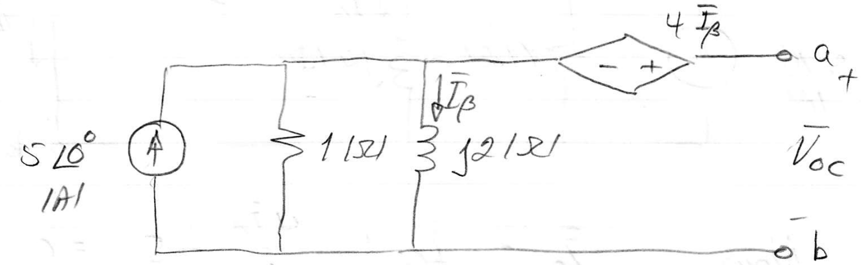
Example 9.6



Find the Thevenin equivalent at a, b.

Open-Circuit Voltage:

Note there is no current in the source $4\bar{I}_\beta$, so we may use the current divider rule to find $\bar{I}_\beta$:

$$\bar{I}_\beta = 5\angle 0^\circ \cdot \frac{1}{1+j2} = 1-j2 \ |A|$$

Then

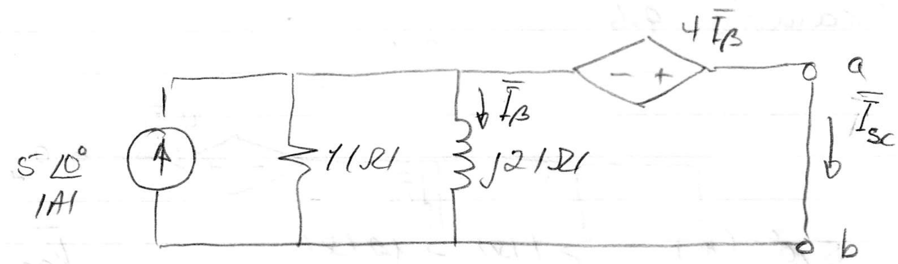
$$\bar{V}_{oc} = 4\bar{I}_\beta + j2\bar{I}_\beta$$

$$= (4+j2)(1-j2) = 8-j6$$

$$\bar{V}_{oc} = \bar{V}_{Th} = 10\angle -36.9^\circ \ |V|$$

Short-Circuit Current:

Now there is a current in the $4\bar{I}_\beta$ source: it is $\bar{I}_{sc}$:



Now $\quad \bar{I}_{sc} + \bar{I}_\beta + \dfrac{4\bar{I}_\beta}{1} - 5 = 0$

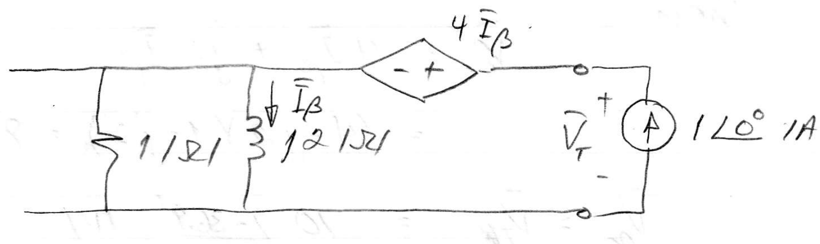
But $\quad \dfrac{-4\bar{I}_\beta}{j2} = \bar{I}_\beta \Rightarrow \bar{I}_\beta = 0$!

$\therefore \quad \bar{I}_{sc} = 5\angle 0° \text{ [A]}$

$\Rightarrow \quad Z_{Th} = \bar{V}_{oc}/\bar{I}_{sc} = 2\angle -36.9° \text{ [Ω]}$

$\underline{Z_{Th} = 1.6 - j1.2 \text{ [Ω]}}$

Just for practice, we use a test source as well:



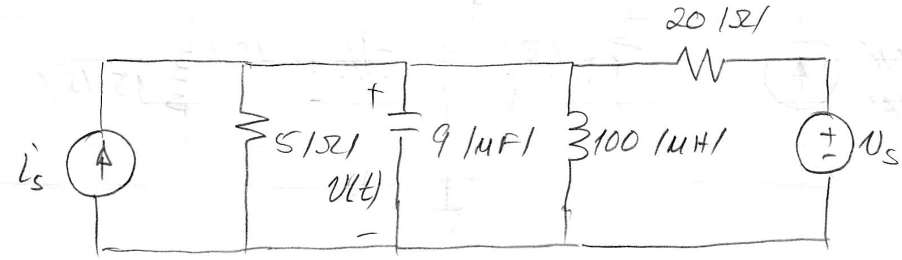
$\bar{I}_\beta = 1 \cdot \dfrac{1}{1+j2} = 0.2 - j0.4 \text{ [A]}$

$\bar{V}_T = 4\bar{I}_\beta + j2\bar{I}_\beta = (4+j2)\bar{I}_\beta$

$= 1.6 - j1.2 \text{ [V]}$

$\therefore \quad \underline{Z_{Th}} = \bar{V}_T/\bar{I}_T = \underline{1.6 - j1.2 \text{ [Ω]}}$ ✓

Example 9.7



GIVEN: $i_s = 10 \cos \omega t$ |A|

$v_s = 100 \sin \omega t$ |V|

$\omega = 50 \times 10^3$ |rad/s|

FIND: $v(t)$ using the Node-Voltage method.

We must transform to the phasor domain.

Note that $v_s = 100 \sin(\omega t)$ |V|

$= 100 \cos(\omega t - 90°)$ |V|

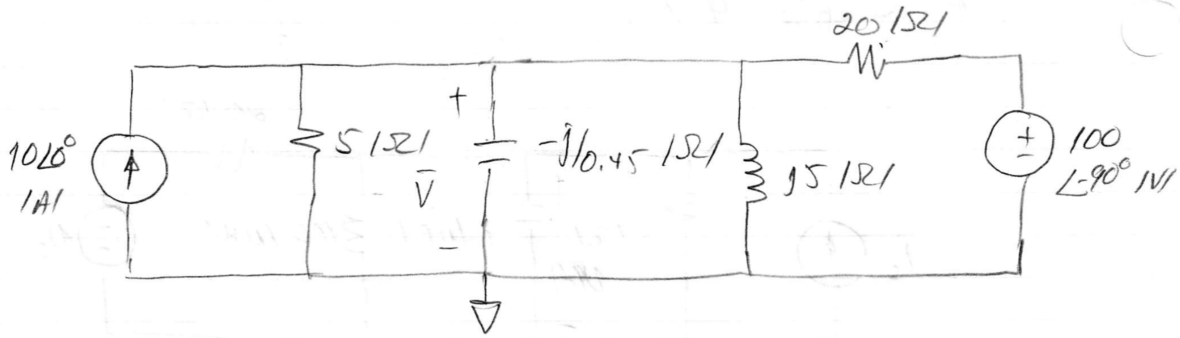
So... $\bar{I}_s = 10 \angle 0°$ |A|

$\bar{V}_s = 100 \angle -90°$ |V|

Also: $j\omega L = j(50\times10^3)(100\times10^{-6}) = j5$ |Ω|

$1/j\omega C = -j/(50\times10^3)(9\times10^{-6}) = -j/0.45$ |Ω|

So in the phasor domain this circuit is:



$$-10 + \frac{\bar{V}}{5} + \frac{\bar{V}}{(-j/0.45)} + \frac{\bar{V}}{j5} + \frac{\bar{V} - 100\angle -90^\circ}{20} = 0$$

$$\bar{V}\left(\frac{1}{5} + \frac{1}{20} + j0.45 - \frac{j}{5}\right) = 10\angle 0^\circ + 5\angle -90^\circ$$

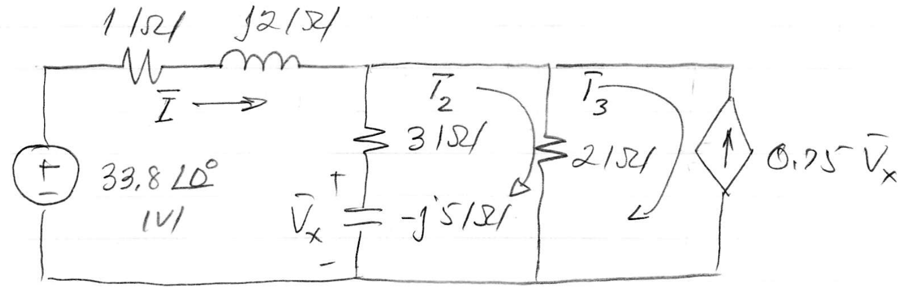
$$\bar{V} = \frac{10 + 5\angle -90^\circ}{0.25 + j0.25} = \frac{10 - j5}{0.25 + j0.25}$$

$$\bar{V} = 10 - j30 \ |V| = 31.6\angle -71.6^\circ \ |V|$$

$$\underline{v(t) = 31.6\cos(\omega t - 71.6^\circ) \ |V|}$$

Example 9.8

Use the mesh current method to find $\bar{I}$.



$$-33.8 + (1+j2)\bar{I} + (3-j5)(\bar{I}-\bar{I}_2) = 0 \qquad (1)$$

$$(3-j5)(\bar{I}_2-\bar{I}) + 2(\bar{I}_2-\bar{I}_3) = 0 \qquad (2)$$

$$\bar{I}_3 = -0.75\,\bar{V}_x \qquad (3)$$

$$\bar{V}_x = -j5(\bar{I}-\bar{I}_2) \qquad (4)$$

(3), (4) : $\bar{I}_3 = j\,3.75\,(\bar{I}-\bar{I}_2)$

(2) : $(5-j5)\,\bar{I}_2 - (3-j5)\,\bar{I} - j7.5\,(\bar{I}-\bar{I}_2) = 0$

$$\Rightarrow \left.\begin{aligned} &(5+j2.5)\,\bar{I}_2 - (3+j2.5)\,\bar{I} = 0 \\ (1) :\quad &-(3-j5)\,\bar{I}_2 + (4-j3)\,\bar{I} = 33.8 \end{aligned}\right\}$$

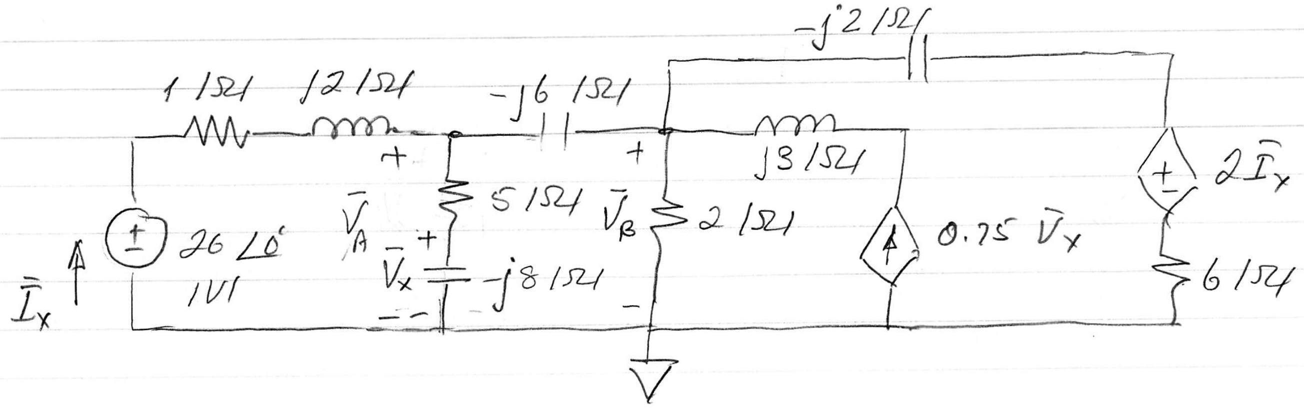
$$\bar{I}_2 = -\bar{I}\,\frac{(3+j2.5)}{5+j2.5} = (0.68+j0.16)\,\bar{I}\ \ |A|$$

$$-(3-j5)(0.68+j0.16)\,\bar{I} + (4-j3)\,\bar{I} = 33.8$$

$$\bar{I} = \frac{33.8}{-(3-j5)(0.68+j0.16)+(4-j3)}$$

$$\boxed{\bar{I} = 29.0 + j2\ |A| = 29.07\angle 3.95^\circ\ |A|}$$

EXAMPLE 9.9



This circuit is already in the phasor domain so no transformation is needed. We write the node voltage equations for $\bar{V}_A$, $\bar{V}_B$:

$$\frac{\bar{V}_A}{5-j8} + \frac{\bar{V}_A - 26\angle 0°}{1+j2} + \frac{\bar{V}_A - \bar{V}_B}{-j6} = 0$$

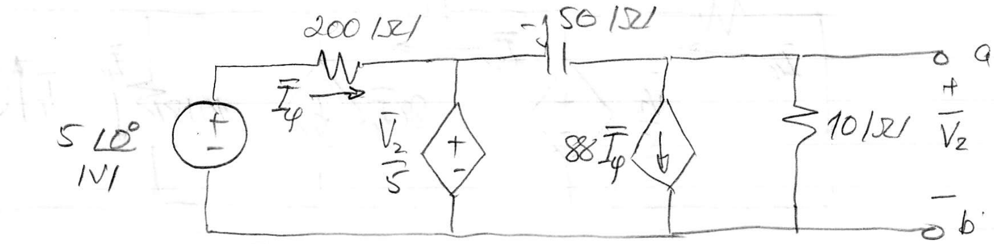
$$\frac{\bar{V}_B}{2} + \frac{\bar{V}_B - \bar{V}_A}{-j6} - 0.75\,\bar{V}_x + \frac{\bar{V}_B - 2\bar{I}_x}{6-j2} = 0$$

auxiliary:

$$\bar{I}_x = \frac{26\angle 0° - \bar{V}_A}{1+j2}$$

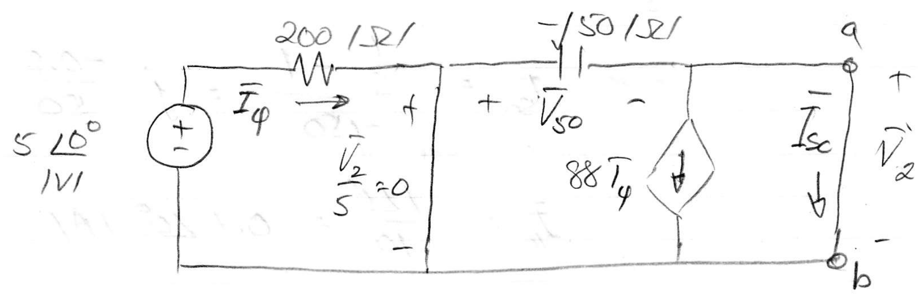
$$\bar{V}_x = \bar{V}_A \cdot \frac{-j8}{5-j8}$$

EXAMPLE 9.10



<u>Norton Equivalent</u> at a, b:

Find short-circuit current:



If we short-circuit a, b then 10 |Ω| has no current in it and $\bar{V}_2 = 0$, so the dependent voltage source is also a short,

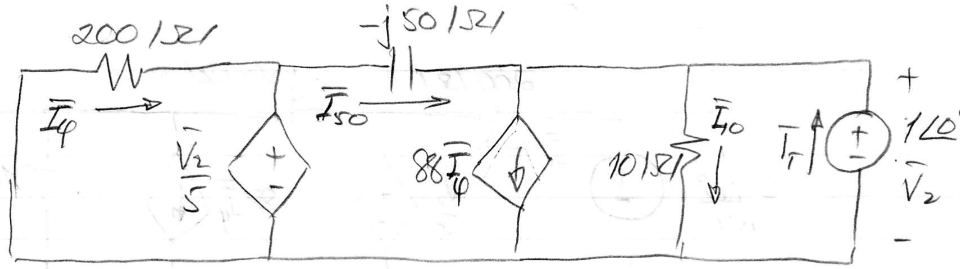
$$\therefore \quad \bar{I}_\varphi = \frac{5\angle 0°}{200} = 0.025 \angle 0° \text{ |A|}$$

Also, $\bar{V}_{50} = 0$ (KVL through shorts).

$$\therefore \quad \bar{I}_{sc} = -88\,\bar{I}_\varphi = -2.2 \angle 0° \text{ |A|}$$

Thevenin equivalent impedance:

We will use a test voltage:



$$\bar{V}_2 = 1\angle 0^\circ\ [V] \quad \Rightarrow \quad \bar{I}_\varphi = \frac{-\bar{V}_2}{1000} = -0.001\angle 0^\circ\ [A]$$

KVL: $$-\frac{\bar{V}_2}{5} + \bar{I}_{50}(-j50) + 1 = 0$$

$$\bar{I}_{50} = \frac{\frac{1}{5} - 1}{-j50} = j\,\frac{-0.8}{50} = -j0.016\ [A]$$

$$\bar{I}_{10} = \frac{1\angle 0^\circ}{10} = 0.1\angle 0^\circ\ [A]$$

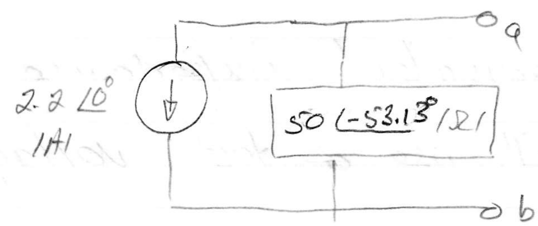
KCL: $$-\bar{I}_T - \bar{I}_{50} + 88\bar{I}_\varphi + \bar{I}_{10} = 0$$

$$\bar{I}_T = -\bar{I}_{50} + 88\bar{I}_\varphi + \bar{I}_{10} = 0.012 + j0.016\ [A]$$

$$\therefore \quad Z_{Th} = Z_N = \frac{\bar{V}_T}{\bar{I}_T} = \frac{1}{0.012 + j0.016}$$

$$= 50\angle -53.13^\circ\ [\Omega]$$

$$= 30 - j40\ [\Omega]$$



(EXAMPLE 9.10 con't)

There is something else we can do here:

The general form of the impedance is

$$Z = R + jX$$

so if $Z_N = 30 - j\,40\,|\Omega|$, we have

$$R = 30\,|\Omega| \qquad X = -40\,|\Omega|$$

Now the reactance of a capacitor (the "capacitive reactance") is

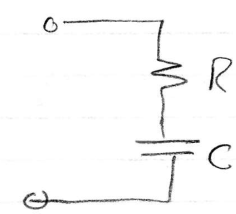
$$Z_C = -j/\omega C \quad \Rightarrow \quad X_C = -1/\omega C$$

and the "inductive reactance" is

$$Z_L = j\omega L \quad \Rightarrow \quad X_L = \omega L$$

Since X_C is negative, we can interpret Z_N as the impedance of a resistor in series with a capacitance:

$$Z_N = 30 - j\,40\,|\Omega|$$



$$X = -4 = -1/\omega C$$

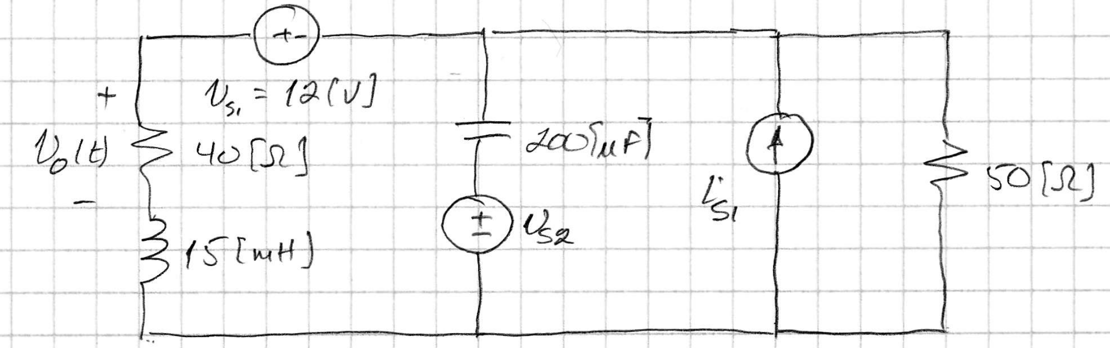
$$\therefore \; C = 1/\omega \cdot 40$$

We were not given a frequency here, but say

$$\omega = 10{,}000\,|\text{rad}/\text{s}| \quad \Rightarrow \quad C = 2.5\,|\mu F|$$

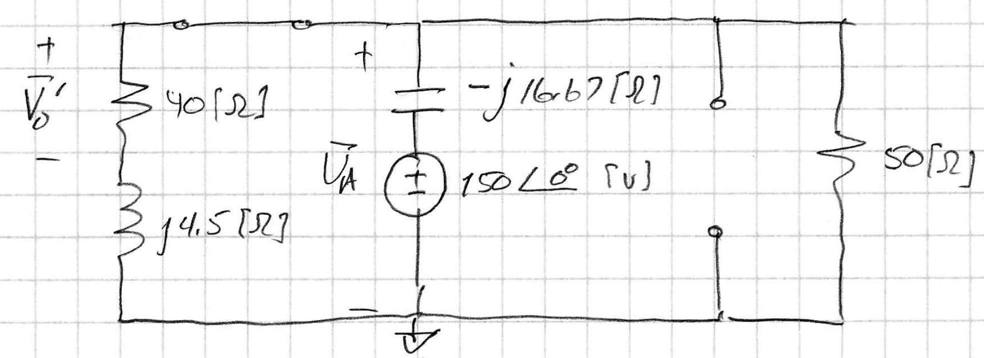
Problem 9.11

This problem requires superposition to solve, because the sources are at different frequencies.



$v_{S2} = 150\cos(300t)$ [V] $\qquad i_{S1} = 2\cos(100t)$ [A]

Consider v_{S2} first $\Rightarrow v_{S1} \to$ short, $i_{S1} \to$ open ckt



For $v_s(t)$, $\omega = 300\left[\frac{rad}{s}\right] \Rightarrow L \to j\omega L = j4.5[\Omega]$

$C \to -j/\omega C = -j16.67[\Omega]$

$$\frac{\bar{V}_A - 150}{-j16.67} + \frac{\bar{V}_A}{50} + \frac{\bar{V}_A}{40+j4.5} = 0$$

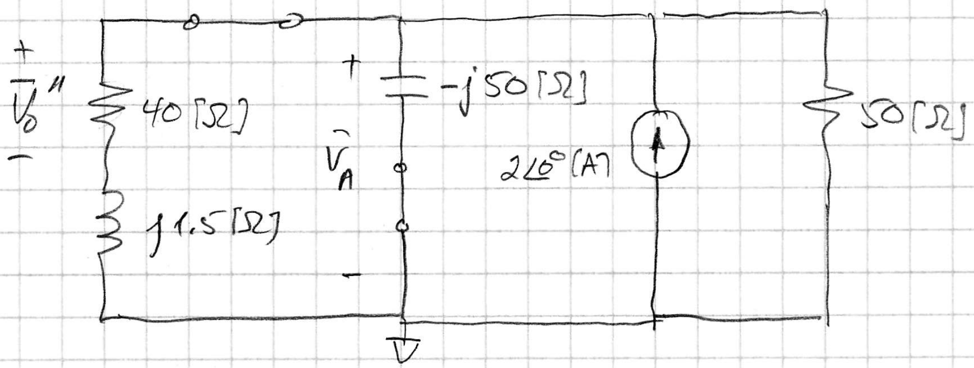
$\bar{V}_A = 123.96\angle 38.00°$ [V] $\qquad \bar{V}_o' = \bar{V}_A \dfrac{40}{40+j4.5} = 123.2\angle 31.58°$ [V]

↗

Consider now $i_{s1}(t) \Rightarrow \omega = 100\,[\frac{rad}{s}]$

$L \rightarrow j\omega L = j1.5\,[\Omega]$ $\quad C \rightarrow -j/\omega C = -j50\,[\Omega]$

$v_{s2} \rightarrow$ short, $\quad v_{s1} \rightarrow$ short

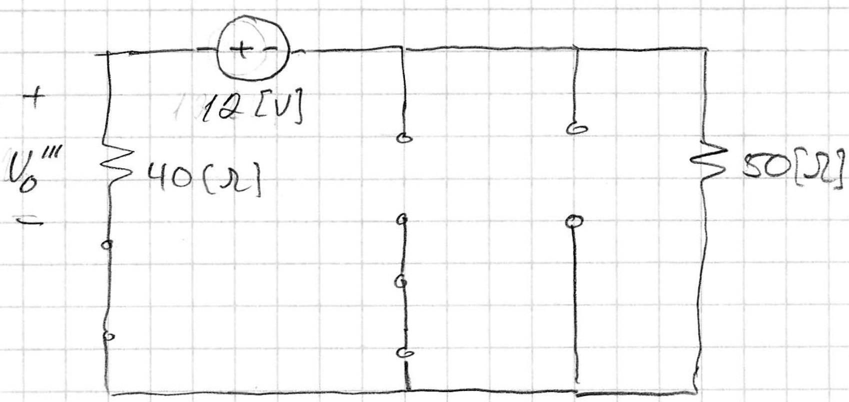


$$\frac{\bar{V}_A}{-j50} + \frac{\bar{V}_A}{40 + j1.5} - 2 + \frac{\bar{V}_A}{50} = 0$$

$\bar{V}_A = 40.95\angle -22.98^\circ\,[V]$ $\qquad \bar{V}_o'' = \bar{V}_A \cdot \dfrac{40}{40+j1.5} = 40.92\angle -25.13^\circ\,[V]$

v_{s1} is a dc source, for which we take $\omega = 0$. In that case, $L \rightarrow$ short, $C \rightarrow$ open circuit.

Also, $v_{s2} \rightarrow$ short, $i_{s1} \rightarrow$ open.



M

Since there are no reactive components left, we can think of this entirely as a time-domain circuit, but we will contine to phasor notation for consistency.

$$\overline{V}_o''' = 12 \cdot \frac{40}{40+50} = 5.33 \text{ [V]}$$

We now transform back to the time domain, but we have to do it separately for each component of V_o. Adding phasors does not make sense because they were calculated using different frequencies. So...

$v_o(t) = 123.2 \cos(300t + 31.58°)$ [V]

from v_{s2} $\quad$ $+ 40.92 \cos(100t - 25.13°)$ [V]

from i_{s1} $\quad$ $+ 5.33$ [V]