EXAMPLE 9.1

For the function $v(t)$, find the indicated quantities.

$$v(t) = 40 \cos(2513.27t + 36.87°) \; [V]$$

frequency a) $f = \omega/2\pi = 2513.27/2\pi = 400 \; [Hz]$

period b) $T = 1/f = 0.002500 = 2.5 \; [ms]$

amplitude c) $V_m = 40 \; V$

$v(t=0)$ d) $v(0) = 40 \cos(36.87°) = 32 \; [V]$

phase e) $\varphi = 36.87° \longrightarrow \frac{2\pi}{360} \times 36.87 = 0.643 \; [rad]$

f) time at which $v(t) = 0$

$$\cos(2513.27t + 36.87°) = 0$$

$$2513.27t + 0.643 = 1.5708$$

$$t = \frac{1.5708 - 0.643}{2513.27} = 368.96 \; [\mu s]$$

g) time at which $\frac{dv}{dt} = 0$

$$dv/dt = -40(2513.27) \sin(2513.27 + 36.87°) = 0$$

$$2513.27t + 0.643 = 0$$

$$t = -255.84 \; [\mu s]$$

or

$$2513.27t + 0.643 = 3.1415$$

$$t = 994.16 \; [\mu s]$$

Example 9.2

Find the phasor transform of each time-domain function.

a) $v(t) = 170 \cos(377t - 40°)$ [V]

$v(t) = \text{Re}\{170 e^{j377t} e^{-j40°}\}$

$\therefore \quad \bar{V} = 170 \angle -40°$ [V]

b) $i(t) = 10 \sin(1000t + 20°)$ [A]

Since our transform are being defined in terms of cosine and not sine, we need to convert:

$i(t) = 10 \cos(1000t + 20° - 90°)$ [A]

$= 10 \cos(1000t - 70°)$ [A]

$\therefore \quad \bar{I} = 10 \angle -70°$ [A]

c) $i(t) = 5\cos(\omega t + 36.87°) + 10 \cos(\omega t - 53.13°)$ [A]

$\bar{I} = 5\angle 36.87° + 10 \angle -53.13°$ [A]

We could add these directly with a calculator but let's do it manually to see what's involved.

$5 \angle 36.87° = 5 \cos 36.87° + j5 \sin 36.87°$

$= 4.0 + j3.0$

$10\angle{-53.13^\circ} = 10\cos(-53.13^\circ) + j\,10\sin(-53.13^\circ)$

$= 6.0 - j\,8.0$

Adding real and imaginary parts gives

$\bar{I} = (4.0 + j\,3.0) + (6.0 - j\,8.0) = 10.0 - j\,5.0 \quad [A]$

Now we transform back:

$\phi = \tan^{-1} {-5}/{10} = -26.57^\circ$

$r = \sqrt{10^2 + 5^2} = 11.18 \ [A]$

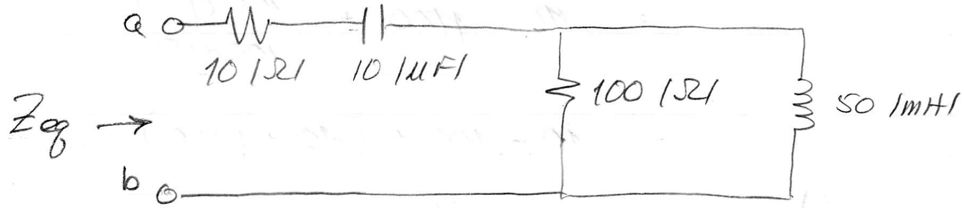
$\therefore \quad \bar{I} = 11.18\angle{-26.57^\circ} \quad [A]$

we can go back to the time domain:

$i(t) = 11.18\cos(\omega t - 26.57^\circ) \ [A]$

Example 9.3

Find the equivalent impedance at a, b.



<u>Solution</u> : Transform to phasor domain:

$$C \rightarrow 1/j\omega C = -j/\omega C$$

$$L \rightarrow j\omega L$$

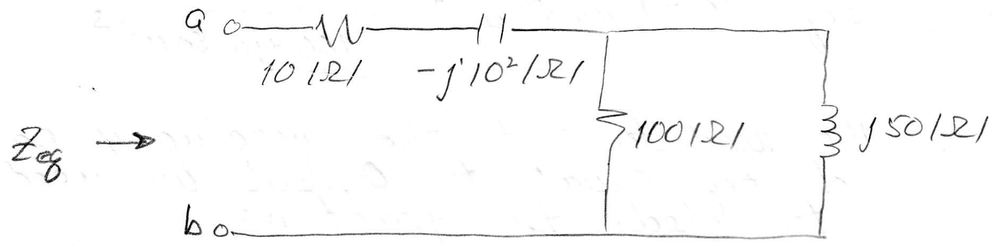
$$R \rightarrow R$$

a) $\omega = 1000$ [rad/s]

10 [μF] : $-j/\omega C = -j/(1000 \cdot 10\times10^{-6}) = -j\,10^2$ [Ω]

50 [mH] : $j\omega L = j(1000 \cdot 50\times10^{-3}) = j\,50$ [Ω]

so we have



and $Z_{eq} = 10 + (-j10^2) + 100 \,||\, j50$

$= 10 - j100 + \dfrac{100(j50)}{100 + j50}$

$= 10 - j100 + (20 + j40)$

$= 30 - j60 \; [\Omega]$

$\omega = 4000 \left[\frac{rad}{s}\right]$

b) $10\,[\mu F]$: $-j/\omega C = -j/(4000 \cdot 10\times10^{-6}) = -j25 \; [\Omega]$

$50\,[mH]$: $j\omega L = j(4000 \cdot 10\times10^{-6}) = j200 \; [\Omega]$

$Z_{eq} = 10 - j25 + \dfrac{100(j200)}{100 + j200}$

$= 10 - j25 + (80 + j40)$

$= 90 + j15 \; [\Omega]$

At what ω is Z_{eq} purely resistive?

c) Purely resistive $\Rightarrow$ $Z = R + \cancel{jX}^{\,0}$ i.e. there is no imaginary part. Thus

$Z_{eq} = 10 - j/10\times10^{-6}\omega + \dfrac{100(j\omega\, 50\times10^{-3})}{100 + j\omega\, 50\times10^{-3}}$

We need to set the imaginary part of Z_{eq} equal to 0, but we need to clear the fraction:

(Example 9.3 con't)

$$\frac{100(j\omega 50\times10^{-3})}{100+j\omega 50\times10^{-3}}\cdot\frac{100-j\omega 50\times10^{-3}}{100-j\omega 50\times10^{-3}}$$

$$= \frac{j\omega 5\cdot(100-j\omega 50\times10^{-3})}{100^2+(\omega 50\times10^{-3})^2}$$

$$= \frac{j\omega 500+\omega 0.250}{10^4+\omega^2 2.5\times10^{-3}}$$

So $\Im m\{Z_{eq}\} = -1/\omega 10\times10^{-6} + \dfrac{\omega 500}{10^4+\omega^2 2.5\times10^{-3}} = 0.$

$$-(10^4+\omega^2 2.5\times10^{-3}) + \omega^2 5.0\times10^{-3} = 0$$

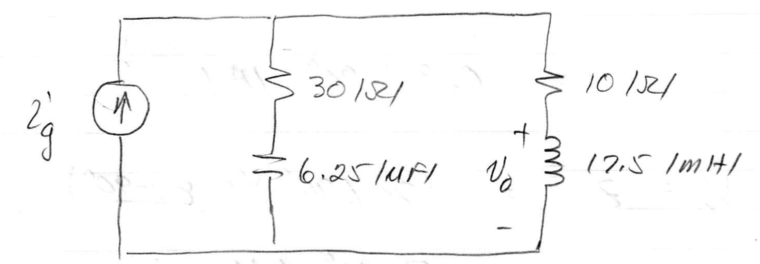
$$\omega^2(2.5\times10^{-3}) = 10^4$$

$$\omega^2 = 4\times10^6$$

$$\underline{\underline{\omega = 2\times10^3\ \mathrm{rad/s}}}$$

Example 9.4

Find $v_o(t)$.



GIVEN: $i_g = 0.8 \cos 4000t$ [A]

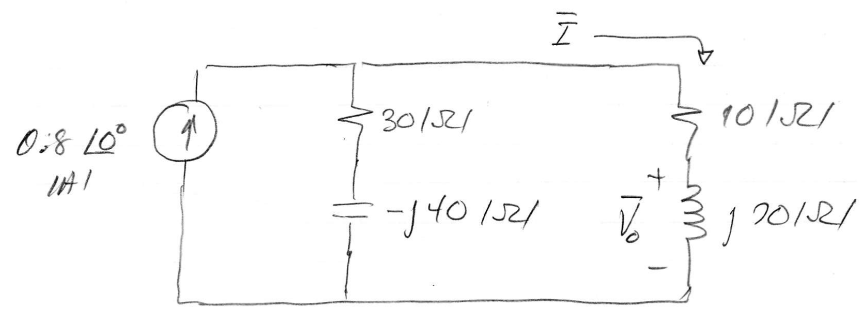
SOLUTION: Transform to phasor domain:

i_g: $\bar{I}_g = 0.8 \angle 0^\circ$ [A]

C : $-j/\omega C = -j/(4000 \times 6.25 \times 10^{-6}) = -j40$ [Ω]

L : $j\omega L = j(4000 \times 17.5 \times 10^{-3}) = j70$ [Ω]

so we have:



$$\bar{I} = 0.8\angle 0^\circ \cdot \frac{30 - j40}{(10 + j70) + (30 - j40)}$$

$$\bar{I} = 0.8 \angle 0^\circ \cdot \frac{30 - j40}{40 + j30}$$

$$= 0.8 \angle -90^\circ \; |A|$$

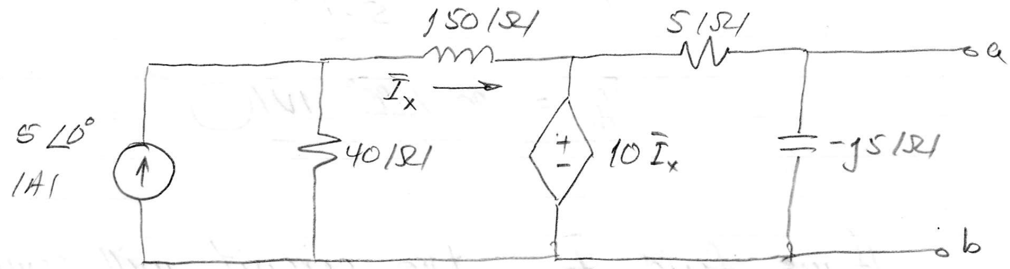
$$\bar{V}_o = Z\bar{I} = (70\angle 90^\circ)(0.8\angle -90^\circ)$$

$$= 56 \angle 0^\circ \; |V|$$

Inverse Transform:

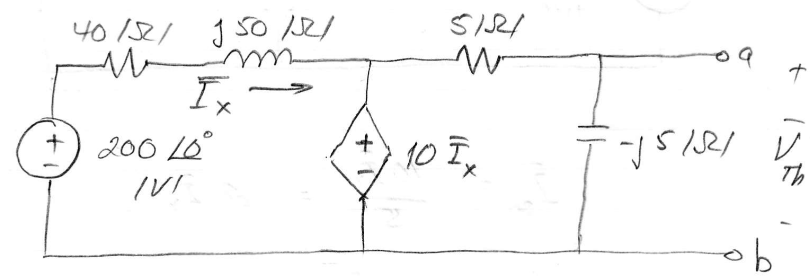
$$\underline{v_o(t) = 56 \cos(4000t + 0^\circ) \; |V|}$$

Example 9.5



Find the Thevenin Equivalent circuit at a, b.

Source transformation:



Voltage divider rule applies here:

$$\bar{V}_{Th} = 10\,\bar{I}_x \cdot \frac{-j5}{5-j5} \qquad (1)$$

$$-\bar{I}_x = \frac{10\,\bar{I}_x - 200}{40+j50} \qquad (2)$$

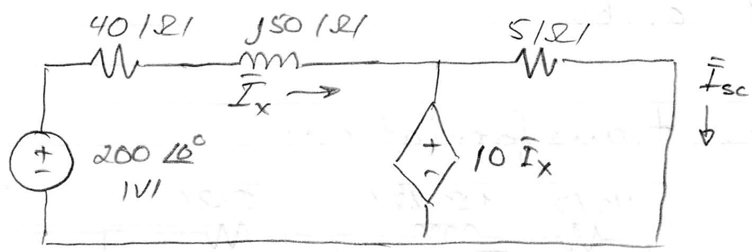
$$(2) \Rightarrow -(40+j50)\,\bar{I}_x = 10\,\bar{I}_x - 200$$

$$\bar{I}_x = \frac{80-j100}{45-j5} = 2.83\ \angle{-45^\circ}\ |A|$$

Then $\qquad \bar{V}_{Th} = \dfrac{(28.3\angle{-45^\circ})(5\angle{-90^\circ})}{5-j5}$

$$\underline{\bar{V}_{Th} = 20\angle{-90^\circ}\ |V|}$$

If we find $\bar{I}_{sc}$, the circuit will simplify because $-j5$ will be shorted out:



$$\bar{I}_{sc} = \frac{10\bar{I}_x}{5} = 2\bar{I}_x$$

$$-\bar{I}_x = \frac{10\bar{I}_x - 200}{40 + j50} \quad \text{(same as before)}$$

$$= 2.83\angle{-45^\circ}\ |A|$$

$$\bar{I}_{sc} = 5.66\angle{-45^\circ}\ |A|$$

$$\therefore \quad \underline{\underline{Z_{Th}}} = \frac{\bar{V}_{Th}}{\bar{I}_{sc}} = \frac{20\angle{-90^\circ}}{5.66\angle{-45^\circ}} = \underline{2.5 - j2.5\ |\Omega|}$$

Note the use of Z_{Th} as a generalization of R_{Th}.