Signature
Name (print, please)
Student No.

ELEE 2300 Circuit Analysis
Summer 2011
Final Exam

DO NOT OPEN THIS EXAM BOOKLET UNTIL INSTRUCTED TO DO SO

This exam has 14 pages including this cover page. If you are missing any pages, raise your hand. You have 180 minutes to complete the exam.

Notes

1. Be sure your name and signature appear above.
2. The exam is closed-book; you may have a single 8 ½ x 11" crib sheet and a calculator.
3. To receive full credit for a problem, you must:
   - Show all work necessary to solve the problem;
   - Write clearly and legibly;
   - Indicate clearly whether power being calculated is absorbed or delivered;
   - Define all variables and parameters and label them on circuit diagrams;
   - Show all units explicitly in intermediate steps as well as final solutions;
   - Use the proper notation for all variables.

1. ________/14	4. ________/30
2. ________/18	5. ________/15
3. ________/18	6. ________/20

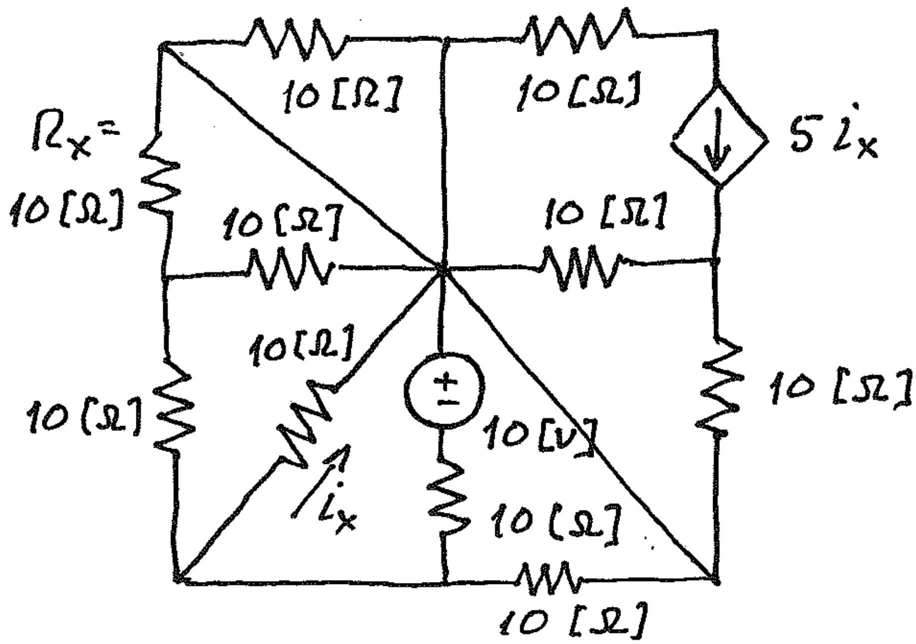
Total________/115

A perfect score on the exam will result in 115 points, so there is an opportunity for extra credit.

Room for extra work

1. (<u>14 points</u>) In the circuit below, find the following.

i) The power delivered to the circuit by each source.
ii) The power dissipated by resistor R_x.

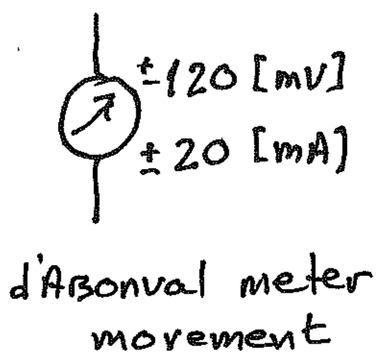


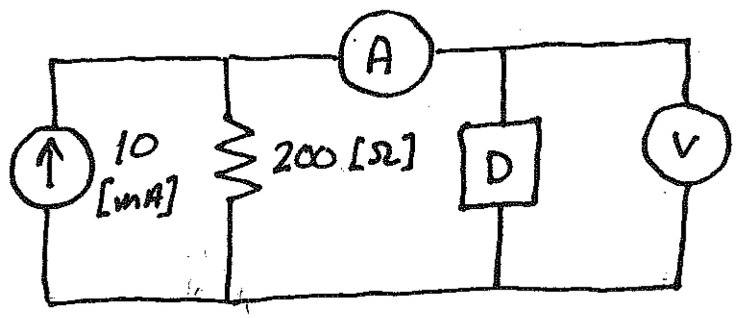
Room for extra work

2. (<u>18 points</u>) The d'Arsonval meter movement shown below on the left is used to construct an ammeter and a voltmeter that can measure both positive and negative values. These meters are then connected into the circuit on the right in order to measure the resistance of the device D by dividing the voltmeter reading by the ammeter reading. The ammeter is set to a full scale reading of 30 [mA]. The voltmeter can be set to full scale values of 1, 2, 5, 10, 20, 50, 100, 200, 500, ...(etc.) [V].

It is known that the device D has a resistance of -75.00 [Ω], but when the meters are used to measure this value, the result is -81.08 [Ω].

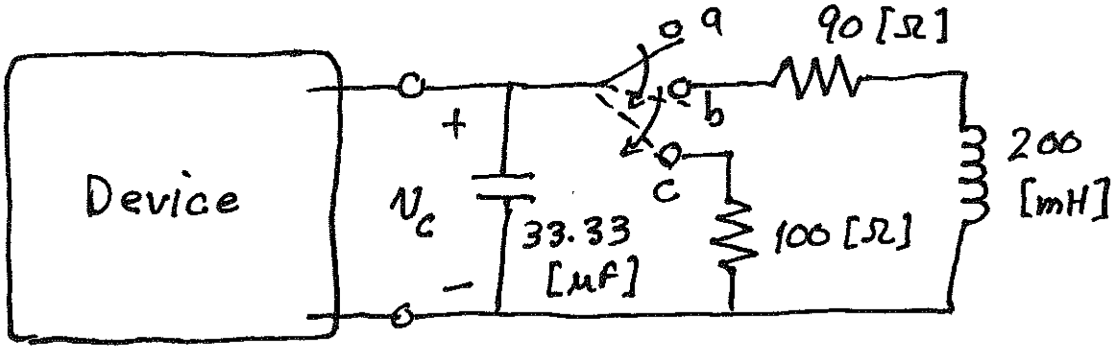
i) What is the reading on the ammeter?
ii) What is the full scale voltage setting of the voltmeter? (Note that the possible settings are given in the statement of the problem above.)
iii) What is the smallest full scale voltage setting of the voltmeter that would reduce the error in the measurement to 5% or less?





Room for extra work

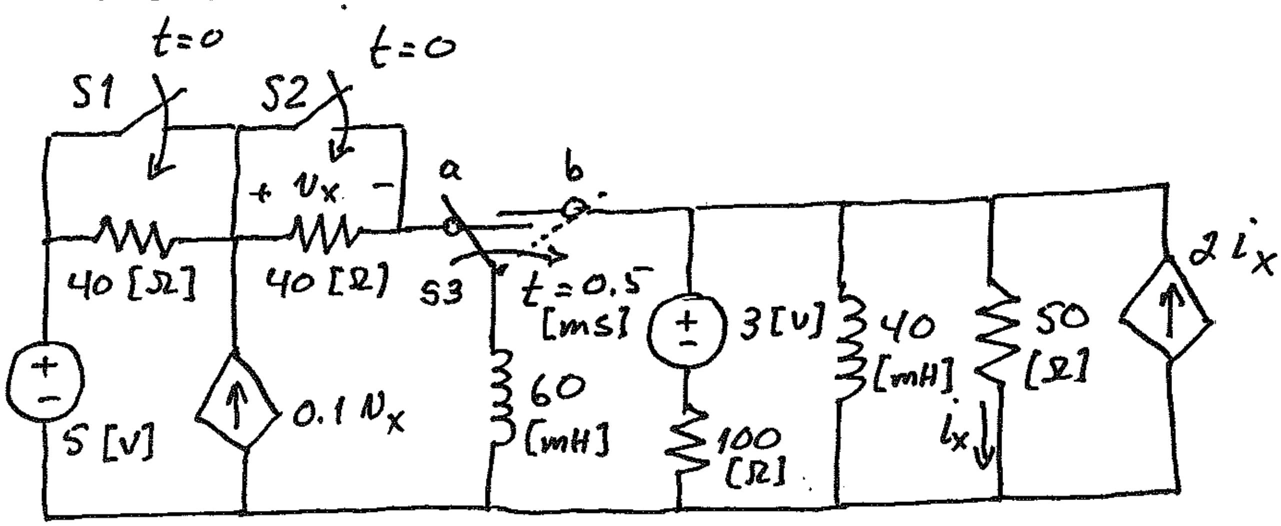
3. (18 points) The device inside the box can be modeled as a sinusoidal voltage source $v_s(t)$ in series with an impedance Z_s. The switch was in position "a" for a long time, and the steady-state voltage v_C was $v_C(t) = 40.2 \cos(300t - 35^{\circ})$ [V]. The switch then moved to position "b" at $t = 0$, and after a long time, the steady-state value voltage v_C was $v_C(t) = 21.59 \cos(300t - 74.72^{\circ})$ [V]. If the switch is then moved to position "c" at $t = t_0$, what is the steady-state voltage $v_C(t)$?



Room for extra work

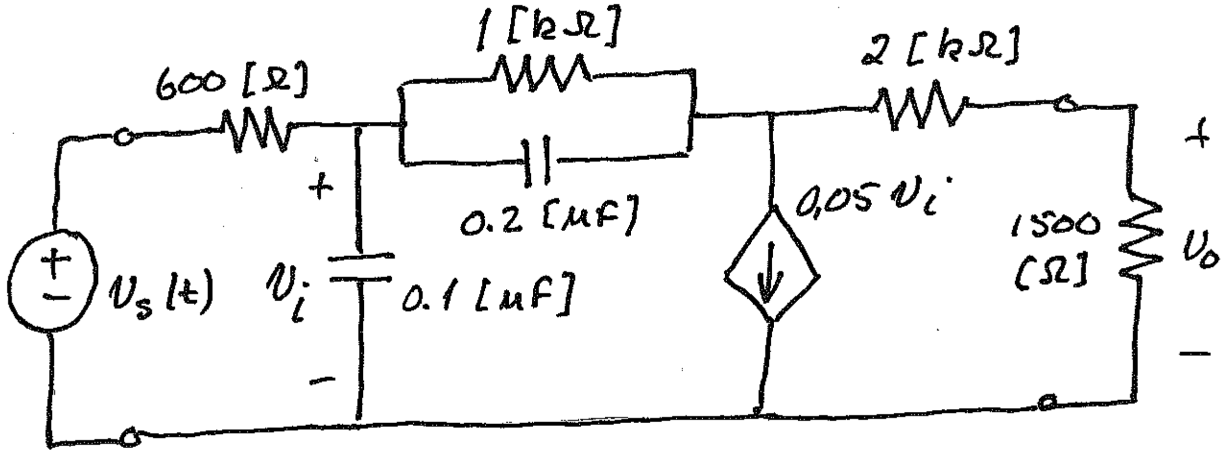
4. (<u>30 points</u>) The switches S1 and S2 in the circuit below were open for a long time before t = 0. At t = 0 they closed simultaneously. Then, switch S3 moved from position "a" to position "b" at t = 0.5 [ms]. Find the current i_x at t = 0.6 [ms].

You can maximize partial credit for this problem by clearly indicating what you are trying to find in each step of the problem.



Room for extra work

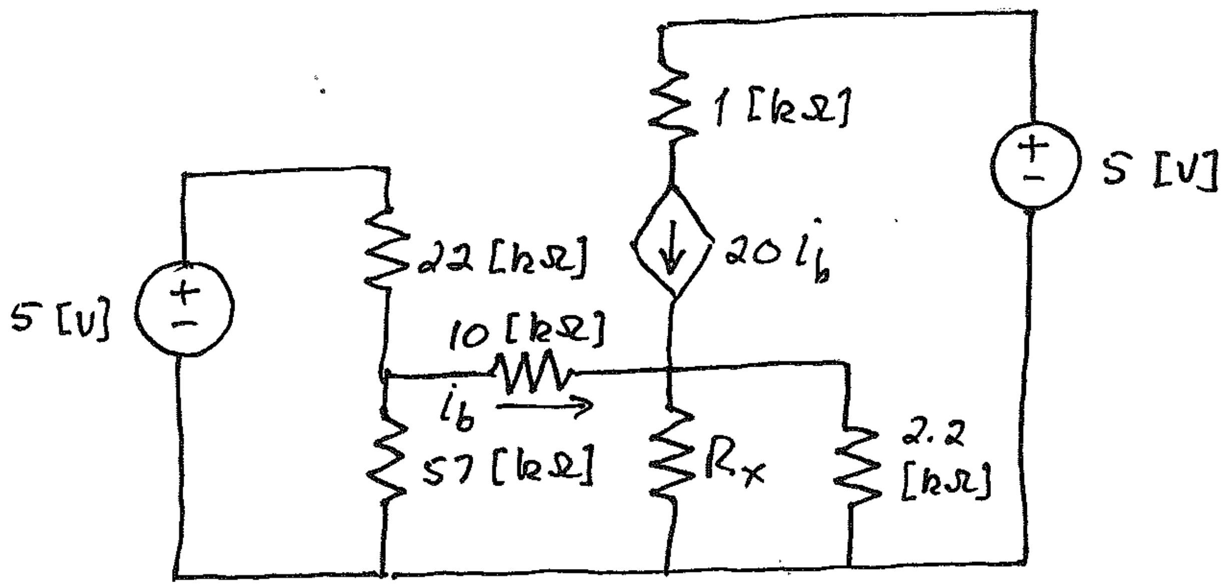
5. (15 points) The input $v_s(t)$ to the circuit shown below is given in the figure. Note that it has an ac and a dc component. Find $v_o(t)$.



$v_s(t) = 1{,}5 \sin(2000t) + 10.0$ [mV]

Room for extra work

6. (20 points) In the circuit below, the resistance R_x was adjusted so that the power delivered to R_x is a maximum. What is the maximum power being delivered to R_x?



Room for extra work

Signature
Name (print, please)
Student No.

ELEE 2300 Circuit Analysis
Summer 2011
Final Exam

DO NOT OPEN THIS EXAM BOOKLET UNTIL INSTRUCTED TO DO SO

This exam has 14 pages including this cover page. If you are missing any pages, raise your hand. You have 180 minutes to complete the exam.

Notes

1. Be sure your name and signature appear above.
2. The exam is closed-book; you may have a single 8 ½ x 11" crib sheet and a calculator.
3. To receive full credit for a problem, you must:
   - Show all work necessary to solve the problem;
   - Write clearly and legibly;
   - Indicate clearly whether power being calculated is absorbed or delivered;
   - Define all variables and parameters and label them on circuit diagrams;
   - Show all units explicitly in intermediate steps as well as final solutions;
   - Use the proper notation for all variables.

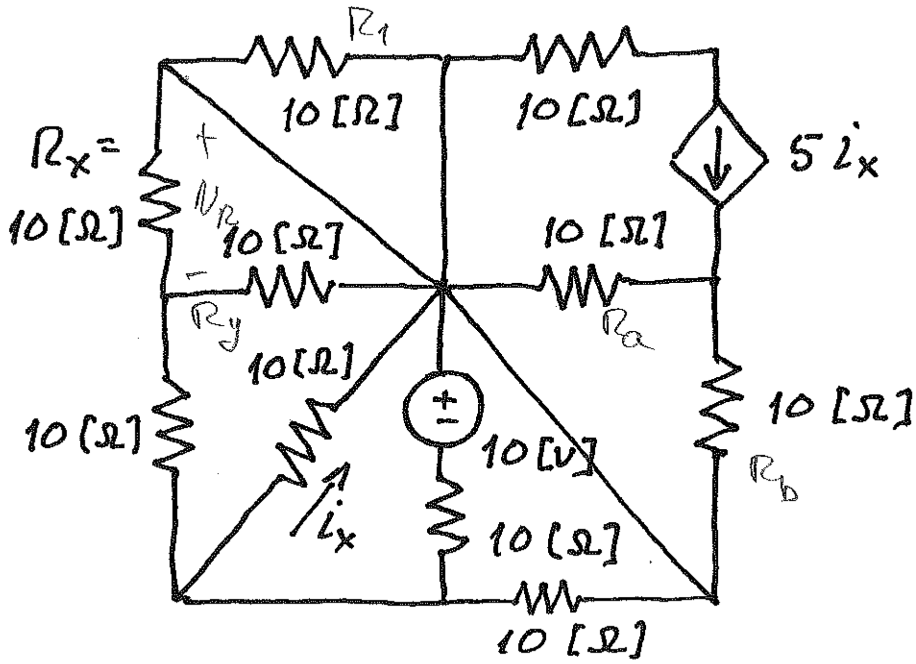
1.	______/14	4.	______/30
2.	______/18	5.	______/15
3.	______/18	6.	______/20

Total ______/115

A perfect score on the exam will result in 115 points, so there is an opportunity for extra credit.

1. (14 points) In the circuit below, find the following.

i) The power delivered to the circuit by each source.
ii) The power dissipated by resistor R_x.

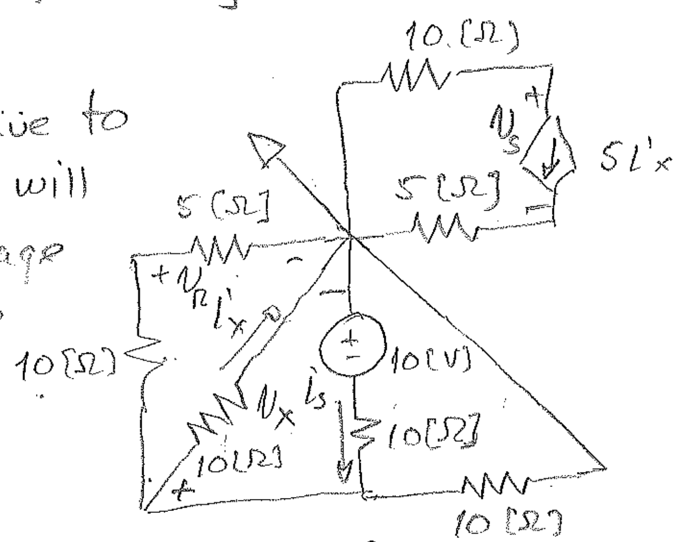


We could of course tackle this with node voltage or mesh current, but there are some simplifications to make first:

R_1 is in parallel with a short, so we remove it. Also, $R_x // R_y = 5\,[\Omega]$ and $R_a // R_b = 5\,[\Omega]$.

This leaves the following:

We could continue to simplify, but we will go with node voltage from here since we have only two essential nodes.



Room for extra work

NVM: $\frac{v_x}{15} + \frac{v_x}{10} + \frac{v_x+10}{10} + \frac{v_x}{10} = 0 \Rightarrow v_x = -2.727$ [V]

+3 Now v_R is the voltage across R_x: $v_R = v_x \frac{5}{15} = -0.9091$ [V]

+3 Also, $i_x = \frac{v_x}{10} = -0.2727$ [A] $i_s = -\frac{v_x+10}{10} = -0.7273$ [A]

+3 Finally, $v_s = -10(5i_x) - 5(5i_x) = 20.45$ [V]

So $P_{del\ 5i_x} = -v_s \cdot 5i_x = 27.89$ [W]

+2 $P_{del\ 10[V]} = -10 i_s = 7.273$ [W]

+2 $P_{diss,R_x} = v_R^2/10 = 82.65$ [mW]

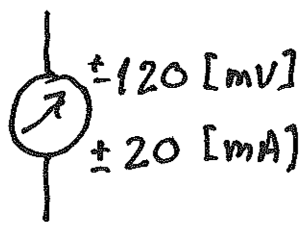
+1

NVEqns only: +7

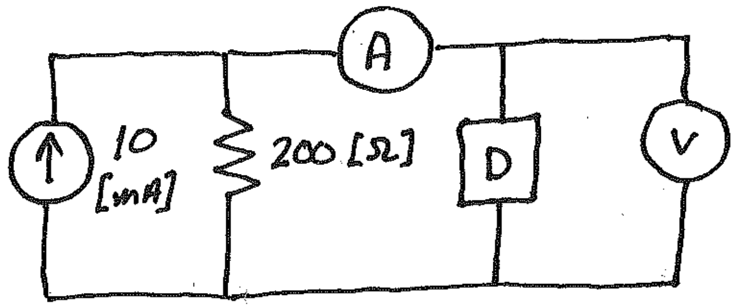
2. (<u>18 points</u>) The d'Arsonval meter movement shown below on the left is used to construct an ammeter and a voltmeter that can measure both positive and negative values. These meters are then connected into the circuit on the right in order to measure the resistance of the device D by dividing the voltmeter reading by the ammeter reading. The ammeter is set to a full scale reading of 30 [mA]. The voltmeter can be set to full scale values of 1, 2, 5, 10, 20, 50, 100, 200, 500, ...(etc.) [V].

It is known that the device D has a resistance of -75.00 [Ω], but when the meters are used to measure this value, the result is -81.08 [Ω].

i) What is the reading on the ammeter?
ii) What is the full scale voltage setting of the voltmeter? (Note that the possible settings are given in the statement of the problem above.)
iii) What is the smallest full scale voltage setting of the voltmeter that would reduce the error in the measurement to 5% or less?



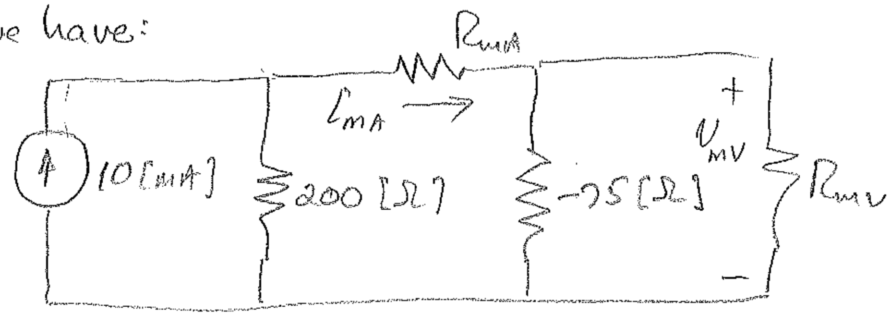
d'ABonval meter movement



The ammeter resistance is + 1

$$R_{mA} = \frac{120 \text{ [mV]}}{30 \text{ [mA]}} = 4 \text{ [Ω]}$$ + 4

So we have:



We are given that

$$\frac{v_{MV}}{i_{mA}} = -81.08 \text{ [Ω]}$$

which means that

$$R_{MV} \parallel (-75 \text{ [Ω]}) = -81.08 \text{ [Ω]}$$

$$\Rightarrow R_{MV} = 1000 \text{ [Ω]}$$

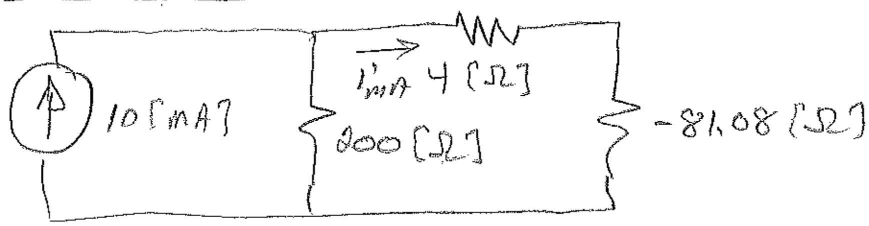
+ 4

Room for extra work

But $R_{mv} = \frac{V_{fs}}{i_{d'A\,fs}} \Rightarrow V_{fs} = -1000\,(0.02) = -20$ [V]

ii) So the full scale voltmeter voltage is -20 [V]. We can

redraw as



+ 3

So the ammeter current is

i) $$i'_{mA} = 0.01\,\frac{200}{200+4-81.08} = 16.193 \text{ [mA]}$$

iii) We require that $\frac{R_{mv}\,//\,(-75) - (-75)}{-75} = 0.05$ + 6

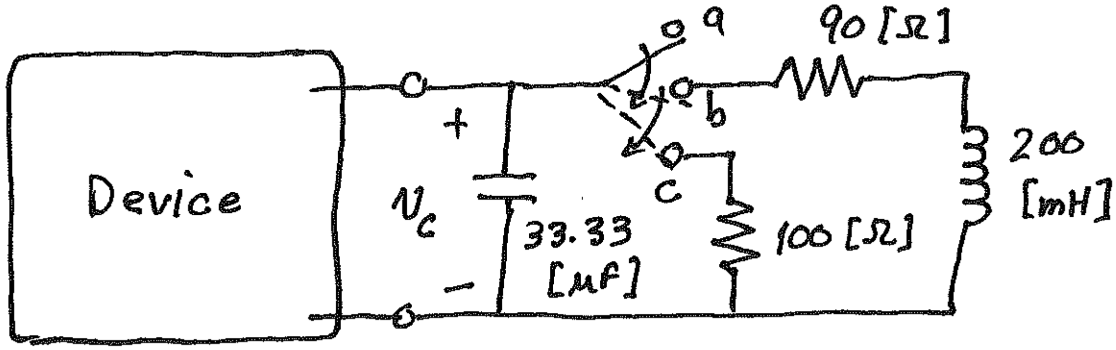
Since $R_{mv}//(-75)$ is what we will measure. Solving this equation for R_{mv} gives

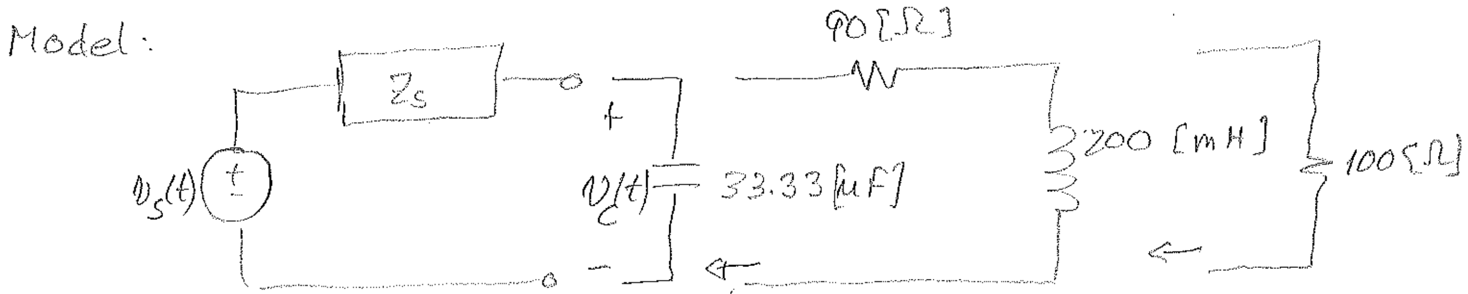
$R_{mv}//(-75) = -78.75$ [Ω] $\Rightarrow R_{mv} = 1575$ [Ω]

$\Rightarrow V_{fs} = -31.5$ [V]

So we go to -50 [V] full-scale as the smallest full-scale value that will give no more than 5% error.

3. (18 points) The device inside the box can be modeled as a sinusoidal voltage source $v_s(t)$ in series with an impedance Z_S. The switch was in position "a" for a long time, and the steady-state voltage v_C was $v_C(t) = 40.2 \cos(300t - 35^o)$ [V]. The switch then moved to position "b" at t = 0, and after a long time, the steady-state value voltage v_C was $v_C(t) = 21.59 \cos(300t - 74.72^o)$ [V]. If the switch is then moved to position "c" at $t = t_0$, what is the steady-state voltage $v_C(t)$?





With just the capacitor connected and in steady state,

$$\bar{V}_c = 40.2 \angle -35^o = \bar{V}_S \frac{Z_C}{Z_C + Z_S}$$ +3

With the resistor and inductor connected, we have:

$$\frac{\bar{V}_c - \bar{V}_S}{Z_S} + \frac{\bar{V}_c}{-j100} + \frac{\bar{V}_c}{90 + j60} = 0$$ $Z_C = -j100\ [\Omega]$, $Z_L = j60\ [\Omega]$ +4

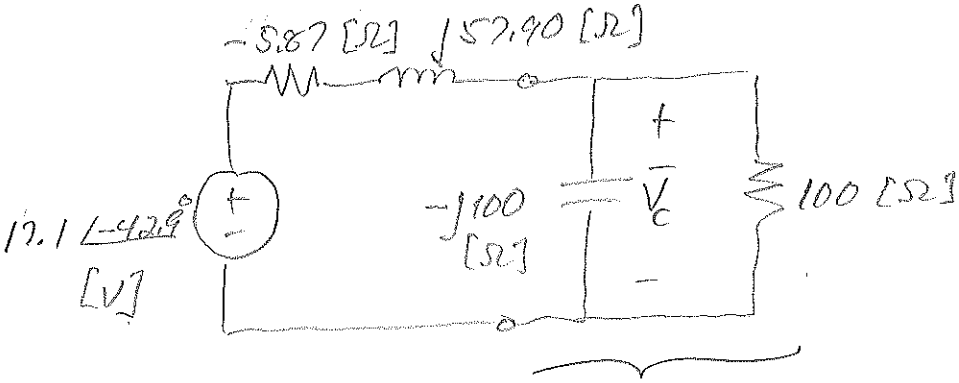
$$\bar{V}_c = 21.59 \angle -74.72^o\ [V]$$

$\Rightarrow \bar{V}_S = 17.09 \angle -42.94^o\ [V]$ $\quad Z_S = 58.20 \angle 95.79^o\ [\Omega]$ +2

$= 12.51 - j11.64\ [V]$ $\quad = -5.87 + j57.90\ [\Omega]$

Room for extra work

Now with resistor and capacitor we have



$$Z_L = -j100 \,//\, 100 = 70.71 \angle -45^\circ \ [\Omega]$$
$$= -50 - j50 \ [\Omega]$$

+5

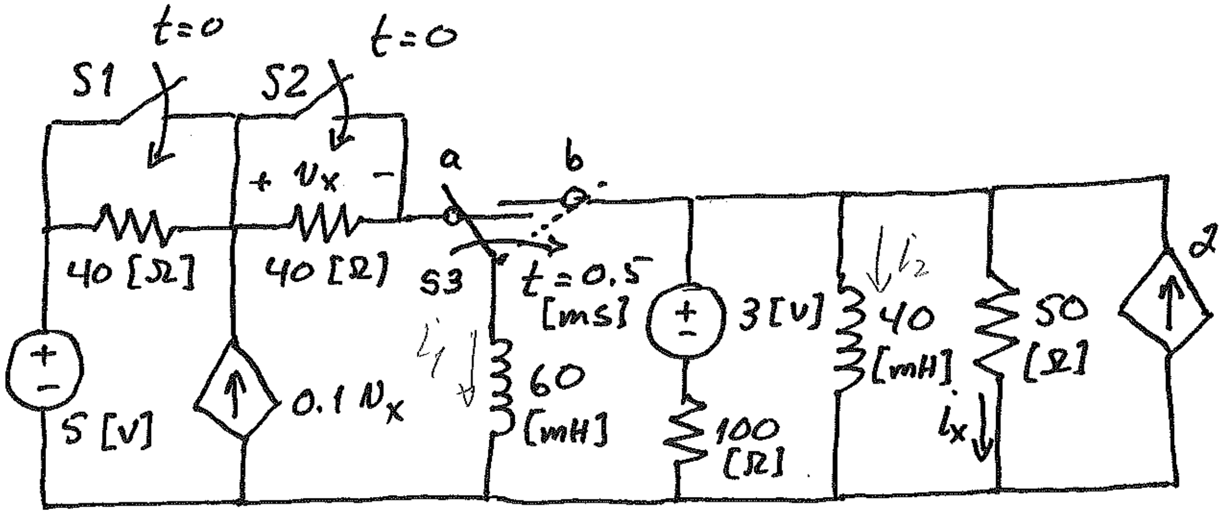
$$\bar{V}_c = 17.09 \angle -42.91^\circ \ \frac{Z_L}{Z_s + Z_L} = 26.96 \angle -98.09^\circ \ [V]$$
$$= -3.795 + j26.69 \ [V]$$

+4 (transforms both ways)

$$\therefore \quad v_c(t) = 26.96 \cos(300t - 98.09^\circ) \ [V]$$

4. (<u>30 points</u>) The switches S1 and S2 in the circuit below were open for a long time before t = 0. At t = 0 they closed simultaneously. Then, switch S3 moved from position "a" to position "b" at t = 0.5 [ms]. Find the current i_x at t = 0.6 [ms].

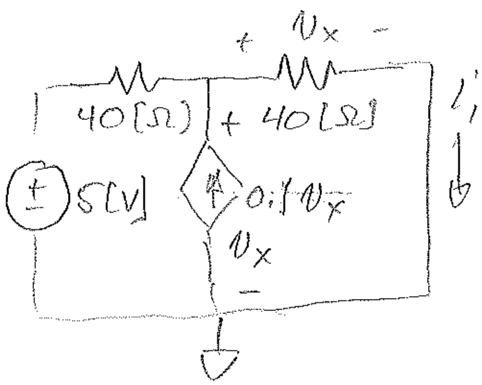
You can maximize partial credit for this problem by clearly indicating what you are trying to find in each step of the problem.

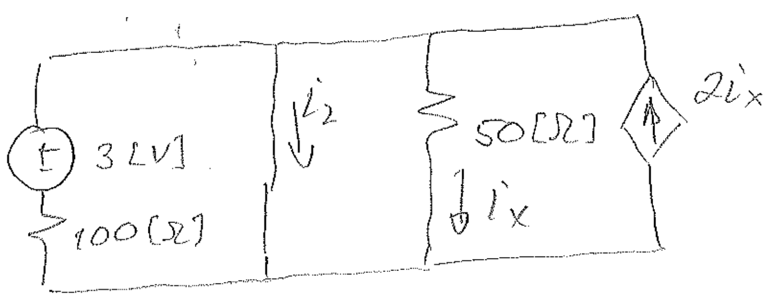


We have initial conditions on both inductors:

t<0

INITIAL CURRENTS





$$\frac{v_x}{40} - 0.1 v_x + \frac{v_x - 5}{40} = 0$$

$v_x = -2.5$ [V]

$i_1(0^-) = i_1(0^+) = -0.0625$ [A]

$i_2(0^-) = i_2(0^+) = 0.03$ [A]

This remains the same during $0 \le t \le 0.5$ [ms]

$\Rightarrow i_2(0.5 \text{ [ms]}^+) = 0.03$ [A]

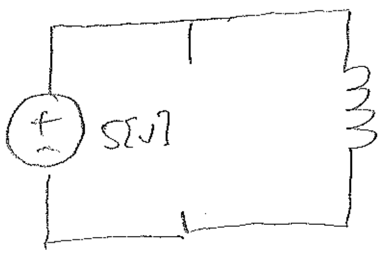
+3 +2

Room for extra work

FIRST SWITCHING EVENT

$0 < t < 0.5$ [ms]

Both 40 [Ω] resistors are shorted, and $v_x = 0$, so we have:



$$i_1 = \frac{1}{0.06}\int_0^{0.5\times10^{-3}} 5\,dt - 0.0625$$

$$= 83.33 \times t\Big|_0^{0.0005} - 0.0625$$

0.04167

$$= -0.02083 \text{ [A]} = i_1(0.5[ms]^+)$$

+ 6

$t > 0.5$ [ms] we now have two inductors in parallel: $L_{eq} = \left(\frac{1}{0.06} + \frac{1}{0.04}\right)^{-1} = 24$ [mH]

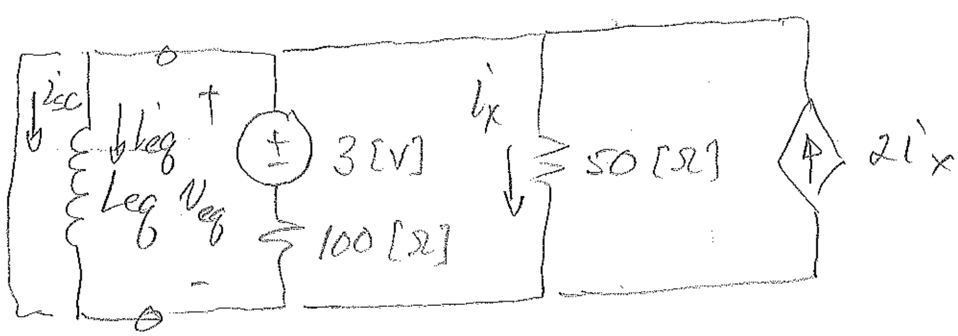
+ 4

$$i_{eq}(0.5[ms]^+) = i_1(0.5[ms]^+) + i_2(0.5[ms]^+)$$

$$= -0.02083 + 0.03 = 0.00917 \text{ [A]}$$

SECOND SWITCHING EVENT

$L_{eq} =$ 24 [mH]



We need the Thevenin resistance and final (steady-state) current at the inductor terminals.

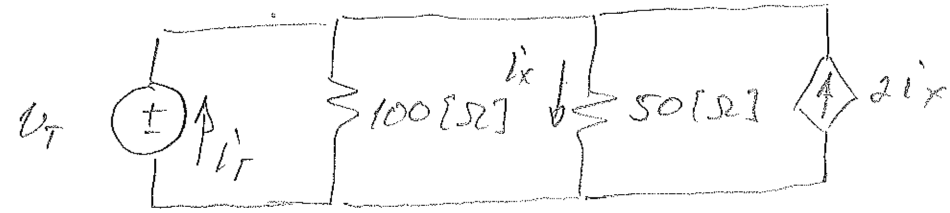
Replacing L_{eq} with a short to find the steady-state value...

... the 50 [Ω] resistor is shorted out so $i_x = 0$.

Then $i'_{sc} = \frac{3}{100} = 0.03$ [A] (same as $i'_2(0^-)$)

+3 $= i'_{eq,f}$

Thevenin resistance:



$$i'_T = \frac{v_T}{100} + \frac{v_T}{50} - 2\frac{v_T}{50} = -\frac{1}{100}$$

+5 $\Rightarrow R_{Th} = \frac{v_T}{i'_T} = -100$ [Ω] $\qquad \tau_L = \frac{L_{eq}}{R_{Th}} = -2.4\times10^{-4}$ [s]

So

+4 $$i'_{eq}(t) = i'_{eq,f} + (i'_{eq}(0.5[ms]^+) - i'_{eq,f})\, e^{\frac{(t-5\times10^{-4})}{2.4\times10^{-4}}} \text{ [A]} \quad t \geq 5\times10^{-4} \text{ [s]}$$

$$= 0.03 + (0.00917 - 0.03)\, e^{\frac{t-5\times10^{-4}}{2.4\times10^{-4}}} \text{ [A]}$$

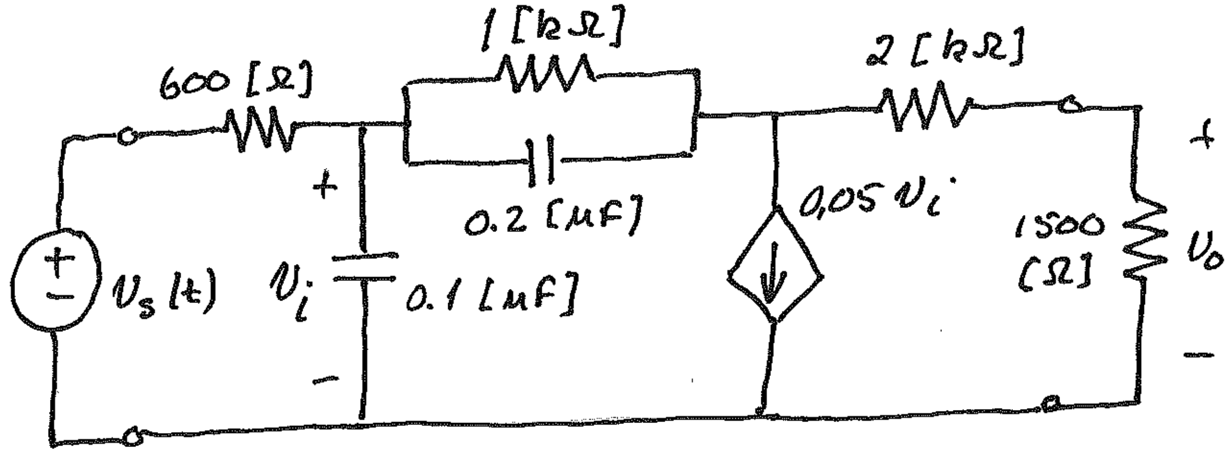
$$v_{eq}(t) = L_{eq}\frac{di'_{eq}}{dt} = 24\times10^{-3}\,\frac{(-0.0283)}{2.4\times10^{-4}}\, e^{\frac{t-5\times10^{-4}}{2.4\times10^{-4}}} \text{ [V]} \quad t > 5\times10^{-4} \text{ [s]}$$

$$i_x(t = 0.6[ms]) = \frac{v_{eq}(t = 0.6\,[ms])}{50}$$

$$= -0.0566\, e^{\frac{1\times10^{-4}}{2.4\times10^{-4}}} \text{ [A]}$$

+3 $$= -0.08586 \text{ [mA]}$$

5. (15 points) The input $v_s(t)$ to the circuit shown below is given in the figure. Note that it has an ac and a dc component. Find $v_o(t)$.



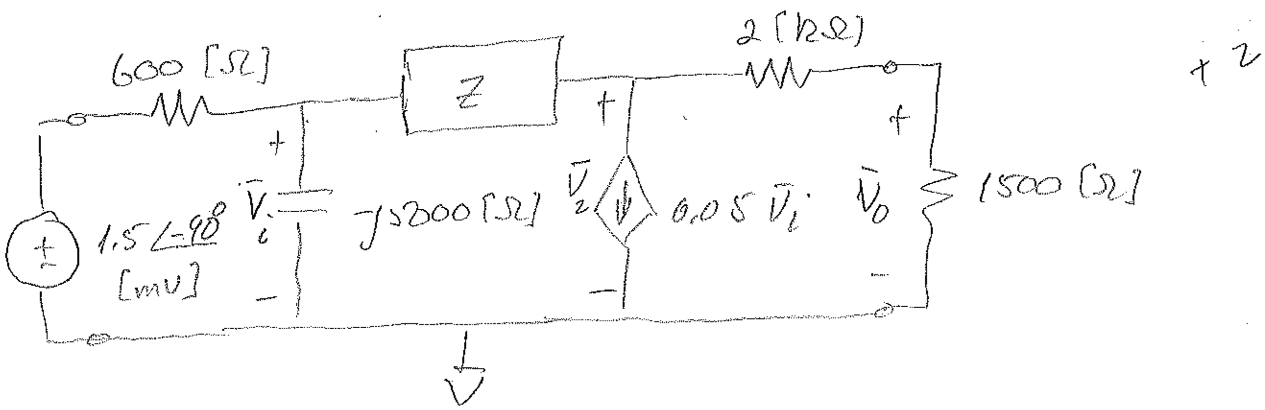
$v_s(t) = 1.5 \sin(2000t) + 10.0$ [mV]

We need superposition here... we'll do the ac portion first. Transforming to phasor domain:

0.1 [μF] $\rightarrow \frac{1}{j(2000)(10^{-7})} = -j\,5000$ [Ω]

0.2 [μF] $\rightarrow -j\,2500$ [Ω]

$Z \equiv 1000 \,//\, (-j2500) = \frac{1000(-j2500)}{1000 - j2500} = 928.5 \angle -21.80°$ [Ω] $= 862.1 - j\,344.8$ [Ω]



Room for extra work

$$0.05\bar{V}_i + \frac{\bar{V}_2}{3500} + \frac{\bar{V}_2 - \bar{V}_i}{Z} = 0$$

$$\frac{\bar{V}_i - 0.0015\angle -90^\circ}{600} + \frac{\bar{V}_i - \bar{V}_2}{Z} + \frac{\bar{V}_i}{-j5000} = 0$$

+3

$\bar{V}_i = 6.072\times10^{-5}\angle -94.93^\circ$ [V] $= -5.222\times10^{-6} - j\,6.0500\times10^{-5}$ [V] +2

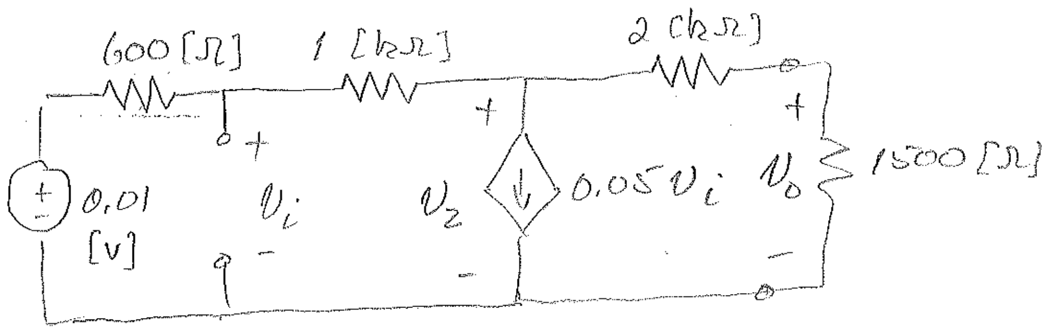
$\bar{V}_2 = 2.168\times10^{-3}\angle 67.65^\circ$ [V] $= 8.246\times10^{-4} + j\,2.006\times10^{-3}$ [V]

$\bar{V}_o = \bar{V}_2 \cdot \frac{1500}{3500} = 9.2914\times10^{-4}\angle 67.65^\circ$ [V]

$= 3.533\times10^{-4} + j\,8.593\times10^{-4}$ [V] +2

$\therefore v_{o\,ac}(t) = 0.9291\cos(2000t + 67.65^\circ)$ [mV]

dc component: capacitors are open ($\omega = 0 \Rightarrow Z_c = \infty$)



+2

$$0.05 v_i + \frac{v_2 - v_i}{1000} + \frac{v_2}{3500} = 0$$

$$\frac{v_i - 0.01}{600} + \frac{v_i - v_2}{1000} = 0$$

$v_i = 0.409$ [mV] +2

$v_2 = -15.58$ [mV]

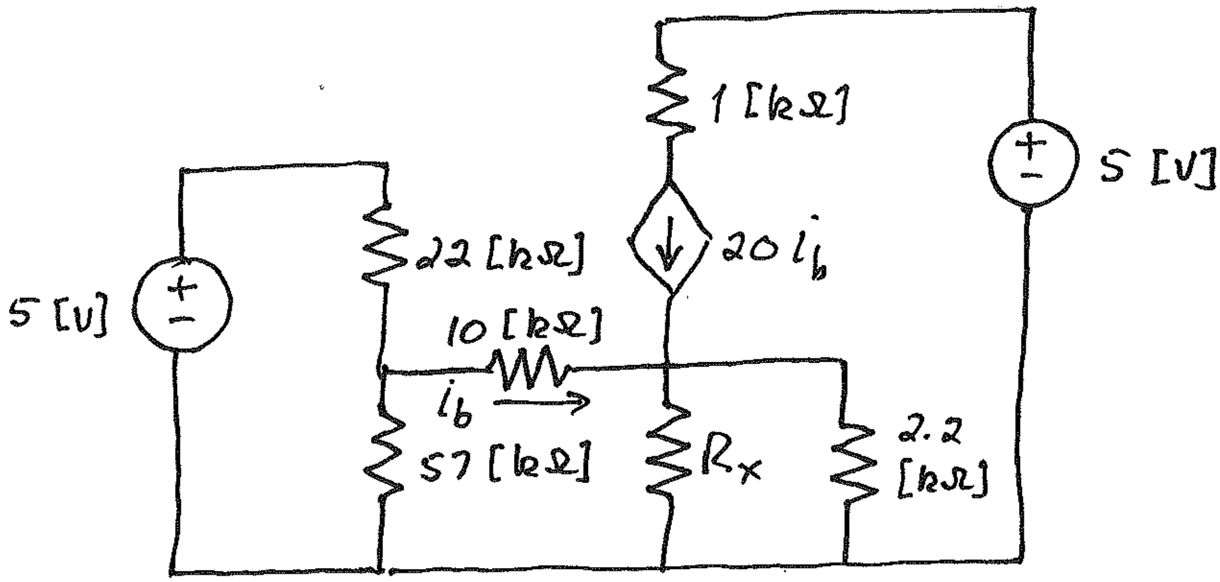
$\therefore v_{o\,dc} = v_2 \cdot \frac{1500}{3500}$

$= -6.677$ [mV]

$\therefore v_o(t) = 0.9304\cos(2000t + 157.6^\circ) - 6.677$ [mV]

+2

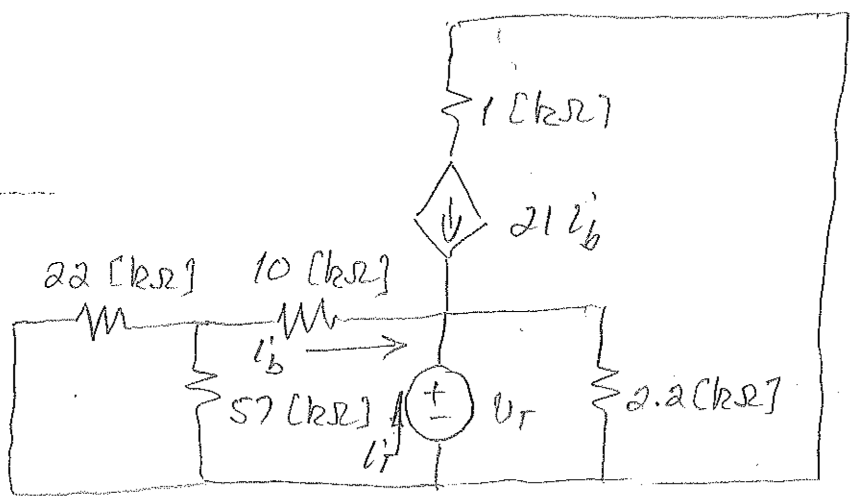
6. (<u>20 points</u>) In the circuit below, the resistance R_x was adjusted so that the power delivered to R_x is a maximum. What is the maximum power being delivered to R_x?



We need the Thevenin equivalent of the circuit as seen by R_x.

Test Source:

$10 + 22 // 57 = 25.87 [k\Omega]$



$$i_T' = -21\, i_b' + \frac{v_T}{2200} \qquad i_b' = \frac{-v_T}{10k + 22k // 57k} = -3.865 \times 10^{-5}\, v_T$$

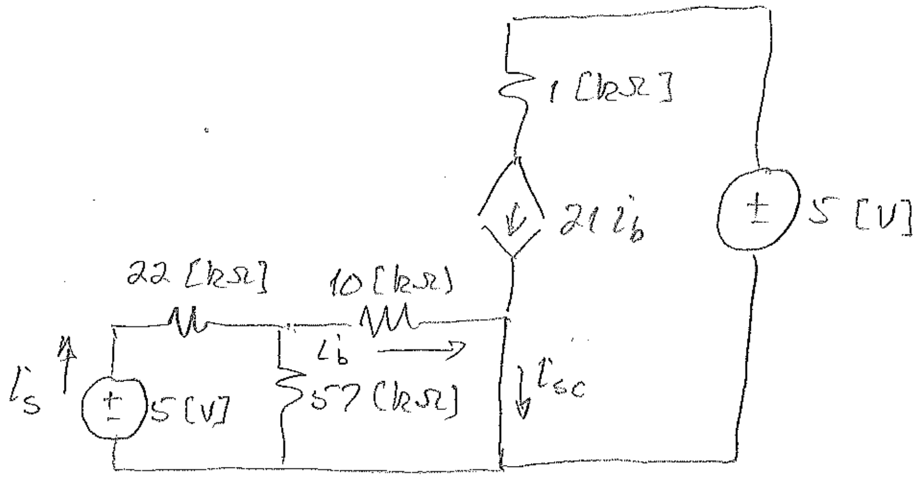
$$= 1.2662 \times 10^{-3}\ [A]$$

$$\therefore R_{Th} = 789.8\ [\Omega]$$

+8

Room for extra work

Short circuit current:



$i'_{sc} = 21\, i'_b$ $\quad$ $i'_s = \dfrac{5}{22k + 10k // 57k} = 0.1639$ [mA]

+8 $\quad$ $i'_b = i'_s \cdot \dfrac{57k}{57k + 10k} = 0.1394$ [mA]

$i'_{sc} = 21\, i'_b = 2.9281$ [mA] $\Rightarrow$ $V_{oc} = V_{Th} = i'_{sc} \cdot R_{Th} = 2.316$ [V]

+4 $\quad$ $\Rightarrow$ $P_{del\ to\ R_x} = \dfrac{(2.316)^2}{4 \cdot (789.8)} = 1.693$ [mW]

Include R_x : -3