Signature
Name (print, please)

ELEE 2300 Circuit Analysis
Summer 2011
Mid Term Exam

DO NOT OPEN THIS EXAM BOOKLET UNTIL INSTRUCTED TO DO SO

This exam has 9 pages including this cover page. If you are missing any pages, raise your hand. You have 90 minutes to complete the exam.

Notes

1. Be sure your name and signature appear above.
2. The exam is open book and open notes.
3. To receive full credit for a problem, you must:

- Show all work necessary to solve the problem;
- Write clearly and legibly;
- Indicate clearly whether power being calculated is absorbed or delivered;
- Define all variables and parameters and label them on circuit diagrams;
- Show all units explicitly in intermediate steps as well as final solutions;
- Use the proper notation for all variables.

1. ____________/22

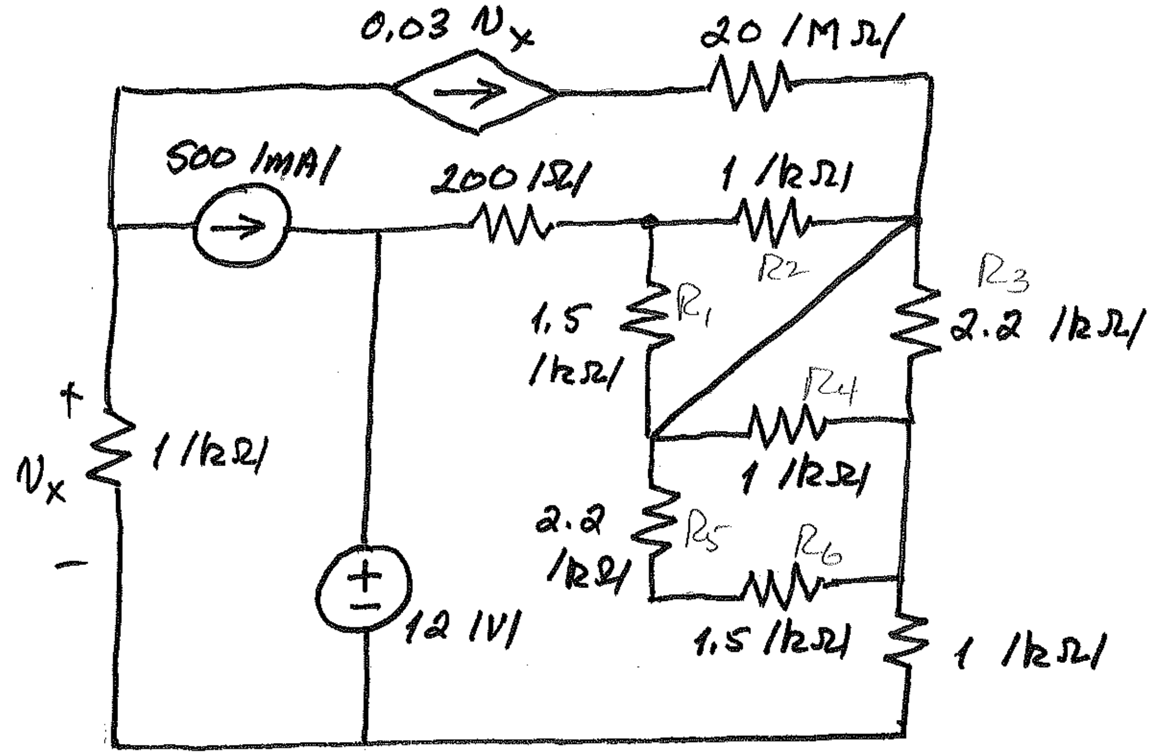
2. ____________/28

3. ____________/25

4. ____________/25

Total____________/100

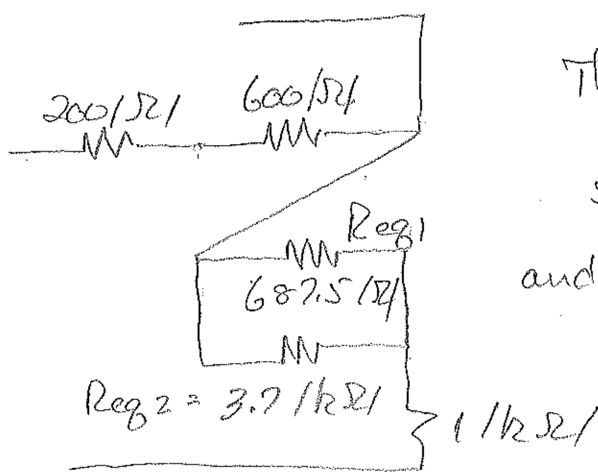
1. (22 points) In the circuit below, find the power delivered to the circuit by the dependent current source.



We could attack this problem directly with the node voltage or mesh current method, but it will be easier to reduce the resistor network first.

$R_1 \parallel R_2 = 600\ |\Omega|$ $\quad R_3 \parallel R_4 = 687.5\ |\Omega|$ $\quad R_5 + R_6 = 3.7\ |k\Omega|$

After those reductions we have:



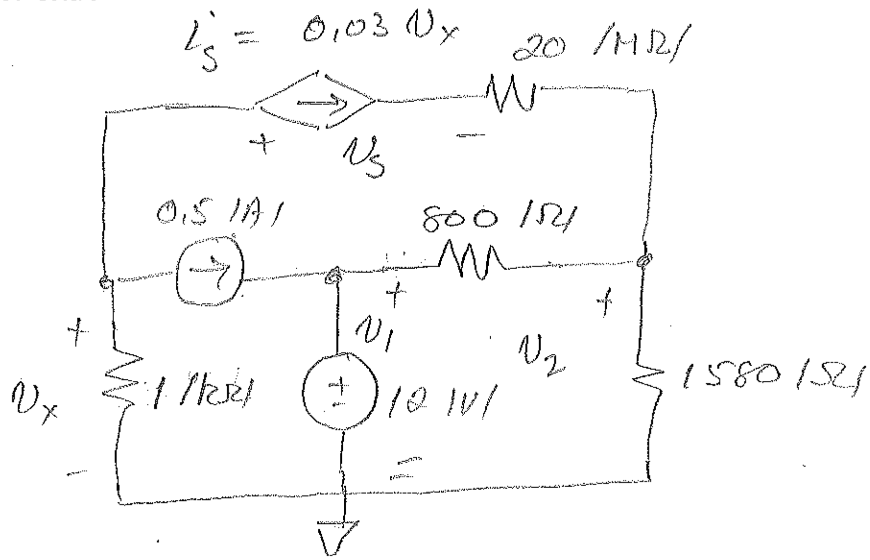
Then $R_{eq1} \parallel R_{eq2} \cong 580\ |\Omega|$

$580 + 1000 = 1580\ |\Omega|$

and $200 + 600 = 800\ |\Omega|$

... which leaves...

Room for extra work



$v_1 = 12$ [V]

$\frac{v_x}{1000} + 0.5 + 0.03\,v_x = 0 \Rightarrow v_x = -16.13$ [V]

$\frac{v_2}{1580} + \frac{v_2 - 12}{800} - 0.03(-16.13) = 0$

$\Rightarrow v_2 = -249.03$ [V]

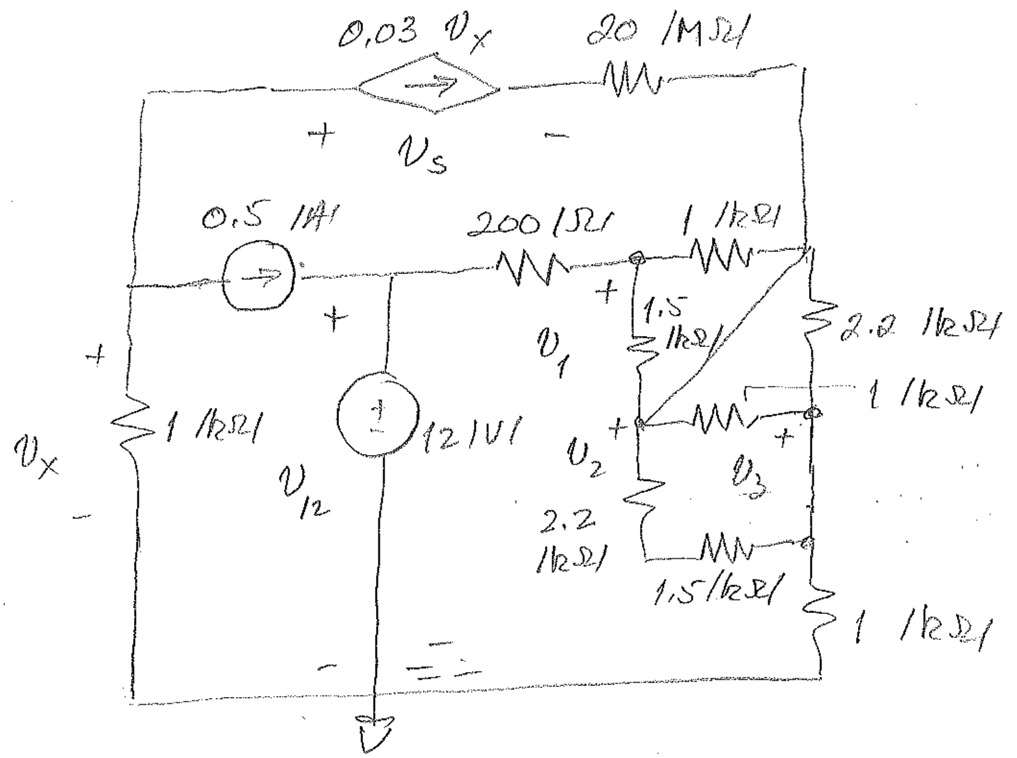
$v_s + 0.03\,v_x(20\times10^6) + v_2 - v_x = 0$

$v_s = v_x - v_2 + 9.678\times10^6$

$= 9.67823\times10^6$ [V]

$P_{del,\,i_s} = -(0.03\,v_x)\cdot v_s = 4.6833$ [MW]

Using NVM directly :



$$\frac{v_x}{1000} + 0.5 + 0.03 v_x = 0 \quad \Rightarrow \quad v_x = -16.13 \text{ V}$$

$$v_{12} = 12 \text{ V}$$

$$\frac{v_1 - 12}{200} + \frac{v_1 - v_2}{1500} + \frac{v_1 - v_2}{1000} = 0$$

$$\frac{v_2 - v_3}{3700} + \frac{v_2 - v_1}{1500} + \frac{v_2 - v_3}{1000} + \frac{v_2 - v_1}{1000} + \frac{v_2 - v_3}{2200} - 0.03 v_x = 0$$

$$\frac{v_3 - v_2}{1000} + \frac{v_3 - v_2}{2200} + \frac{v_3 - v_2}{3700} + \frac{v_3}{1000} = 0$$

$v_1 = -53.25$ V

$v_2 = -249.02$ V

$v_3 = -157.63$ V

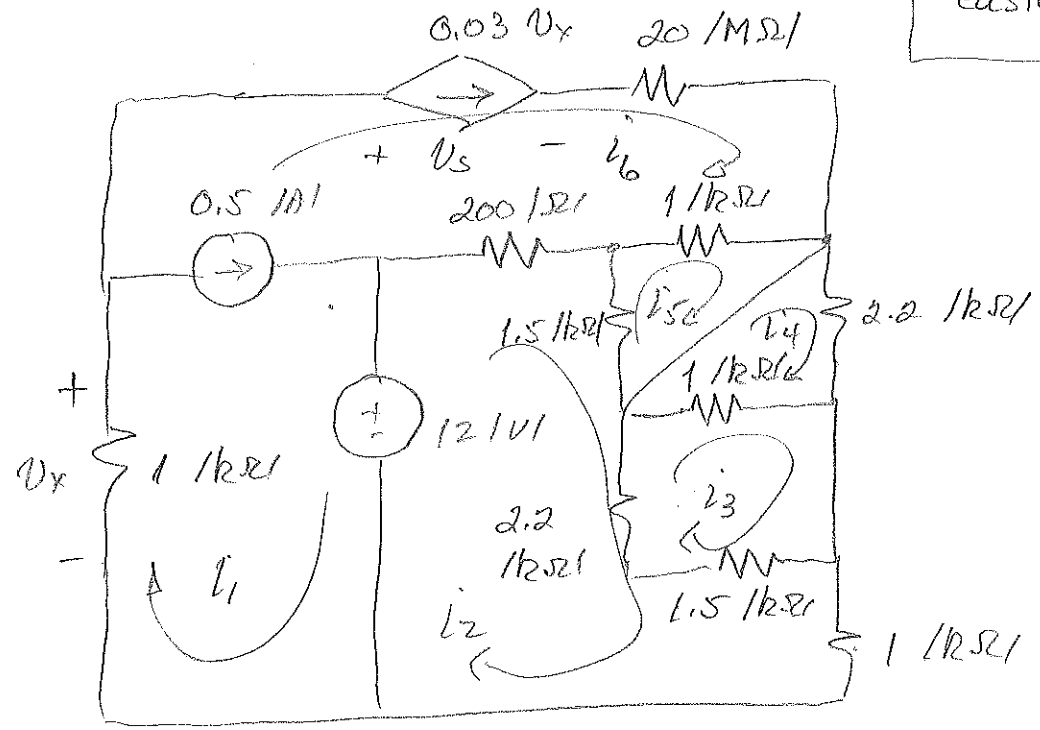
$$-v_x + v_s + v_2 + 0.03 v_x (20 \times 10^6) = 0$$

$$\Rightarrow v_s = 9.67823 \times 10^6 \text{ V}$$

same as before !

MCM Directly.

Oh, yeah, this is easier. NOT!



① $i_6 = 0.03\, v_x$ ② $v_x = -1000\, i_1$ ③ $i_1 - i_6 = 0.5$

④ $200(i_2 - i_6) + 1500(i_2 - i_5) + 2200(i_2 - i_3) + 1500(i_2 - i_3) + 1000\, i_2 - 12 = 0$

⑤ $1500\,(i_5 - i_2) + 1000\,(i_5 - i_6) = 0$

⑥ $2200\, i_4 + 1000(i_4 - i_3) = 0$

⑦ $1000\,(i_3 - i_4) + 1500(i_3 - i_2) + 2200\,(i_3 - i_2) = 0$

$i_1 = 0.01613$ /A/ $i_4 = -0.0415$ /A/ $v_x = -16.13$ /V/

$i_2 = -0.1576$ /A/ $i_5 = -0.2881$ /A/

$i_3 = -0.1329$ /A/ $i_6 = -0.4839$ /A/

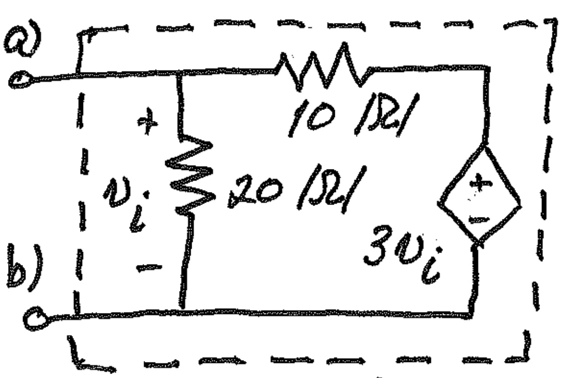
$v_s + 0.03\, v_x\,(20 \times 10^6) + 1000\,(i_6 - i_5) + 200\,(i_6 - i_2) + 12 - v_x = 0$

$\Rightarrow v_s = 9.678 \times 10^6$ /V/ ✓

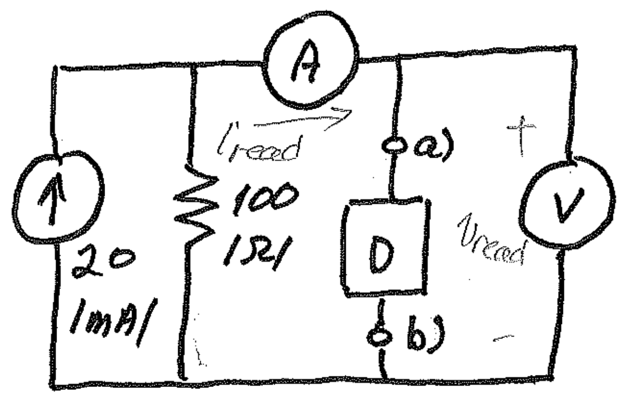
2. (<u>28 points</u>) The voltmeter and ammeter in the circuit below are constructed from d'Arsonval meter movements rated at +/- 100 /mV/ and +/- 20 /mA/. The designation "+/-" means that these meter movements can read both positive and negative values. At full positive deflection they have + 20 /mA/ going through them and +100 /mV/ across them, and at full negative deflection they have – 20 /mA/ going through them and – 100 /mV/ across them.

The voltmeter is constructed from a d'Arsonval movement like the one described above, and has a full scale reading of +/- 20 /V/. The ammeter is constructed from another d'Arsonval meter like the one described above, and has a full scale reading of +/- 50 /mA/.

The ammeter and voltmeter are used to measure the equivalent resistance of the device D. This is done by inserting D into the circuit at the terminals a) and b) as indicated, and reading the voltage on the voltmeter and the current on the ammeter. Find the equivalent resistance of D determined using the readings on the voltmeter and the ammeter.



device D



$R_D = v_{read}/i_{read}$

There are a couple of ways to approach this. One is to find the meter resistances and insert them along with "D" into the circuit. We'll do that first.

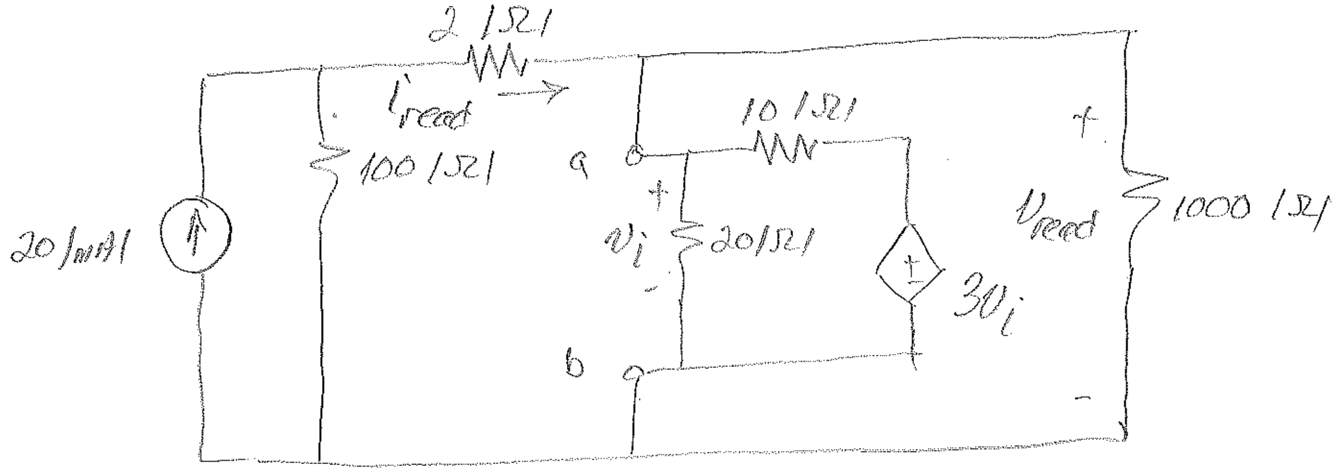
$$R_{m,A} = \frac{v_{d'A,fs}}{i_{fs}} = \frac{100\ /mV/}{50\ /mA/} = 2\ /\Omega/$$

$$R_{m,V} = \frac{v_{fs}}{i_{d'A,fs}} = \frac{20\ /V/}{20\ /mA/} = 1000\ /\Omega/$$

Note that these resistances are the same whether we choose positive or negative full-scale values.

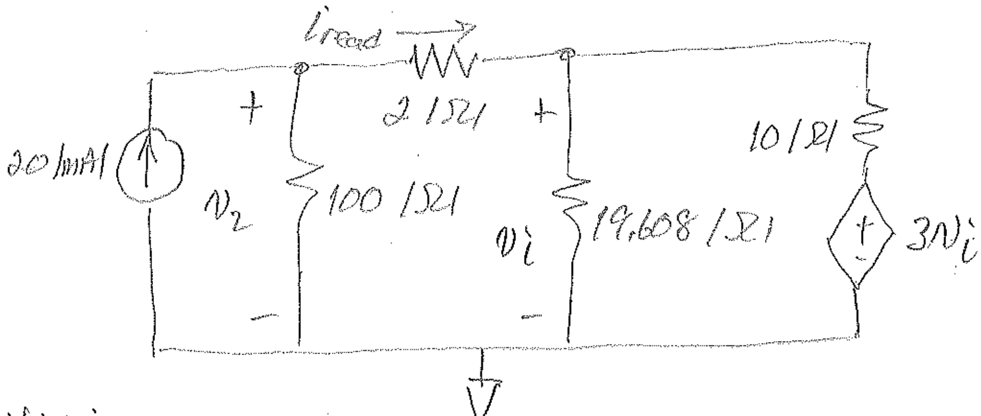
Room for extra work

The equivalent circuit is then:



If we want the equivalent resistance of "D" we need to find $R_D = \frac{v_{read}}{i_{read}}$. These quantities are labeled on the original circuit as well.

We now re-draw the circuit, noting that $1000\ k\Omega \parallel 20\ k\Omega = 19.608\ k\Omega$.



NVM:

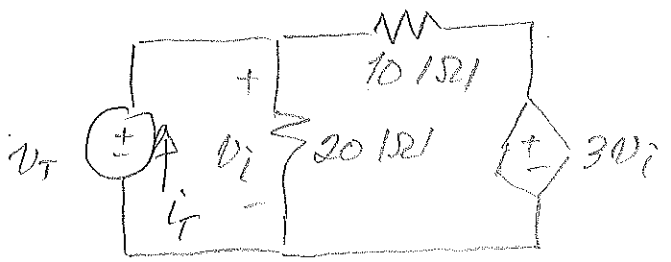
$$\left.\begin{aligned} &\frac{v_2}{100} - 0.02 + \frac{v_2 - v_i}{2} = 0 \\ &\frac{v_i}{19.608} + \frac{v_i - v_2}{2} + \frac{v_i - 3v_i}{10} = 0 \end{aligned}\right\} \Rightarrow \begin{aligned} v_i &= -0.140864\ [V] \\ v_2 &= -0.098867\ [V] \end{aligned}$$

Thus $v_{read} = v_i = -0.140864$ [V] and

$i_{read} = \frac{v_2 - v_i}{2} = 0.020988$ [A]

Thus $R_D = \frac{-0.140864}{0.020988} = -6.712$ [Ω]

Another way to do this is to find the equivalent resistance of "D" directly using a test source, and then to recognize that what is actually being measured is the resistance of "D" in parallel with R_{mv}.



$v_T = v_i$

$\frac{v_T}{20} + \frac{v_T - 3v_T}{10} = i_T$

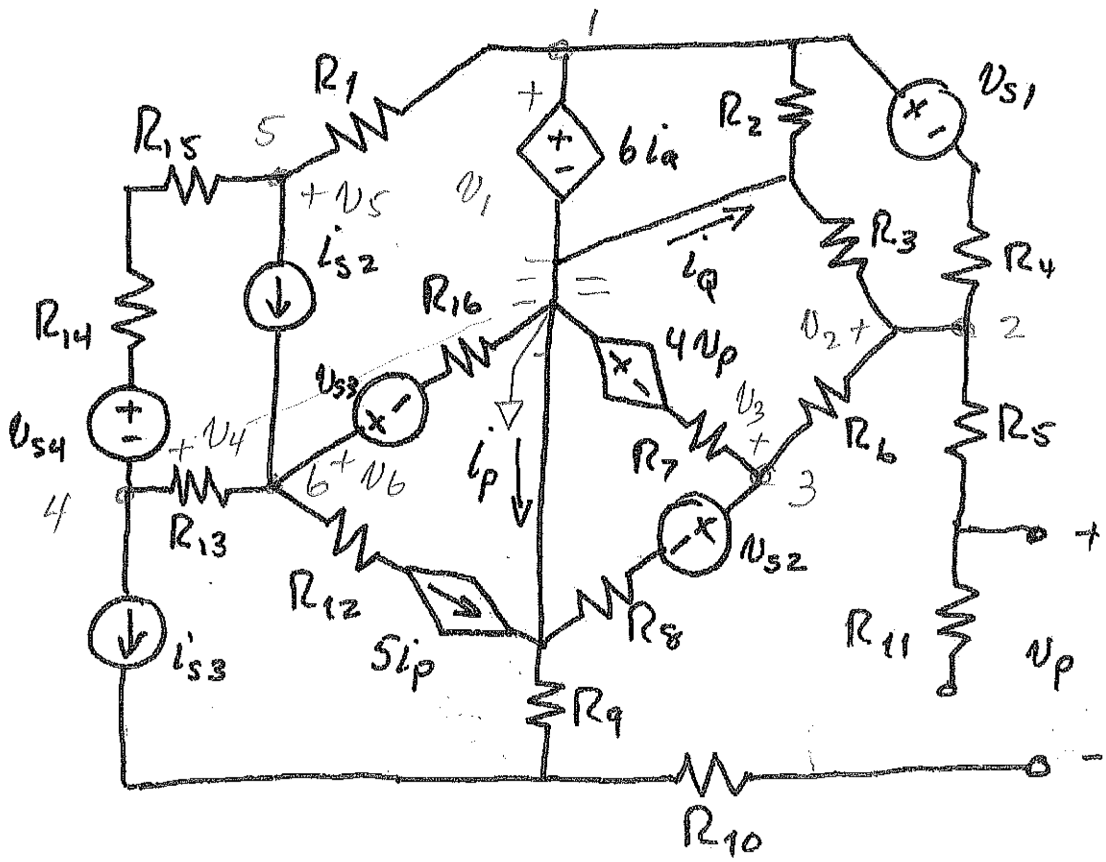
$R_{D,true} = \frac{v_T}{i_T} = -\frac{20}{3} = -6.667$ [Ω]

Now $R_{D,true} \parallel R_{mv} = \frac{(-6.667)(1000)}{1000 - 6.667} = -6.711$ [Ω]

which is the same (within round-off error) as we got above.

3. (<u>25 points</u>) Write a set of equations using the node voltage method that could be used to solve the circuit below. Do not attempt to simplify or solve the equations.

Full credit will be given only if (i) all node voltages are clearly labeled and (b) there are no more or fewer equations than necessary to solve the problem.



There are 6 essential nodes other than the reference. None are supernodes. There are 3 dependent variables so we need 9 equations.

Note that the node connecting i_{s3}, R_9, and R_{10} is not an essential node because there is no current in R_{10}.

So: 6 node voltage equations

3 auxiliary equations

As it happens, this is not correct. Essential nodes are not defined in terms of current. They are defined in terms of the number of components connected there. So, the node connecting R9 and R10 is indeed an essential node, and should have been included here as such, despite R10 having no current through it. DPS

Room for extra work

(1) $v_1 = 6 i_Q$

(2) $\frac{v_2}{R_3} + \frac{v_2 - v_3}{R_6} + \frac{v_2 - v_1 + v_{s1}}{R_4} = 0$

(3) $\frac{v_3 - v_{s2}}{R_8} + \frac{v_3 + 4v_p}{R_7} + \frac{v_3 - v_2}{R_6} = 0$

(4) $i_{s3} + \frac{v_4 - v_6}{R_{13}} + \frac{v_4 - v_5 + v_{s4}}{R_{14} + R_{15}} = 0$

(5) $\frac{v_5 - v_4 - v_{s4}}{R_{14} + R_{15}} + i_{s2} + \frac{v_5 - v_1}{R_1} = 0$

(6) $\frac{v_6 - v_4}{R_{13}} + 5 i_p - i_{s2} + \frac{v_6 - v_{s3}}{R_{16}} = 0$

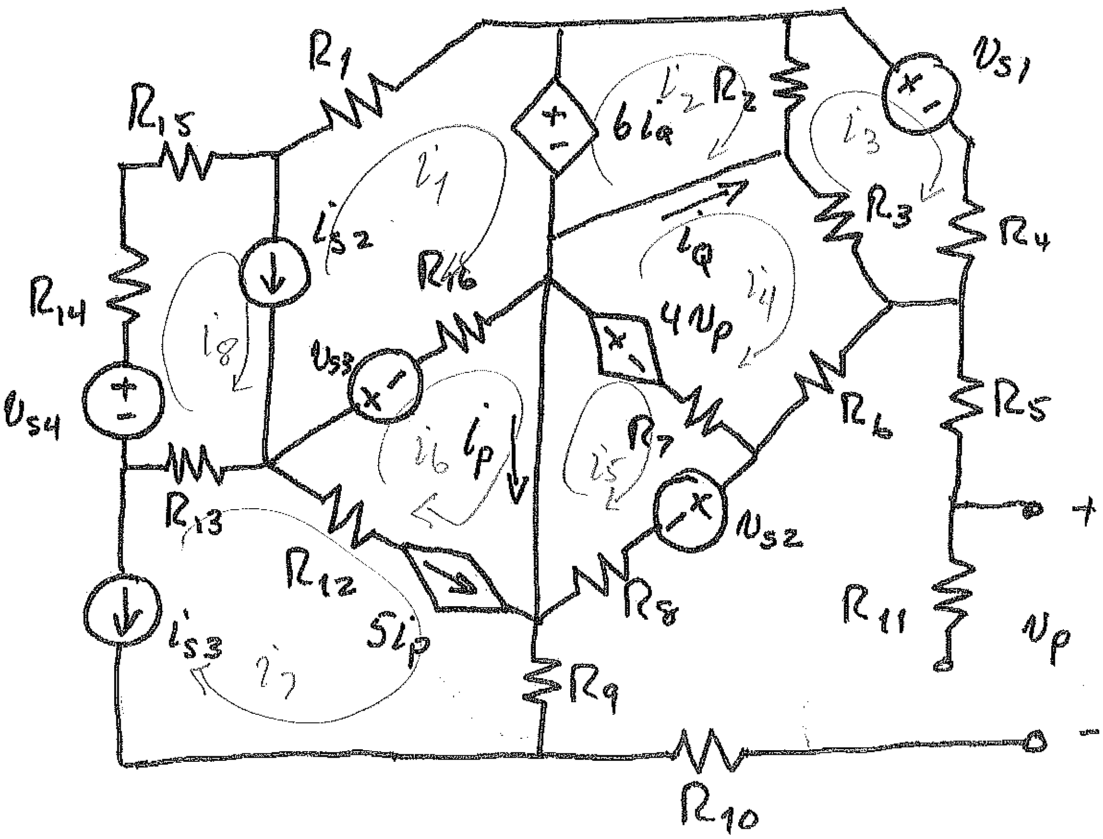
v_p : $v_p + i_{s3} R_9 - v_2 = 0$ (no current in R_5 or R_{11})

i_Q : $-i_g - \frac{6 i_Q}{R_2} - \frac{v_2}{R_3} = 0$

i_p : $-i_p - i_{s3} - 5 i_p - \frac{v_3 - v_{s2}}{R_8} = 0$

4. (28 points) Write a set of equations using the mesh current method that could be used to solve the circuit below. Do not attempt to simplify or solve the equations.

Full credit will be given only if (i) all mesh currents are clearly labeled and (b) there are no more or fewer equations than necessary to solve the problem.



There are 8 mesh currents. There are 2 supermeshes. There are 3 dependent variables. So:

6 mesh current and 2 constraint equations.

3 auxiliary equations.

Room for extra work

SM 1

(1) $i_1 R_1 + 6 i_\varphi + R_{16}(i_1 - i_6) - v_{s3} + R_{13}(i_8 - i_7) - v_{s4} + (R_{14} + R_{15}) i_8 = 0$

(2) Constraint: $i_8 - i_1 = i_{s2}$

SM 2 : none required because :

(3) (4) $i_7 = -i_{s3}$ $\quad i_7 - i_6 = 5 i_p$

(5) $4 v_p + R_7(i_5 - i_4) + v_{s2} + R_8 i_5 = 0$

(6) $R_3(i_4 - i_3) + R_6 i_4 + R_7(i_4 - i_5) - 4 v_p = 0$

(7) $v_{s1} + R_4 i_3 + R_3(i_3 - i_4) + R_2(i_3 - i_2) = 0$

(8) $R_2(i_2 - i_3) - 6 i_\varphi = 0$

(9) i_φ : $\quad i_\varphi = i_4 - i_2$

(10) v_p : $\quad v_p = R_6 i_4 + v_{s2} + R_8 i_5 + R_9 i_7$

(11) i_p : $\quad i_p = i_6 - i_5$