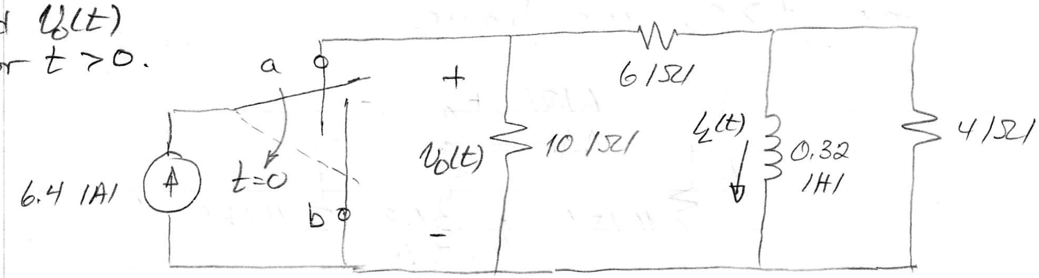
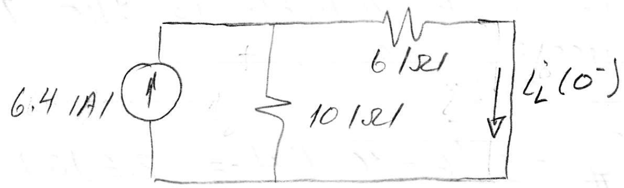
PROBLEM 7.1

a) Find $v_o(t)$ for $t > 0$.



a) For $t < 0$ the inductor acts like a short, so the equivalent circuit is

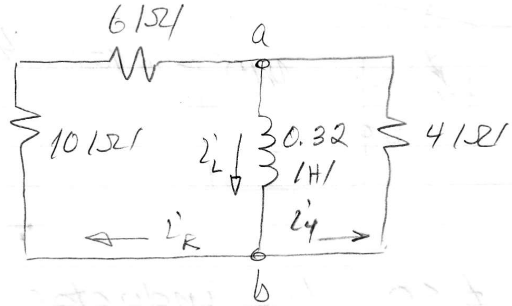


We solve for current in the inductor first, then find $v_o(t)$.

Reason: We can find inductor current for $t = 0^+$ because we know there is no instantaneous change in inductor current. We have no such restriction on v_o so finding v_o for $t = 0^+$ is tricky.

So, $i_L(0^-) = i_L(0^+) = 6.4 \cdot \frac{10}{10+6} = 4\ [A]$

For $t > 0$, we have



Inductor "sees" a resistance at the terminals a, b which is the Thevenin resistance:

$$R_{Th} = (6+10) \parallel 4 = 3.2 \,|\Omega|$$

$$\therefore \quad \tau = L/R_{Th} = 0.32/3.2 = 0.1 \,|s|$$

$$i_L(0^+) = i_L(0^-) = I_0, \quad \text{and} \quad i'_L(t) = I_0 e^{-t/\tau}$$

$$\therefore \quad i'_L(t) = 4e^{-10t} \,|A| \quad t \geq 0 \,|s|$$

Now, CDR $\Rightarrow$

$$i'_R = i'_L \cdot \frac{4}{16+4} = 0.8e^{-10t} \,|A|$$

So $\underline{\underline{v_o(t) = -10\,i'_R = -8\,e^{-10t} \,|V| \quad t > 0 \,|s|}}$

(PROBLEM 7.1 con't)

Let's look at $v_o(0^-)$ and $v_o(0^+)$:

Note that before $t = 0$, we had

$$v_o(0^-) = 10\left[6.4 \times \frac{6}{16}\right] = 24.0 \text{ [V]}$$

This is $V_o(0^-)$. From our solution above,

$$V_o(0^+) = -8 \text{ [V]}$$

So clearly $V_o(0^-) \neq V_o(0^+)$!

b) Find the percent of the initial inductor energy that is dissipated in the 4 [Ω] resistor.

$$w_L = \frac{1}{2} L\, i_L(0^+)^2 = \frac{1}{2}(0.32)(4)^2 = 2.56 \text{ [J]}$$

In 4 [Ω] resistor

$$i_4 = i_L \frac{16}{20} = 3.2e^{-10t} \text{ [A]} \quad t \geq 0 \text{ [s]}$$

$$\therefore P_{del\,to\,R} = i_4^2 R = (10.24)(4)e^{-20t} \text{ [W]} \quad t \geq 0 \text{ [s]}$$

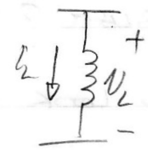
Total energy dissipated in 4 [Ω] is

$$W = \int_0^\infty 40.96e^{-20t}\,dt = 2.048 \text{ [J]}$$

So % dissipated in 4 [Ω] is

$$\frac{2.048}{2.56} = 0.8 \Rightarrow \underline{\underline{80\%}}.$$

Another approach:



$$v_L(t) = L\frac{di_L}{dt} = 0.32\frac{d}{dt}(4e^{-10t})$$

$$= -3.2(4)e^{-10t}$$

$$= -12.8e^{-10t} \quad |V| \qquad t > 0 \ |s|$$

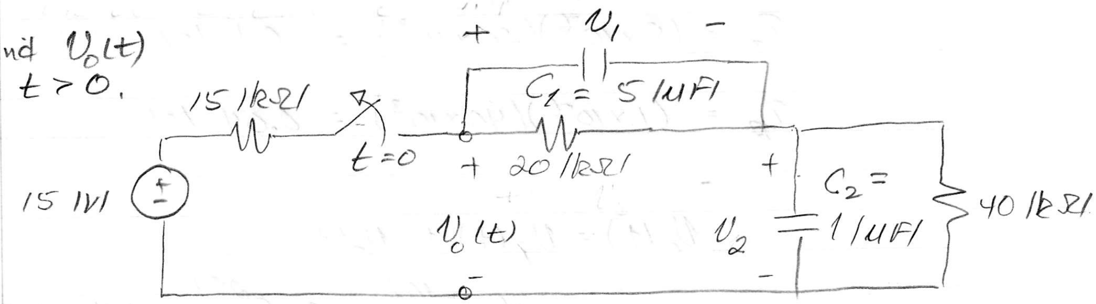
Now by VDR:

$$v_o = v_L \cdot \frac{10}{10+6}$$

$$= -8e^{-10t} \quad |V| \qquad t > 0 \ |s|$$

PROBLEM 7.2

a) Find $v_o(t)$ for $t > 0$.

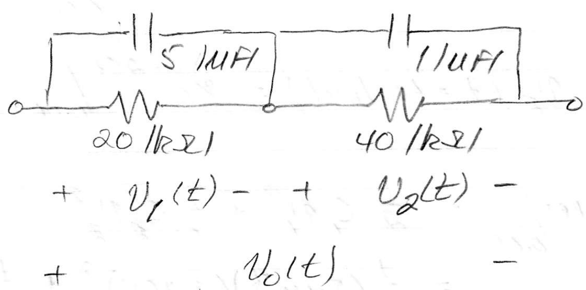


a) For $t<0$, there is no current in the capacitors, so

$$v_1(0^-) = v_1(0^+) = 15 \cdot \frac{20}{20+40+15} = 4 \text{ [V]} \equiv V_{01}$$

$$v_2(0^-) = v_2(0^+) = 15 \cdot \frac{40}{20+40+15} = 8 \text{ [V]} \equiv V_{02}$$

Now for $t \geq 0$, we have



We can treat this problem as two separate RC circuits, i.e.

$\tau_1 = (5\times10^{-6})(20\times10^{3}) = 0.1$ [s]

$\tau_2 = (1\times10^{-6})(40\times10^{3}) = 0.04$ [s]

$$v_o(t) = v_1(t) + v_2(t)$$

$$= 4e^{-10t} + 8e^{-25t} \quad [V] \quad t \geq 0 \text{ [s]}$$

b) Find the percent of the initial energy in the capacitors that is dissipated after 60 [ms].

$$W = \tfrac{1}{2}C_1V_{01}^2 + \tfrac{1}{2}C_2V_{02}^2$$

$$= \tfrac{1}{2}(5\times10^{-6})(4)^2 + \tfrac{1}{2}(1\times10^{-6})(8)^2$$

$$= 7.2\times10^{-5} \text{ [J]}$$

After 60 [ms] :

$$v_1(t=60\text{ [ms]}) = 4e^{-10t}\Big|_{0.06} = 2.195 \text{ [V]}$$

$$v_2(t=60\text{ [ms]}) = 8e^{-25t}\Big|_{0.06} = 1.785 \text{ [V]}$$

$$\therefore\ w\Big|_{0.06} = \tfrac{1}{2}C_1v_1 + \tfrac{1}{2}C_2v_2^2$$

$$= \tfrac{1}{2}(5\times10^{-6})(2.195)^2 + \tfrac{1}{2}(1\times10^{-6})(1.785)^2$$

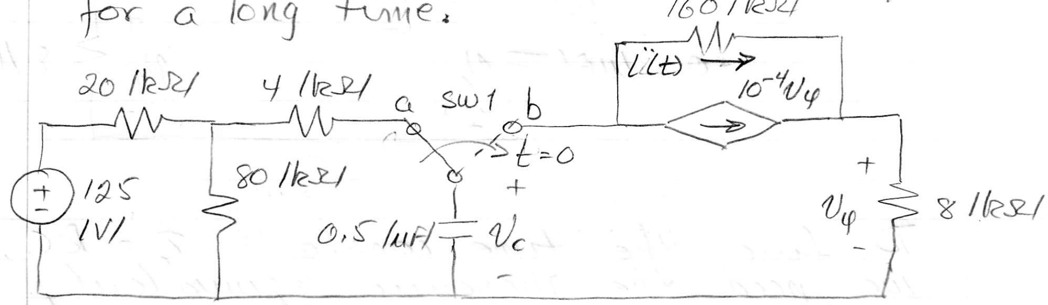
$$= 1.364\times10^{-5} \text{ [J]}$$

So percent dissipated is

$$1 - \frac{1.364}{7.200} = \underline{\underline{81.06\%}}$$

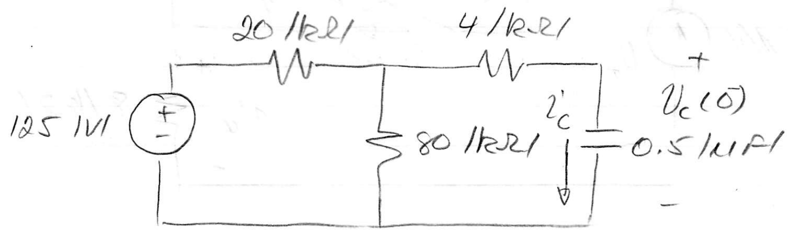
PROBLEM 7.3

For the circuit shown, find $v_c(t)$ and $i(t)$ for $t > 0$. SW1 was in position a for a long time.



We will find $v_c(t)$ first, and use this information to find $i(t)$.

Find initial conditions:

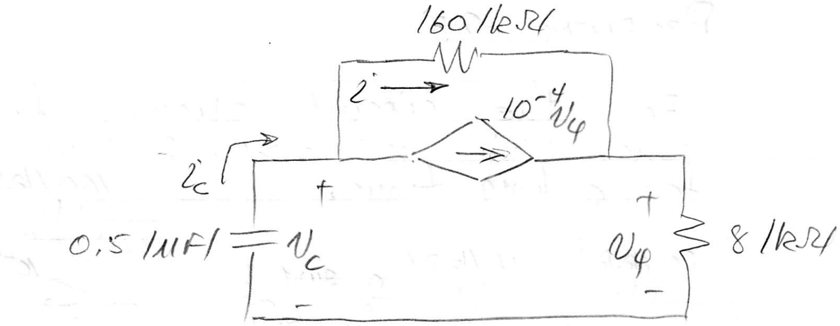


Since SW1 was in position a for a long time, $i_c = 0$, so

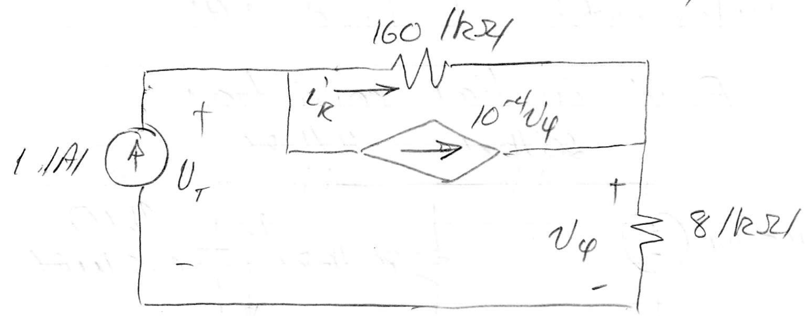
$$v_c(0^+) = v_c(0) = 125 \cdot \frac{80 \times 10^3}{(80+20) \times 10^3} = 100\ [V]$$

Now for $t > 0$ ...

$t > 0$



To find the time constant $\tau_c = RC$, we need the Thevenin equivalent resistance as seen at the capacitor terminals: Use a test source for this:



Thus $v_\varphi = 8 \times 10^3 \cdot [V]$

$$i_R + 10^{-4} v_\varphi = 1$$

$$i_R = 1 - 0.8 = 0.2 \ [A]$$

$$\therefore \ v_T = 160 \times 10^3 \, i_R + v_\varphi$$

$$= 3.2 \times 10^4 + 8 \times 10^3 = 4 \times 10^4 \ [V]$$

$$\therefore \ R_{Th} = v_T/1 = 40 \ [k\Omega]$$

PROBLEM 7.3

$$\tau = R_{Th} \cdot C = (4\times10^4)(0.5\times10^{-6}) = 20 \,[ms]$$

Thus

$$v_c(t) = 100\, e^{-t/20\times10^{-3}} \quad [V]$$

$$\Rightarrow \quad \boxed{v_c(t) = 100\, e^{-50t} \quad [V] \quad t \geq 0\, [s]}$$

Now referring back to the $t > 0$ figure,

$$i_c = -C \frac{dv_c}{dt}$$

$$= -(5\times10^{-7})(-5000)\, e^{-50t}$$

$$= 0.0025\, e^{-50t} \quad [A] \quad t > 0\, [s]$$

Thus $\quad v_\varphi = 8000\, i_c = 20\, e^{-50t} \,[V]$

$$i(t) + 10^{-4} v_\varphi = i_c$$

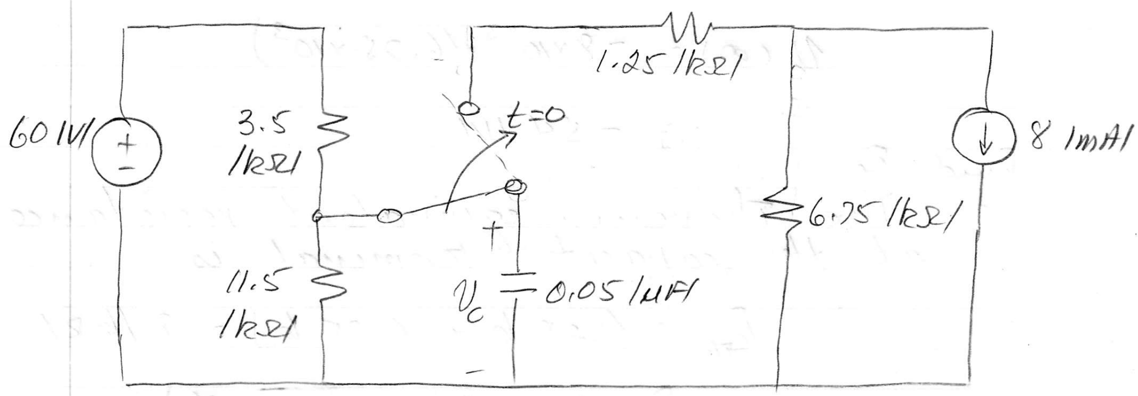
$$\therefore \; i(t) = i_c - 10^{-4}(20\, e^{-50t})$$

$$= (2.5 - 2.0)\times10^{-3} \times e^{-50t}$$

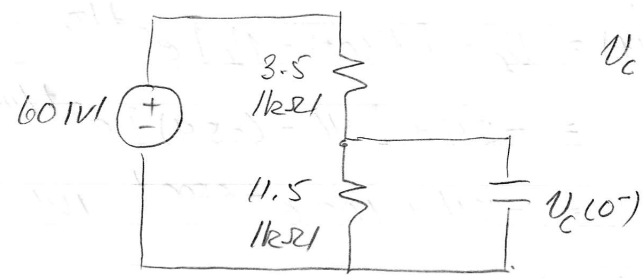
$$\boxed{i(t) = 5\times10^{-4}\, e^{-50t} \quad [A] \quad t \geq 0\, [s]}$$

PROBLEM 7.4

a) Find $v_C(0^+)$.



a) For $t<0$, we have



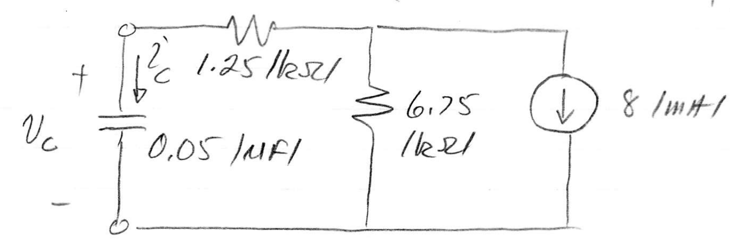
$$v_C(0^-) = 60 \cdot \frac{11.5}{11.5+3.5}$$

$$= 46.0 \ [V]$$

So $v_C(0^+) = v_C(0^-) = 46 \ [V]$

b) Find $v_C(\infty)$, i.e., the steady state value of v_C, after $t=0$.

$t>0$



After a long time, $i_c \to 0$, so

$$v_c(\infty) = -8\times10^{-3}\cdot(6.75\times10^{3})$$

$$= -54 \,|V|$$

c) Find τ_c.

The Thevenin equivalent resistance at the capacitor terminals is

$$R_{Th} = (1.25\,k + 6.75\,k) = 8\,|k\Omega|$$

$$\therefore \quad \tau = R_{Th}C = 8000\,(0.05\times10^{-6})$$

$$= 400\,|\mu s|.$$

d) Find $v_c(t)$.

$$v_c(t) = v_f + [v(0^+) - v_f]\,e^{-t/\tau_c}$$

$$= -54 + [46 - (-54)]\,e^{-t/400\times10^{-6}}$$

$$= -54 + 100\,e^{-2500t} \quad |V| \quad t \geq 0\,|s|$$

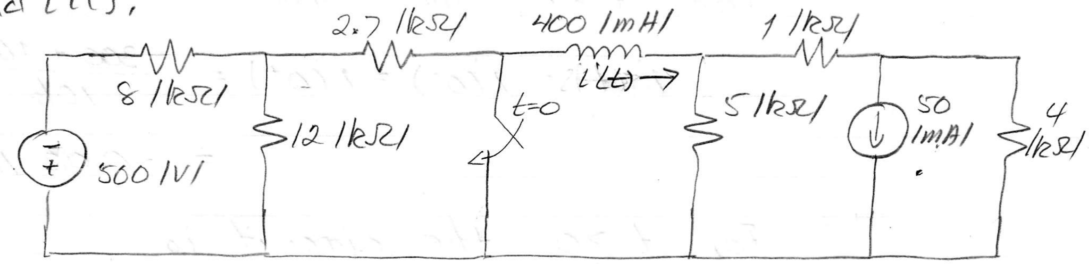
So $\quad v_c(t) = 0$

$$\Rightarrow \quad -54 + 100\,e^{-2500t} = 0$$

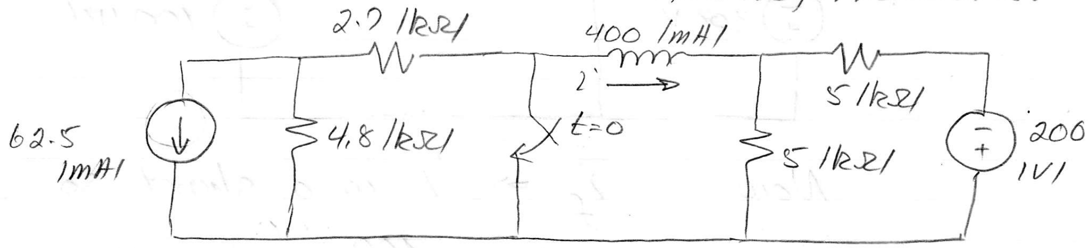
$$t = -\frac{\ln(54/100)}{2500} = 246.5\,|\mu s|$$

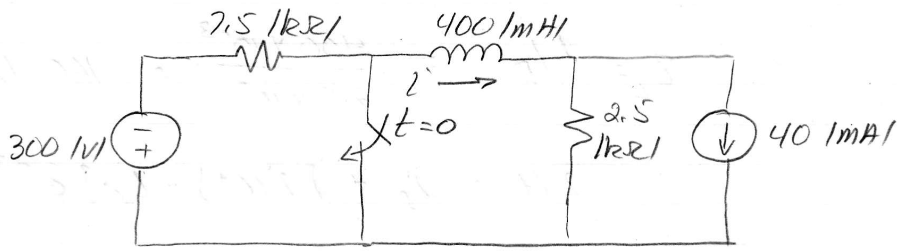
PROBLEM 7.5

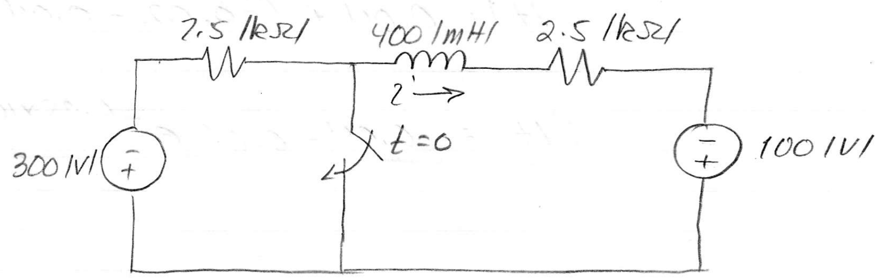
Find i(t).



This looks like a job for source transformations!





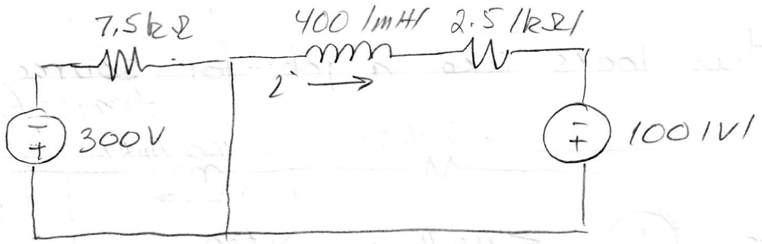


For $t < 0$, we have

$$i(t) = i(0^-) = i(0^+) = \frac{-300 + 100}{10k}$$

$$= -0.02 \ /A/$$

For $t > 0$ the circuit is



Now $i_f \Rightarrow L$ is a short so

$$i_f = \frac{100}{2.5k} = 0.04 \ /A/$$

$$\tau_L = L/R = \frac{400 \times 10^{-3}}{2.5 \times 10^3} = 160 \ /\mu s/$$

$$i(t) = i_f + [i(0^+) - i_f] e^{-t/\tau_L}$$

$$i(t) = 0.04 + (-0.02 - 0.04) e^{-t/160 \times 10^{-6}}$$

$$/A/ \quad t \geq 0 /s/$$

$$\underline{i(t) = 0.04 - 0.06 e^{-6.25 \times 10^3 t} \quad /A/ \quad t \geq 0 /s/}$$