Name: ______________________________ (please print)

Signature: ___________________________

# ECE 2202 – Final Exam
# August 4, 2020
# Online

1. This quiz is open book, open notes.
2. Show all work necessary to complete the problem. A solution without the appropriate work shown will receive no credit. A solution which is not given in a reasonable order will lose credit.
3. Show all units in solutions, intermediate results, and figures. Units in the quiz will be included between square brackets.
4. If the grader has difficulty following your work because it is messy or disorganized, you will lose credit.
5. Do not use red ink. Do not use red pencil.
6. You will have 150 minutes to work on this quiz, and 20 minutes to download/print, scan and submit.
7. You MUST use lower case letters for time domain quantities, and upper-case letters with overbars for phasor quantities.

1. ________________/35

2. ________________/45

3. ________________/45

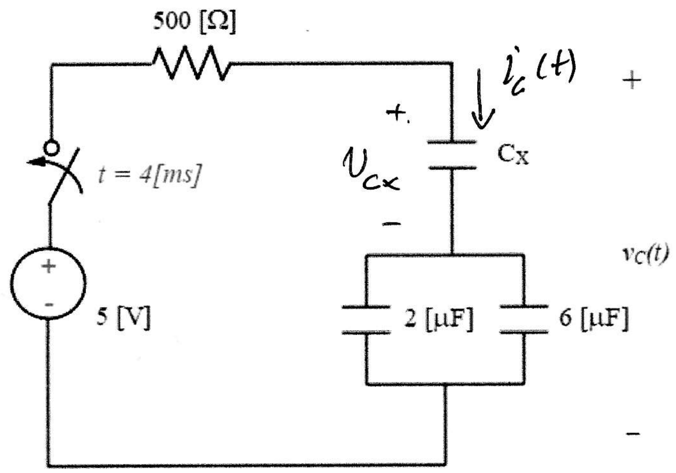
4. ________________/40

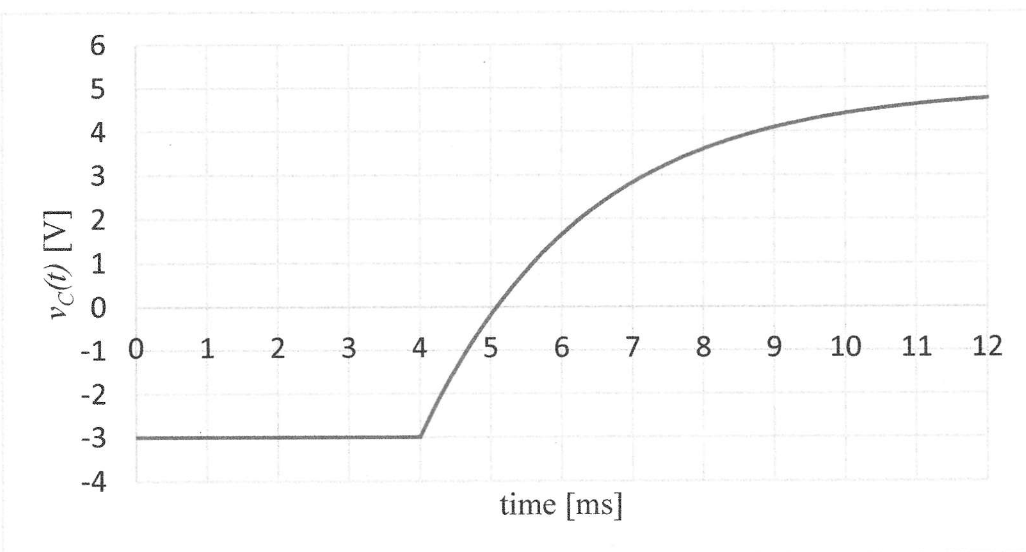
5. ________________/35

________________/200

1. [35 points] The circuit shown was set up as an experiment to determine the value of the unknown capacitance $C_X$. The switch had been open for a long time and closed at t = 4 [ms]. It is known that before the switch closed, the capacitor $C_X$ had no energy stored in it. A plot of $v_C(t)$ is shown in the graph.

a. From the graph, estimate the time constant $\tau_C$.
b. Find the unknown capacitance $C_X$.
c. Write an equation for $v_C(t)$ for $t > 4$ [ms]. Clearly indicate initial and final values, and $\tau_C$.
d. Find the energy stored in $C_X$ for $t \to \infty$.




We define an equivalent capacitance $C_{eq}$, with voltage $v_c(t)$. Then

$$v_c(t) = v_{c,f} + (v_c(t_0) - v_{c,f})\, e^{-(t-4[ms])/\tau_c} \qquad t \geq 4[ms]$$

Room for extra work

So we have $t_0 = 4$ [ms] at which point $V_C(t_0) = -3$ [V].

From the circuit it's also clear that $V_{Cf} = 5$ [V].

a) We can find $\tau_c$ in several ways. Here are two...

1. From the graph we see that $V_C = 0$ at $t \simeq 5.05$ [ms]. ( $t = 5$ [ms] is close enough!) So we have

$$V_C = 0 = 5 + (-3 - 5)\, e^{-(5-4)/\tau_c} \qquad \tau_c \text{ in [ms]}$$

$$\text{So} \quad e^{-1/\tau_c} = \frac{5}{8} \Rightarrow \tau_c \simeq 2.13 \text{ [ms]}$$

2. At the time $(t-4) = \tau_c$, we have

$$V_C = 5 - 8e^{-1} \simeq 2.06 \text{ [V]}.$$

At this voltage, $t \simeq 6.2$ [ms] So $\tau_c \simeq 2.2$ [ms].

So this gives a slightly different answer.

+8

We will use $\tau_c = 2.15$ [ms] (a compromise). For full credit, any answer in the vicinity of 2 [ms] is acceptable.

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b) We have $\tau = 2.15 \text{ [ms]} = R_{TH}\, C_{eq}$

$R_{TH} = 500\ [\Omega] \Rightarrow C_{eq} = 4.4\ [\mu F] \Rightarrow \underline{C_x = 9.8\ [\mu F]}$

+4

c) $\boxed{V_C(t) = 5 - 8e^{-(t-4\text{ [ms]})/2.15\text{ [ms]}}}$ $\quad t \geq 4$ [ms]

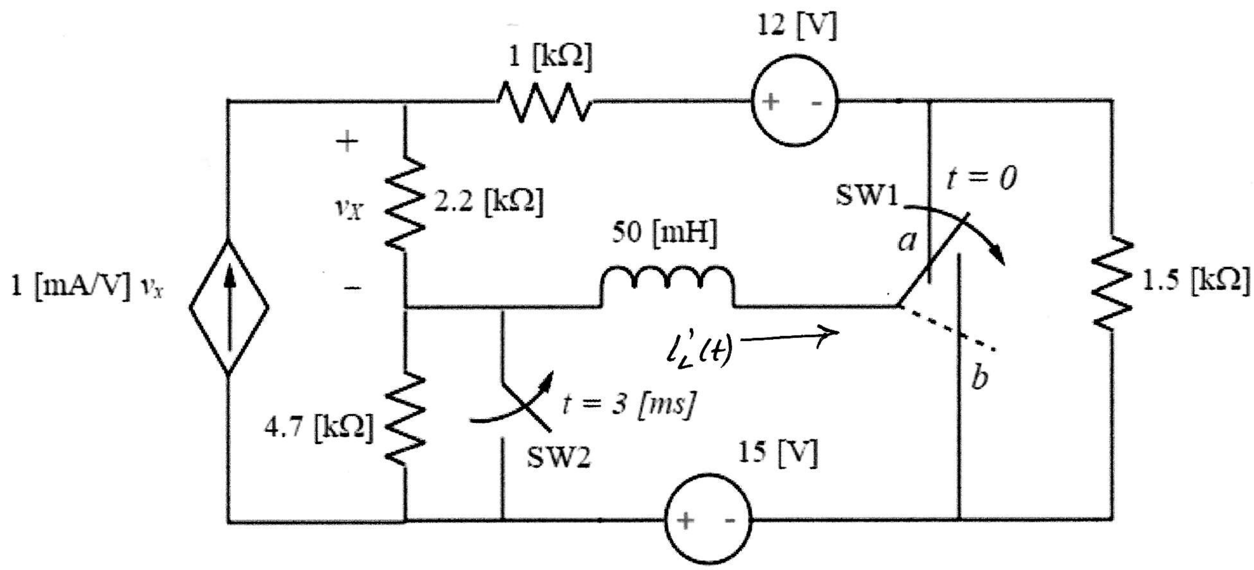
+10 $\qquad$ +3 $\qquad$ +3 $\qquad$ +2 $\qquad$ +2



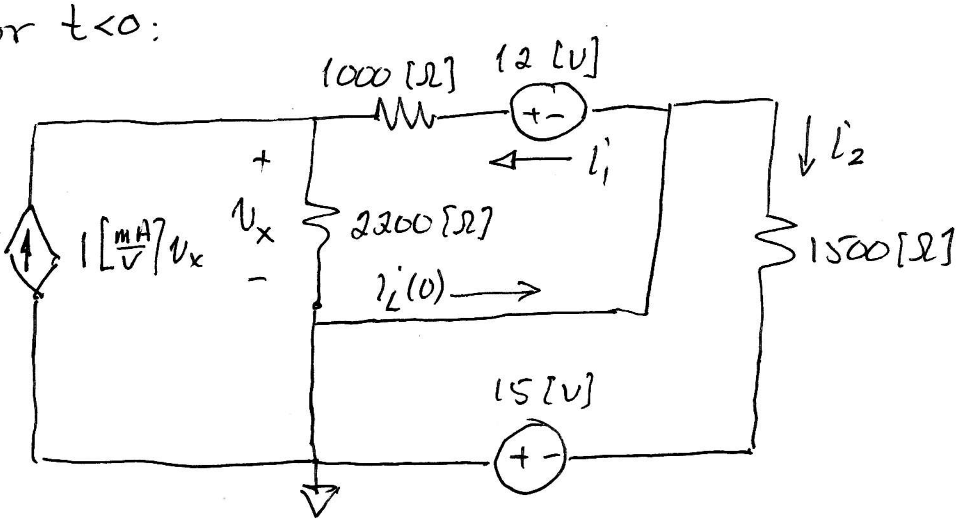
p. 2

2. [45 points] In the circuit below, switch SW1was in position 'a' for a long time, and switch SW2 was closed for a long time. At t = 0, SW1 moved to position 'b' (indicated by the dashed line) and remained there. After 3 [ms], SW2 opened and remained open.

Find the energy stored in the inductor at t = 3.5 [ms].




We will need $i_L'(t)$ ... start by finding the initial current for t<0:




L → short
4700 [Ω] is shorted

+5

+4

$$\frac{v_x}{2200} - 0.001\, v_x + \frac{v_x - 12}{1000} = 0 \quad \Rightarrow \quad v_x = 26.4\ [V]$$

+1

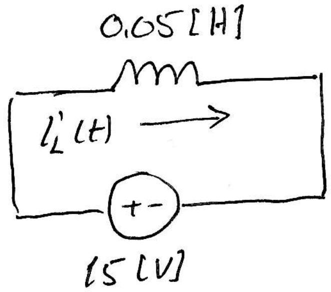
$$i_L'(0) = i_1' + i_2' = -\frac{v_x - 12}{1000} + \frac{15}{1500} = -4.4\ [mA]$$

+10

Room for extra work

For $0 < t < 3\,[ms]$, the inductor is in parallel with a voltage source, so nothing else matters!

+10




+7

$$i_L'(t) = \frac{1}{0.05}\int_0^{3\times10^{-3}\,[s]} 15\,dt - 4.4\,[mA]$$

$i_L(0)$

+7

$$i_L'(t) = \frac{15}{0.005}\,3\times10^{-3} - 4.4 \quad [mA]$$

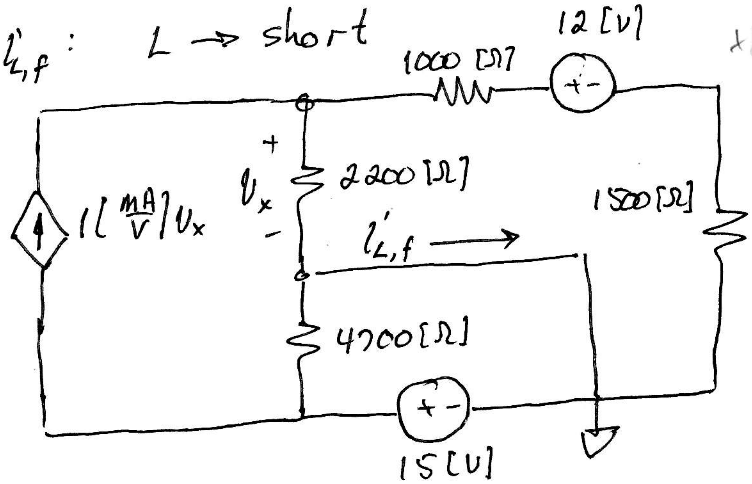
+1

$$i_L'(3\,[ms]) = 895.6\,[mA]$$

For $t > 3\,[ms]$ we need $i_{L,f}$ and $\tau_L$:

$i_{L,f}$: $L \rightarrow$ short

+10




+5

$$-0.001\,v_x + \frac{v_x}{2200} + \frac{v_x - 12}{2500} = 0$$

+4

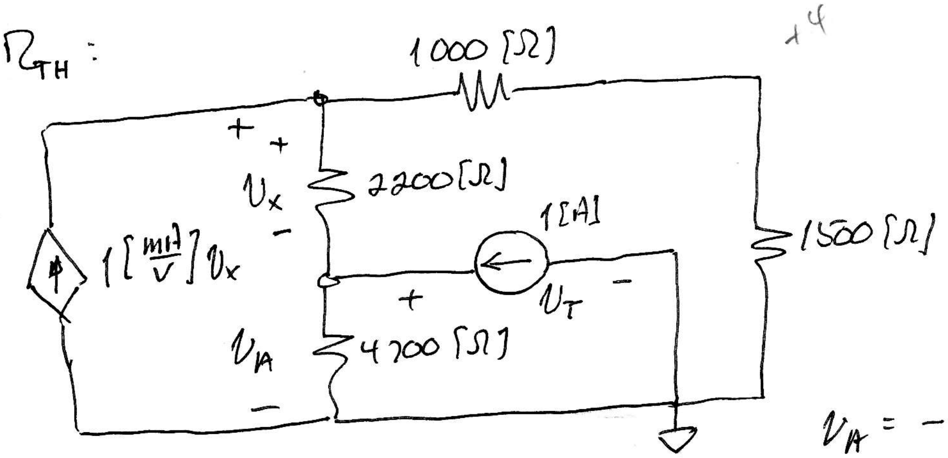
$$v_x = -33.0\,[V]$$

$$\boxed{i_{L,f} = \frac{v_x}{2200} + \frac{15}{4700}} = -11.81\,[mA]$$

+1

$R_{TH}$:

+10




+4

+5

$$\frac{v_T}{4700} - 1 + \frac{v_T - v_A}{2200} = 0$$

$$\frac{v_A - v_T}{2200} + \frac{v_A}{2500} - 0.001\,v_x = 0$$

$$v_x = v_A - v_T$$

$v_A = -3615.4\,[V]$ $\quad v_x = -2651.3\,[V]$

$v_T = -964.1\,[V]$ +1

P2

Room for extra work

Prob 1: d) We need the voltage across $C_x$ ...

+6 $$i_c(t) = C_{eq}\frac{dV_c(t)}{dt} = (4.4\times10^{-6})(-8)\left(e^{-(t-4[ms])/2.15[ms]}\right)\left(\frac{-1}{2.15\times10^{-3}}\right)$$

$$= 0.0164\, e^{-(t-4[ms])/2.15[ms]} \quad [A]$$

$$V_{cx}(t) = \frac{1}{C_x}\int_{4[ms]}^{\infty} i_c(t)\,dt + V_{cx}(4[ms])$$

We were given that there was no energy stored in $C_x$ before the switch closed, so $V_{cx}(4[ms]) = 0$.

+6 $$V_{cx}(t) = \frac{1}{9.8\times10^{-6}}\int_{4[ms]}^{\infty} 0.0164\, e^{-(t-4[ms])/2.15[ms]}\,dt = 3.564\ [V]$$

+1 $\therefore$ $$\boxed{W_{stored\ C_x} = \tfrac{1}{2}(9.8\times10^{-6})(3.564)^2 = 62.26\ [\mu J]}$$

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Prob 2: $V_T = -964.1\ [V] \Rightarrow R_{TH} = -964.1\ [\Omega]$

$$\tau_L = L/R_{TH} = -0.05/964.1 = -0.052\ [ms]$$

So we have... $$i_L(t) = 895.6\ [mA] + (-11.81 - 895.6)[mA]\, e^{+(t-3)/+0.052} \quad t \geq 3[ms]$$

+3

at $t = 3.5\ [ms]$,

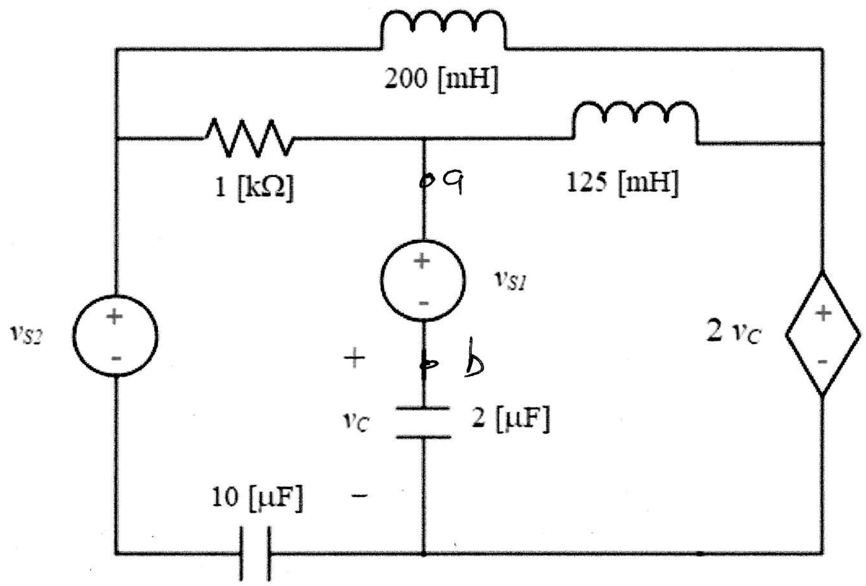
$$i_L(t = 3.5[ms]) = -13.6\ [kA]$$ (unrealistically large, but...)

+2 $$W = \tfrac{1}{2} L i_L^2 = 4.63\times10^{6}\ [J]$$ (yikes!)

3. [45 points] For the circuit below, the voltage sources are as follows.

$$v_{S1}(t) = 110\cos(377\,t + 20^{\circ})\ [\text{V}]; \quad v_{S2}(t) = 52\cos(1000\,t)\ [\text{V}]$$

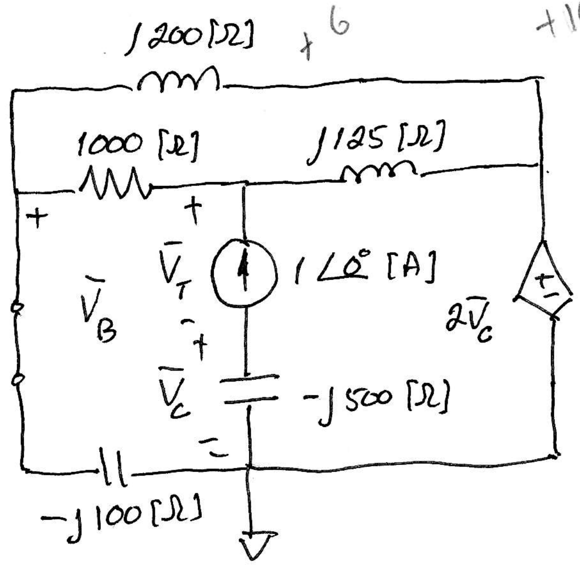
a) Find the Thevenin equivalent impedance of the circuit below **as seen by the voltage source** $v_{S1}$ at $\omega = 1000$ [rad/s].
b) Find the Thevenin equivalent voltage seen by the voltage source $v_{S1}$.
c) Draw the Thevenin equivalent circuit **in the time domain**, using appropriate time domain circuit elements to represent the Thevenin impedance. Clearly label the values of the circuit elements, and the terminals at which you are calculating the equivalent circuit.




Test current source:

a)

+6 +16




+20

$$\frac{\bar{V}_T + \bar{V}_C - \bar{V}_B}{1000} - 1 + \frac{\bar{V}_T + \bar{V}_C - 2\bar{V}_C}{j125} = 0$$

$$\frac{\bar{V}_B}{-j100} + \frac{\bar{V}_B - \bar{V}_C - \bar{V}_T}{1000} + \frac{\bar{V}_B - 2\bar{V}_C}{j200} = 0$$

$$\bar{V}_C = -(1)(-j500) = j500$$

$$\bar{V}_B = 1056.5\angle{-66.4^{\circ}}\ [\text{V}]$$

$$\bar{V}_T = 696.8\angle{67.7^{\circ}}\ [\text{V}]$$

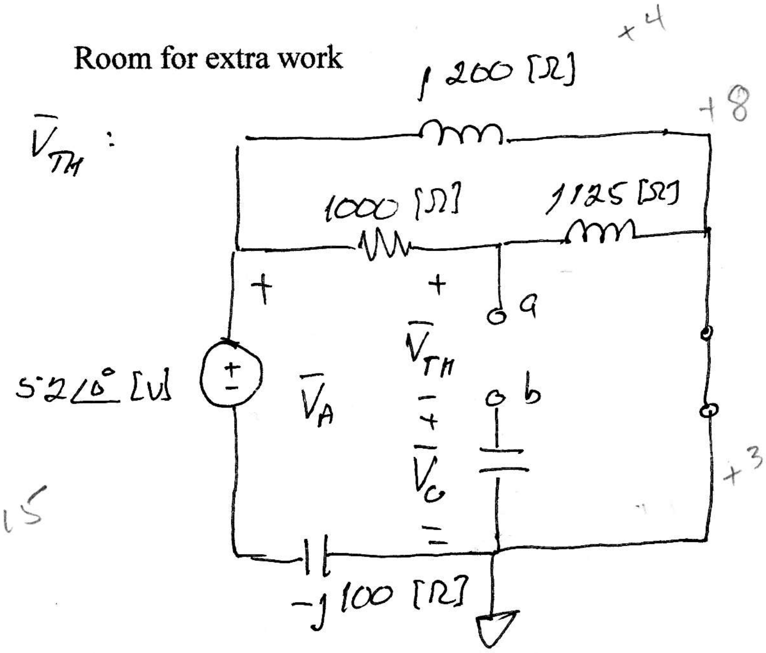
$$\Rightarrow Z_{TH} = 696.8\angle{67.72^{\circ}}\ [\Omega]$$

$$= 264.2 + j644.7\ [\Omega]$$

+4

Room for extra work

b) $\bar{V}_{TH}$ :




+8

+15

$\bar{V}_c = 0.$

$$\frac{\bar{V}_A}{1000+j125} + \frac{\bar{V}_A}{j200} + \frac{\bar{V}_A - 52\angle 0^\circ}{-j100} = 0$$

$\bar{V}_A = 104.5 \angle 11.41^\circ$ (V)

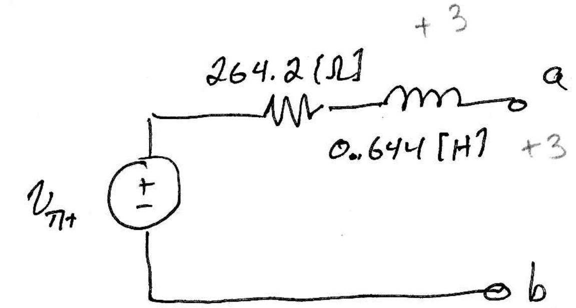
+3

$$\bar{V}_{TH} = \bar{V}_A \cdot \frac{j125}{j125+1000} = 12.96 \angle 94.28^\circ \text{ [V]}$$

$= -0.967 + j12.92$ [V]

c)

+10.




+4

$v_{TH} = 12.96 \cos(1000t + 94.28^\circ)$ [V]

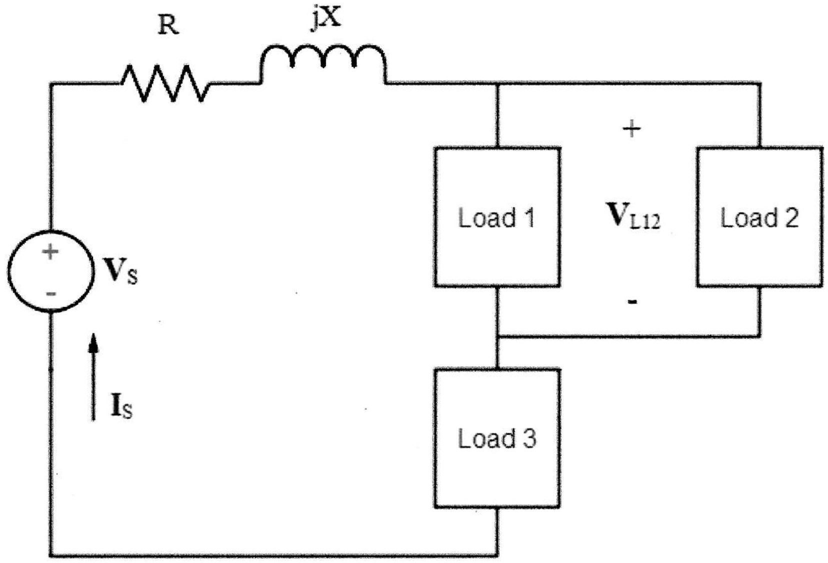
4. [40 points] In this problem, a source $\mathbf{V_S}$ is connected to three loads by way of a transmission line with impedance $Z_{Line} = R + jX$. The following is known about the circuit.

Load 1 is absorbing 600 [W] and delivering 750 [VAR].
Load 2 is absorbing 2500 [VA] at a power factor of 0.7 lagging.
Load 3 is a 50 [Ω] resistor in parallel with an impedance of j200 [Ω].

$= 24586 + j\,88654$ [VA]

The load voltage $\mathbf{V_{L12}}$ is $120\angle{-120^\circ}$ [V]rms. The source $\mathbf{V_S}$ delivers $92\angle 74.5^\circ$ [kVA] of power to the line and the three loads combined, and operates at a frequency of 377 [rad/s].

a) Find $\mathbf{I_S}$.
b) Find the complex power absorbed by the transmission line.
c) Find the values of the line resistance R and inductance L.




a) We will deal with S absorbed unless otherwise stated.

$= 960.5\angle{-5.34^\circ}$ [VA]

+5 $S_1 = 600 - j\,750$ [VA]

+5 $\bar{I}_1^* = \dfrac{600-j\,750}{120\angle{-120^\circ}} = 8\angle 68.87^\circ$ [A]rms $\Rightarrow \bar{I}_1 = 8\angle{-68.87^\circ}$ [A]rms

$(= 2.884 - j\,7.462)$

$S_2$ : $\cos(\theta) = 0.7 \Rightarrow \sin(\theta) = \sqrt{1-0.7^2} = 0.714$

+5 $S_2 = 2500\,(0.7) + j\,2500\,(0.714) = 2500\angle 45.57^\circ$ [VA]

$\bar{I}_2^* = \dfrac{2500\angle 45.57^\circ}{120\angle{-120}} = 20.83\angle 165.6^\circ$ [A]rms

+5 $\bar{I}_2 = 20.83\angle{-165.6^\circ}$ [A]rms

$= -20.18\, j\, 5.18$ [A]rms

+2 Room for extra work

So $\bar{I}_s = \bar{I}_1 + \bar{I}_2 = 21.42 \angle -143.8^\circ$ [A]$_{rms}$

$= -17.28 - j12.65$ [A]$_{rms}$

b) Find $S_3$, then subtract total load power from delivered power to find $S_{abs}$ by Line:

+1 $Z_3 = j200 \parallel 150 = 48.51 \angle 14.04^\circ$ [Ω] $= 47.06 + j11.77$ [Ω]

+5 $S_3 = |\bar{I}_s|^2 Z_3 = 22257.2 \angle 14.04^\circ$ [VA] $= 21592 + j5399.6$ [VA]

+4 $S_{Line} = S_{del\ by\ V_s} - S_1 + S_2 + S_3 = 82{,}222 \angle 89.55^\circ$ [VA]

$= 643.5 + j82219$ [VA]

c) $Z_{Line} = \dfrac{S_{Line}}{|\bar{I}_s|^2} = 179.2 \angle 89.55^\circ$ [Ω] $= 1.403 + j179.2$ [Ω]

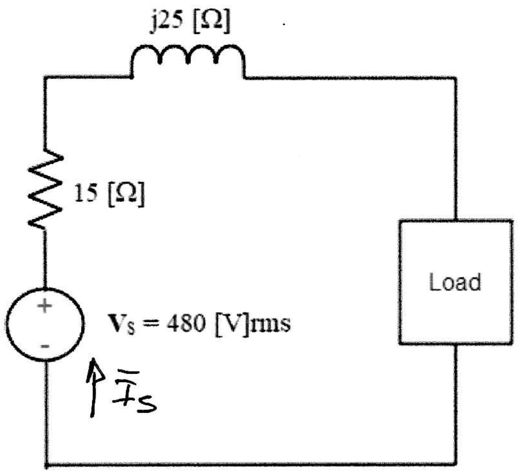
+4

$\Rightarrow \boxed{R = 1.4 \text{ [Ω]}}$ $\boxed{L = \dfrac{179.2}{\omega} = 475 \text{ [mH]}}$

+4

5. [35 points] In the circuit below, the power source **V**S operates at ω = 377 [rad/s]. The Load is a 100 [Ω] resistor in series with a 300 [mH] inductor.

a) Find the complex power delivered by the source **V**S.
b) Find the power factor of the load.
c) A 20 [μF] capacitor is added in parallel with the load to adjust the load power factor. Find the new load power factor.
d) After adding the capacitor in part c, what now is the power supplied by **V**S?




300 [mH] $\rightarrow j(377)(0.3)$ $= j\,113.1$ [Ω]

a) $\bar{I}_s = \dfrac{480}{15 + j25 + 100 + j113.1} = 2.671\angle{-50.22^\circ}$ [A]rms $= 1.709 - j\,2.053$ [A]rms

+5

$\therefore$ $S_{\text{del by } V_s} = \bar{V}_s \bar{I}_s^{\,*} = (480)(2.671\angle{50.22^\circ}) = 1282.1\angle{50.22^\circ}$ [VA] $= 820.3 + j1985.2$ [VA]

+5

b) $Z_{Load} = 100 + j\,113.1$ [Ω] $= 150.97\angle{48.52^\circ}$ [Ω]

$\Rightarrow$ $pf = \cos(48.52^\circ) = 0.6624$

+3

c) 20 [μF] $\rightarrow -j/\omega C = -j\,132.6$ [Ω]

+3 putting this in parallel with $150.97\angle{48.52^\circ}$ [Ω] gives

$Z'_{Load} = \dfrac{(-j132.6)(150.97\angle{48.52^\circ})}{-j132.6 + 150.97\angle{48.52^\circ}} = 196.5\angle{-30.45^\circ}$ [Ω] $= 169.4 - j99.58$ [Ω]

+4

$\Rightarrow pf = \cos(30.45^\circ)$

$= 0.862$



+2

Room for extra work

$$\bar{I}_6' = \frac{480}{15 + j25 + 196.5 \angle -30.45^\circ} = 2.413 \angle 22.02^\circ \; [A]_{rms}$$

$= 2.24 + j\,0.905$

+5

d) $S'_{del\ by\ V_s} = \bar{V}_s \bar{I}_6'^{\,*} = 480\,(2.413 \angle -22.02)$

+6

$= 1158.2 \angle -22.02^\circ \; [VA]$

$= 1073 - j\,434.2 \; [VA]$

e) The power delivered by source has been reduced, so this configuration saves money!

+2