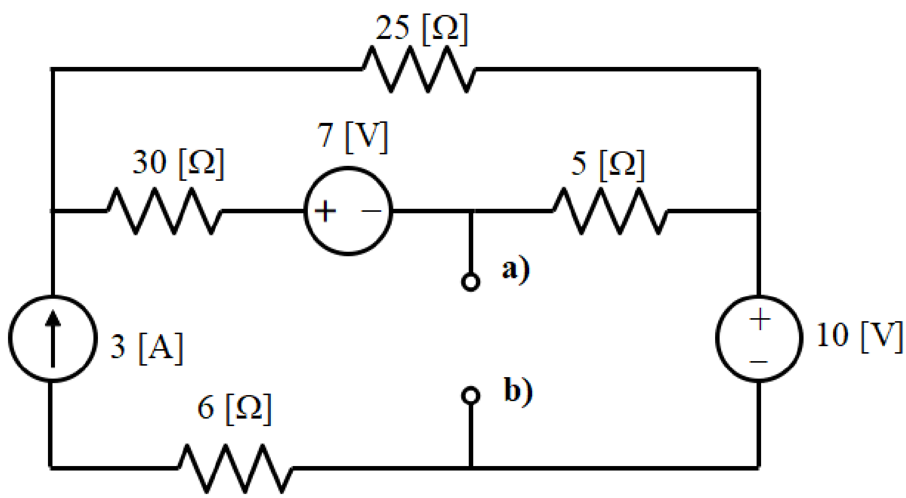
Thevenin and Norton Equivalents

We present here a review of Thevenin and Norton equivalents. We want to refresh our memory on this very important topic, and it will give us a reason to review a lot of basic circuit theory.

The Idea

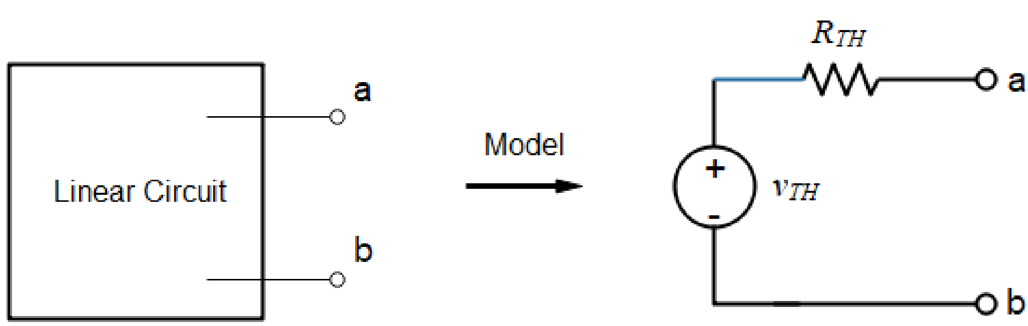
We are often interested in the voltage and/or current in a load that is connected to a certain pair of terminals in a circuit. For example, we may want to connect a load resistor, or maybe several different load resistors, to the terminals labeled a), b) in the circuit shown below.



The following idea is very powerful, and may help in analyzing a case like this, especially if we need to know the voltage across many different load resistors:

> ***Thevenin Equivalent Circuit:*** The behavior of any **linear circuit** at a specific pair of terminals in a circuit may be modeled by a voltage source v_{TH} in series with a resistor R_{TH}.

We will look only at linear circuits in this course. What we are saying is that the circuit below on the left can be modeled by the circuit on the right.



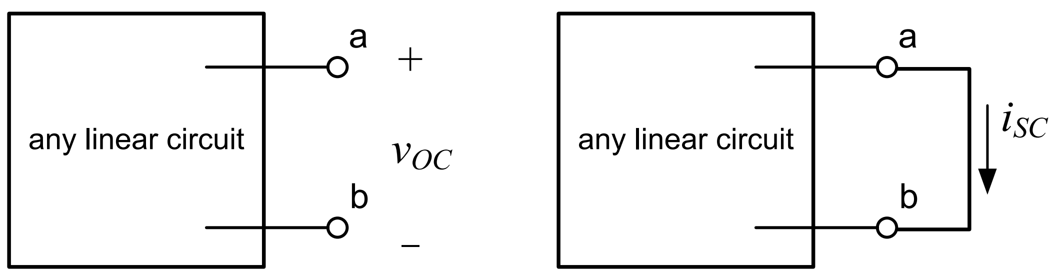
Then, if we know the Thevenin equivalent - that is, if we know v_{TH} and R_{TH} - we can connect as many resistors as we like to the circuit at those terminals, and solve the problem for each of them much more easily.

Important Notes:

- We are "modeling" the circuit at two specific terminals with v_{TH} and R_{TH} – we are not suggesting that the only things inside the box are a resistor and a voltage source.
- The model holds for any load but only at terminals a), b). If we specify different terminals in the original circuit, the values of v_{TH} and R_{TH} will change.
- The circuit must be linear, but it can contain any of the basic circuit elements: voltage sources, current sources (dependent and independent), resistors, capacitors, and inductors.

Finding v_{TH} and R_{TH}

The box in the figure below contains an arbitrary linear circuit. We have labeled terminals a) and b). On the left, we have an open circuit at a), b), resulting in an open-circuit voltage v_{OC}. (We can think of this as an infinite load resistance.) On the right, we have connected a short to the terminals, resulting in a short-circuit current i_{sc}.



By comparing the drawing on the left with the Thevenin equivalent circuit above, it should be clear that

$$v_{OC} = v_{TH} \ .$$

By comparing the drawing on the left with the Thevenin Equivalent circuit, we can see also that

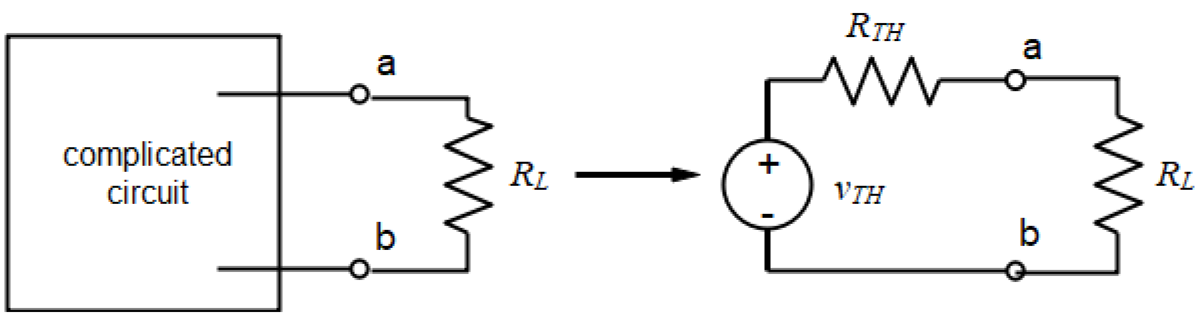
$$i_{SC} = \frac{v_{TH}}{R_{TH}} \ .$$

So we have an algorithm for finding a Thevenin Equivalent: If we know the open-circuit voltage and the short-circuit current at the terminals a), b), we can find the Thevenin Equivalent:

$$v_{TH} = v_{OC} \quad \text{and} \quad R_{TH} = \frac{v_{TH}}{i_{SC}}.$$

If this were an experiment, we could measure v_{OC} and i_{SC}. If it is an analytical problem, we can calculate them using our knowledge of circuit theory.

Is this useful?? Wow, yeah! This idea is used a lot. What it means is that we can talk about a lot of complicated circuits without having to know anything about those circuits except their Thevenin equivalents. Sometimes we don't even need to know the Thevenin equivalent parameters – we just need to know there is a Thevenin equivalent. This is extremely useful.



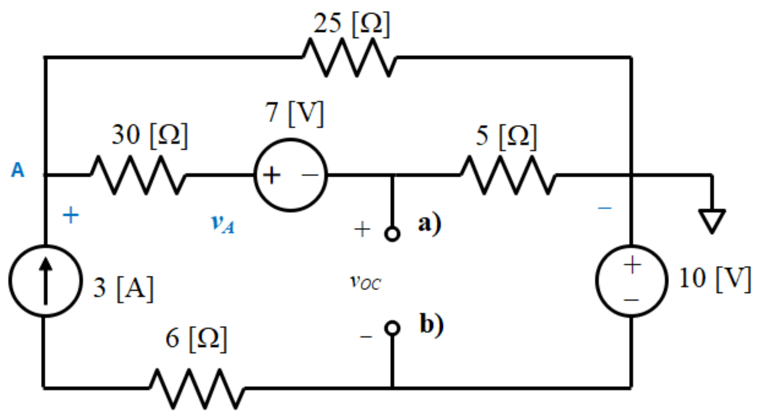
As we pointed out above, for example, if we need to analyze how several different load resistors behave when connected to a circuit at two particular terminals, we only need the Thevenin equivalent, and we can make the calculations much simpler. This idea is shown below.

The load resistor R_L cannot tell the difference between the circuit on the left and the Thevenin equivalent on the right. But it's a lot easier to handle the equivalent than it is to analyze the complicated circuit on the left, especially if we need to test many different R_L values.

An Example

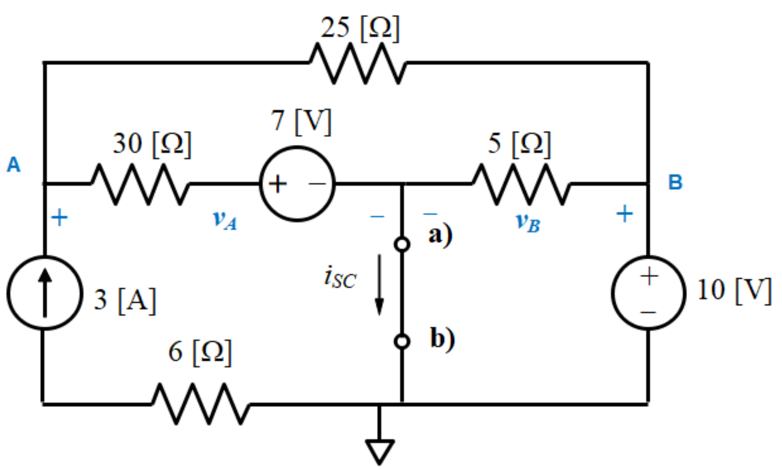
Let's find the Thevenin equivalent of the circuit shown at the beginning of this chapter. We will find the open circuit voltage v_{oc}, and the short circuit current i_{sc} at the terminals a), b). As described above, v_{oc} will be the Thevenin voltage, and the ratio v_{oc}/i_{sc} and will be the Thevenin resistance R_{TH}.

In the drawing below, we have defined the open circuit voltage v_{oc}, the node voltage v_A, and the reference node. We need a node voltage equation for node A, and an equation for v_{oc}.



$$\frac{v_A - 7}{35} - 3 + \frac{v_A}{25} = 0$$
$$\frac{v_{oc} - 10}{5} = \frac{v_A - 7}{35}$$

Solving these equations gives $v_{oc} = 15.667$ [V], $v_A = 46.667$ [V]. Now we need i_{sc}. Below we define i_{sc}, as well as two new node voltages and a new reference node.

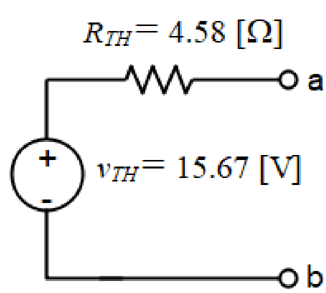


$$\frac{v_A - 7}{30} - 3 + \frac{v_A - v_B}{25} = 0$$
$$v_B = 10 \text{ [V]}$$
$$i_{sc} = \frac{v_B}{5} + \frac{v_A - 7}{30}$$

Solving gives $v_B = 10$ [V], $v_A = 49.54$ [V], $i_{sc} = 3.418$ [A]. Thus, we have

$$v_{TH} = v_{oc} = 15.667 \text{ [V]},$$
$$R_{TH} = \frac{v_{oc}}{i_{sc}} = 4.58 \text{ [}\Omega\text{]}.$$

We should draw the Thevenin equivalent circuit to show the parameters we have calculated.

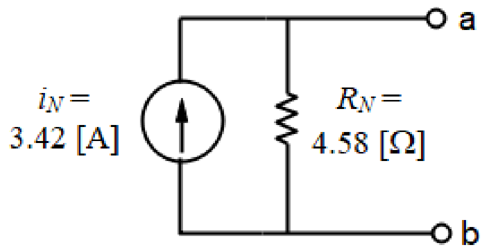


Important Note: The open circuit voltage and short circuit current must be oriented in the passive sign relationship, or we will get the wrong sign for the Thevenin resistance.

The Norton Equivalent

We may want to analyze circuit behavior using a Norton equivalent rather than a Thevenin equivalent. We know from the source transformation theorem that it is a simple matter to convert a voltage source in series with a resistance to a current source in parallel with a resistance. If we make this transformation on a Thevenin equivalent, the result is the Norton equivalent. We could also find the Norton equivalent directly since $i_N = i_{sc}$ and $R_N = R_{TH} = v_{oc}/i_{sc}$.

We will not pursue the Norton equivalent here any further, except that we will transform the Thevenin equivalent we just obtained to a Norton equivalent. This is done in the figure below.

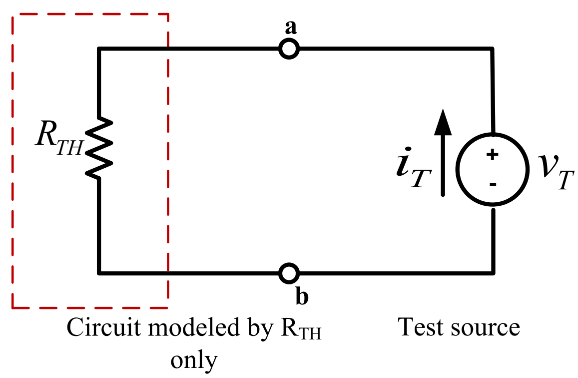


Test Source Method for R_{TH}

There is another method for finding R_{TH} directly.

Suppose we had a circuit that could be modeled using only a resistor. That will be the case if there are no independent sources in the circuit. Then if we were to apply a voltage source v_T and calculate the current i_T through it, we could find R_{TH} as:

$$R_{TH} = \frac{v_T}{i_T}$$



Here, v_T is known as the "test source". If our circuit is simply a resistance, we can use the test source to find that resistance. This will be the Thevenin equivalent resistance R_{TH}.

We can also use a current source as the test source, and then find the test voltage. We show examples of that in *Worked Problems: Thevenin and Norton Equivalents*.

Important Note: The polarity of v_T and the current i_T in the test source method must be in the active sign relationship. Otherwise, we will get the wrong sign for R_{TH}. This can be seen by noting that if in the circuit above, R_{TH} is positive and v_T is positive, i_T will be positive. If we were to reverse the direction of the current and then take the ratio, we would get the wrong sign for R_{TH}.

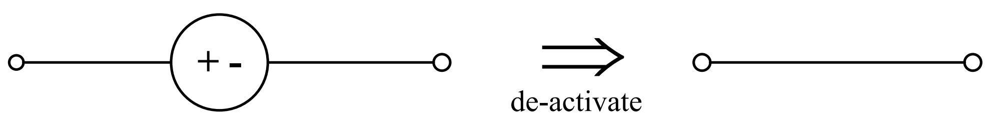
The polarity of the test voltage and the test current must be in the active sign convention whether we use a test voltage source or a test current source.

Resistances Only

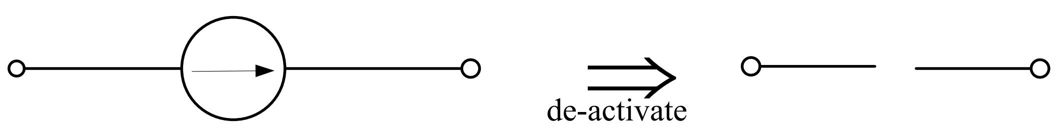
But is this useful? If we are finding the Thevenin equivalent of a circuit that has only resistances, we can simply combine these into one resistance (series, parallel, delta-to-wye), and we have R_{TH}. We don't need a "test source". But what if our circuit is not just a resistance, but contains sources as well? What if some of those sources are dependent sources? We can use the test source idea as follows.

1. De-activate all *independent* sources.

To de-activate an independent voltage source, we replace it with a short. Note that a short is a voltage source of value 0 [V].



To de-activate an independent current source, we replace it with an open circuit. Note that an open circuit is a current source of value 0 [A].



2. Apply a test source of known value; it doesn't matter what the value is. You can even leave the value out and just call it v_T.

3. Calculate the current i_T.

Then $R_{TH} = \dfrac{v_T}{i_T}$. If you have not given your test source a value, just calculate the ratio v_T/i_T.

Notes

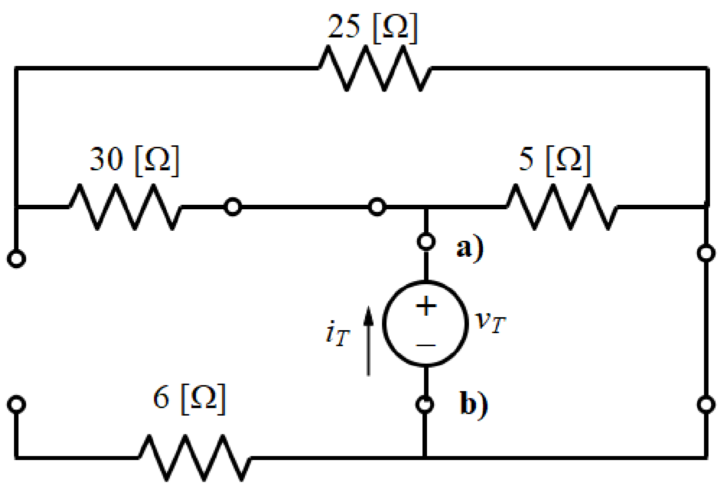
- ***You cannot de-activate dependent sources***. You need to leave them in; they affect the equivalent resistance of the circuit.

- If you have nothing but resistances and independent sources, you don't need the test source: you can simply de-activate the independent sources and find R_{TH} by resistor combinations.

- If there are dependent sources but no independent sources, you must use a test source, because the open circuit voltage and short circuit current will both be 0, so you can't take the ratio. See the example later.

Example

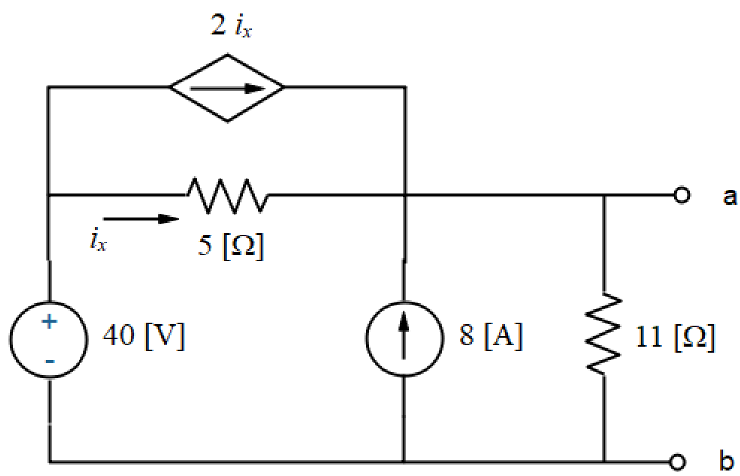
Let's verify the Thevenin resistance of the circuit we solved above. In the circuit below, we have deactivated the two voltage sources (they are replaced by short circuits), and the current source (it has been replaced by an open circuit).We have also applied a test voltage source at the terminals a), b), and indicated v_T and i_T in the active sign convention.



We could find the current i_T and from that calculate the Thevenin resistance, but this problem is easier than that: we have nothing left here but resistances, so we need only combine these in series in parallel. We'll leave it to the student to verify that the equivalent resistance at terminals a), b) is the same as we calculated above.

Example: Resistances and both independent and dependent sources.

Find the Thevenin Equivalent resistance of the circuit below at terminals a), b).



This problem is done in the *Worked Problems: Thevenin and Norton Equivalents* as Problem 4.2.

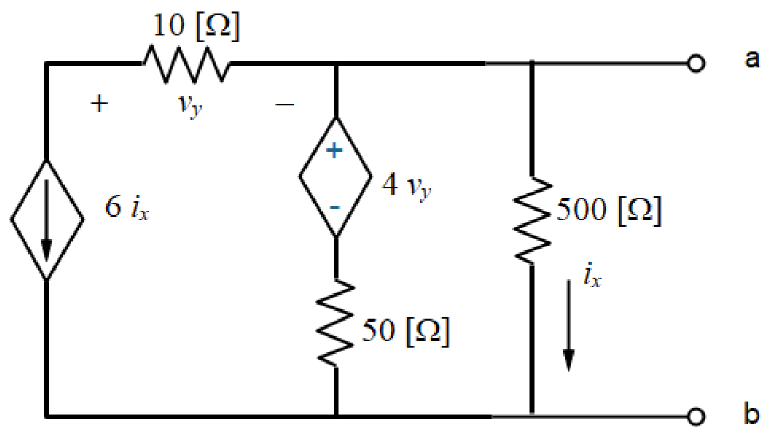
In this problem we have both dependent and independent sources. In the solution shown, we find the open circuit voltage and the short circuit current, and from that we calculate the Thevenin resistance. We then find the Thevenin resistance using a test voltage source, and finally a test current source.

Of course, we did not do you need to use all these methods to solve the circuit. Of the tools we have used here, we needed only two of them to find the two Thevenin parameters. However, it is often the case that one method is preferable to another because it simplifies the circuit. In the example below, the short circuit current is a bit easier to find than the open circuit voltage, because it removes a resistor. Also, application of a test source simplifies the circuit by removing independent sources.

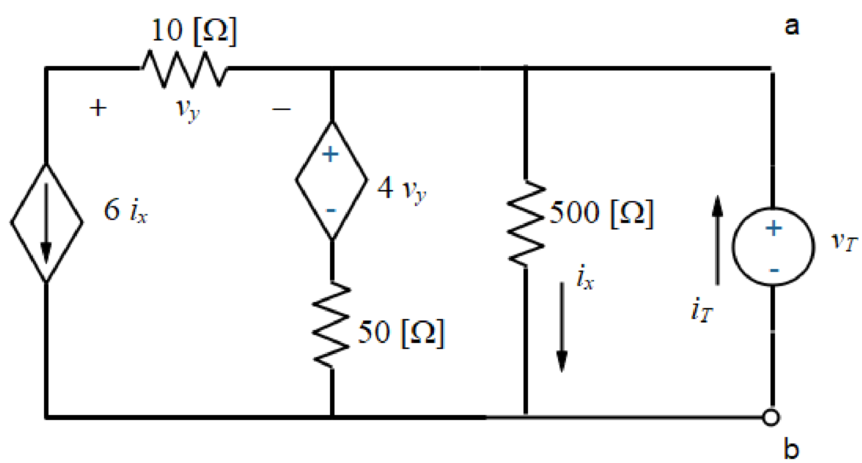
In other circuits, the simplification achieved by a particular method may be more dramatic than in this case. Therefore, before solving for a Thevenin equivalent, you should think about each of these methods to see if one method simplifies the circuit more than another.

Example: Resistances and dependent sources.

Find the Thevenin Equivalent resistance of the circuit below at terminals a), b).



Here we note that at terminals a), b), the open circuit voltage and short circuit current will both be 0, because there are no independent voltage or current sources. Therefore, we must use a test source. In the figure below, we apply a test voltage source. Remember that we do not deactivate dependent sources.



$$i_T = 6i_x + \frac{v_T - 4v_y}{50} + \frac{v_T}{500}$$

$$i_x = \frac{v_T}{500}$$

$$v_y = -60i_x$$

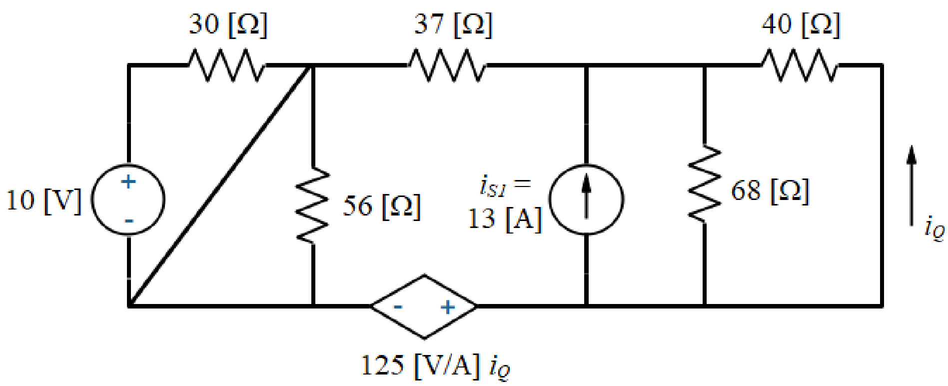
Solving these equations gives $\frac{v_T}{i_T} = R_{TH} = 1.946\ [\Omega]$. We already know that $v_{TH} = 0$.

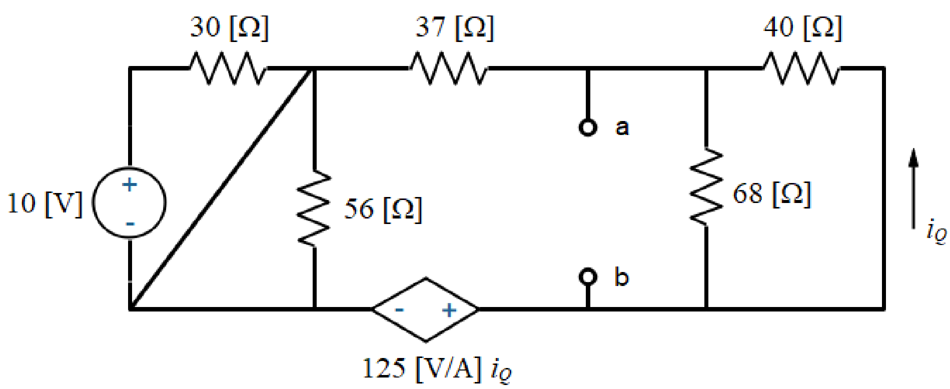
The Thevenin Equivalent "Seen By..."

Sometimes we have a load resistor, or a circuit element, or even a complex device connected to a circuit at two terminals. We may then be interested in the Thevenin equivalent ***seen by*** the load resistor, or by the device, or by the circuit element. In that case we ***remove*** the circuit element or device in question, and find the Thevenin equivalent at the terminals where it was connected.

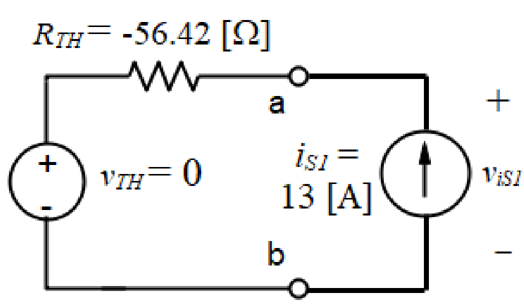
In the circuit below, we want to find the Thevenin equivalent ***seen by*** the current source. In the circuit following, we have removed the current source, and we need to find the Thevenin equivalent at the terminals where it was connected. Those terminals are now labeled a), b).

We work this problem as Problem 4.5 of *Worked Problems: Thevenin and Norton Equivalents*.





Once we have the Thevenin equivalent, we can connect the current source to it and continue with any other analysis we might need to do. In the diagram below, we have done this. We also show the results of the Thevenin equivalent calculation. A KVL then shows that the voltage across the current source is v_{iS1} = -733.5 [V].



Summary for Finding the Thevenin Equivalent

- If the circuit contains independent sources, you can find an open circuit voltage and a short circuit current, or you can use a test source to find R_{TH}. We only need to choose two of these three methods to find the Thevenin equivalent.

- If the circuit contains only resistances, these can be combined into one to find R_{TH}. In that case, the open circuit voltage and short circuit current will be both be 0, which means the Thevenin voltage is 0.

- If the circuit contains only resistances and dependent sources (or only dependent sources), the open circuit voltage and short circuit current will again be 0. In that case, there is no choice but to use a test source.

- It is a smart idea to check to see which of the open circuit voltage, short circuit current, and test source methods is easiest to use: the short circuit current may remove components in parallel with the terminals of interest, for example. The test source method is useful if we want to de-activate independent sources. Before beginning, check the circuit and think ahead about what it will look like using each of the three methods.

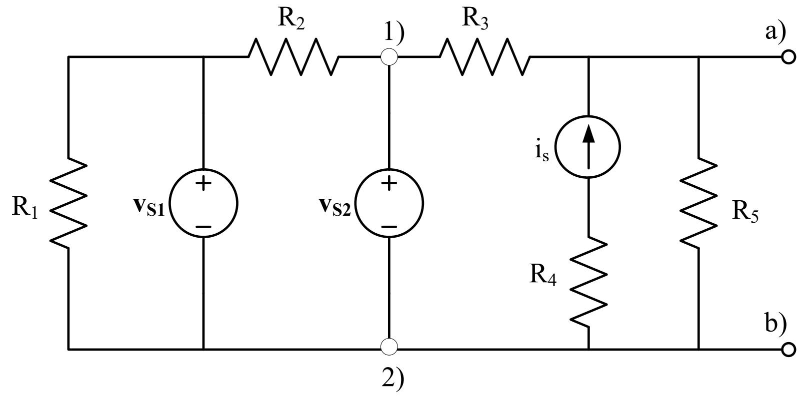
On a Negative Thevenin Equivalent Resistance

We assume that there are no negative-valued resistors (of the type you find in your lab kit, for example). However, ***when modeling a circuit that contains dependent sources***, it is ***possible*** that the Thevenin Equivalent resistance is negative. This does not mean that we can have negative valued resistors. It means that *the circuit **model** includes a negative resistance*. That resistance is simply part of the model; it is not an actual circuit component.

Only circuits with dependent sources can have negative R_{TH}. But just because a circuit has a dependent source does not mean it will have a negative R_{TH}.

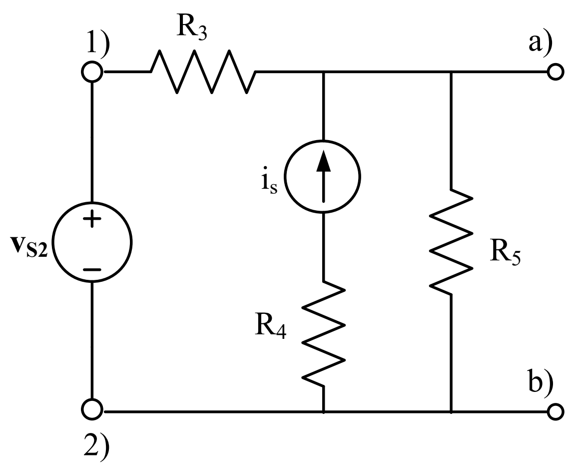
Two interesting cases

Consider the circuit below, where we are interested in terminals a), b).



Any circuit components to the left of the source v_{S2} cannot have any effect on what happens at terminals a), b), because v_{S2} fixes the voltage across those components. So no matter what values R_1 and R_2 or v_{S1} have, the voltage across them is v_{S2}, and something connected to a), b) will see v_{S2} but not those components.

What that means is that *as long as we are interested only in what happens at a), b), which is to the right of terminals 1) and 2)*, we can re-draw the circuit as follows.



Now think about the current source. Nothing outside of R_4 and the current source can "see" R_4, because the current through it is fixed by i_S. In other words, if R_4 doubled in value, nothing different would happen at terminals a), b) – or terminals 1), 2) for that matter. So as long as we are interested in something ***outside*** the branch with i_S and R_4, we can remove R_4 as well.

Bottom line

Circuit components in parallel with a voltage source can be replaced by just the voltage source, provided we are interested only in what is happening outside of those components.

Circuit components in series with a current source can be replaced by just the current source, provided we are interested only in what is happening outside of those components.