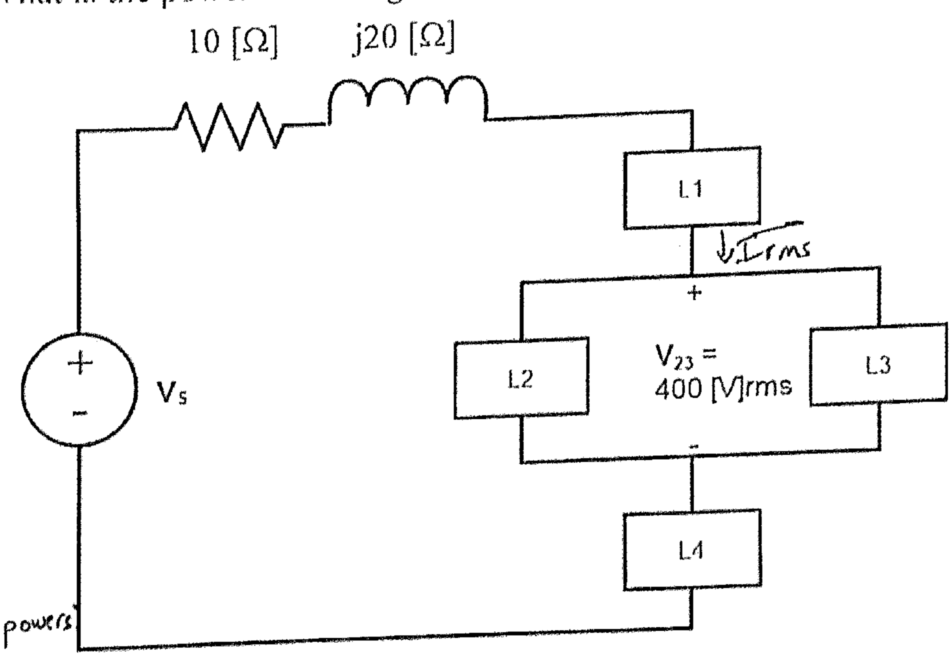
$$= 47.93\ [V]$$

5. {40 Points} In the circuit below, V_S is the source voltage, L1, L2, L3, and L4 are loads, and the impedance (2 +j4) [Ω] represents the line. The following is known.

- L1 absorbs 36000∠17° [VA].
- L2 is an impedance of 40 - j60 [W].
- L3 absorbs 45000 [W] at 0.6 power factor leading.
- L4 absorbs 15000 [W] and delivers 25000 [VAR].

a) What source voltage V_S is needed to provide 400 $[V]_{rms}$ across loads L2 and L3?

b) What is the power factor angle for the four loads combined?



Find complex powers

$S_1 = 36000\angle 17°$ [VA]

$S_2 = \dfrac{|V_{rms_{23}}|^2}{Z_{23}^*} = \dfrac{|400\,(V_{rms})|^2}{40 + j60\,(\Omega)} = \dfrac{|400\,(V_{rms})|^2}{72.1\angle 56.3°} = 2218.8\angle -56.3°$ [VA]

S_3 : $|S_3| = \dfrac{P_3}{pf_3} = \dfrac{45000\,[W]}{0.6} = 75000\,[VA]$

$\theta_3 = -\cos^{-1}(0.6) = -53.13°$ (leading)

$S_3 = 75000\angle -53.13°$ [VA]

S_4 : $S_4 = 15000 - j25000 = 29154.8\angle -59.04°$ (delivered)

Room for extra work

Find $\vec{I_{rms}}$ given the info about $\vec{V_{23,rms}} = 400$ [Vrms] :

Total complex power of Load 2 and Load 3:

$$S_{23} = S_2 + S_3 = 2218.8\angle -56.3° + 75000\angle -53.13° \text{ [VA]}$$

$$= 77215.48\angle -53.22° \text{ [VA]}$$

$$S_{23} = \frac{|\vec{V_{23,rms}}|^2}{Z_{23}^*} = \frac{|400 \text{ [Vrms]}|^2}{Z_{23}^*} = 77215.48\angle -53.22°$$

$$Z_{23}^* = 2.072\angle 53.22° \text{ [}\Omega\text{]}$$

$$Z_{23} = 2.072\angle -53.22° \text{ [}\Omega\text{]}$$

Find $\vec{I_{rms}}$:

$$S_{23} = |\vec{I_{rms}}|^2 Z_{23}$$

$$|I_{rms}| = \sqrt{\frac{S_{23}}{Z_{23}}} = \sqrt{\frac{77215.48\angle -53.22°}{2.072\angle -53.22°}} = 193.04 \text{ [A]}$$

$$S_{23} = \vec{V_{rms,23}}\, \vec{I_{rms}}^* = (400 \text{ [Vrms]} \angle 0°)(193.04\angle -\phi_i)$$

$$53.22° = \phi_i$$

$$\vec{I_{rms}} = 193.04\angle 53.22° \text{ (A]}$$

Find complex power of Line:

$$S_{Line} = |\vec{I_{rms}}|^2 Z_{line} = (193.04)^2 (10 + j20 \text{ [}\Omega\text{]})$$

$$= 833247.1\angle 63.43° \text{ [VA]}$$

$$S_{tot} = S_1 + S_2 + S_3 + S_4 + S_{line} = 816582.6\angle 55° \text{ [VA]}$$

(a) $$\vec{V_s} = \frac{S_{tot}}{I_{rms}^*} = 4230.15\angle 108.2° \text{ [Vrms]}$$

(b) $S_{load} = S_1 + S_2 + S_3 + S_4 = 122373.5\angle -38.6°$. Power Factor Angle is $-38.6°$