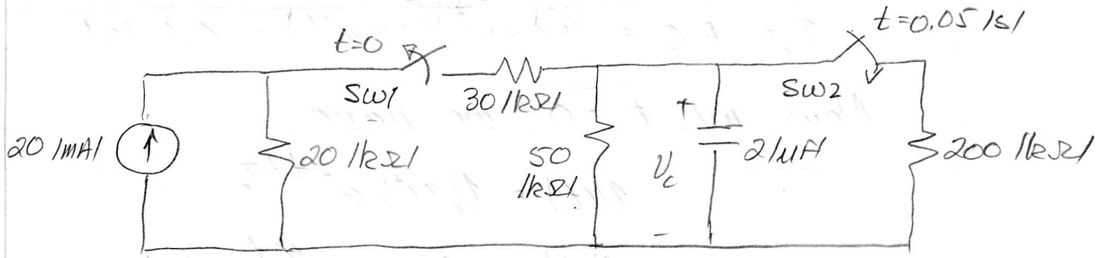
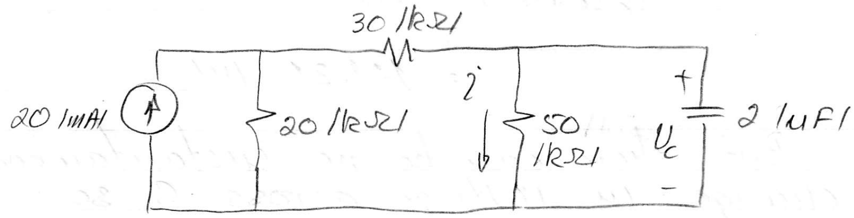
PROBLEM 7.6



SW1 has been closed and SW2 open for a long time.

a) Find $V_c(t)$ for $0 \le t \le 0.05$ |s|

Draw circuit for $t < 0$:



C is an open circuit, so $V_c = 50 \times 10^3 \cdot i$

CDR: $\quad i = (20 \times 10^{-3}) \cdot \dfrac{20k}{20k + 80k} = 4 \times 10^{-3}$ |A|

$\therefore \quad V_c(0^-) = V_c(0^+) = i \cdot 50$ |kΩ| $= 200$ |V|

Re-draw for $0 \le t \le 0.05$ |s| :

$\tau_c = RC = (50\times10^3)(2\times10^{-6}) = 0.1\ |s|$

Now w/ $t_0 = 0$ we have

$$v(t) = v_c(0^+)\, e^{-t/\tau}$$

$$\underline{v(t) = 200\, e^{-10t}}\ |V|,\quad 0 \le t \le 0.05\ |s|$$

b) Find $v_c(t)$ for $t \ge 0.05\ |s|$

Just before $t = 0.05\ |s|$, we have

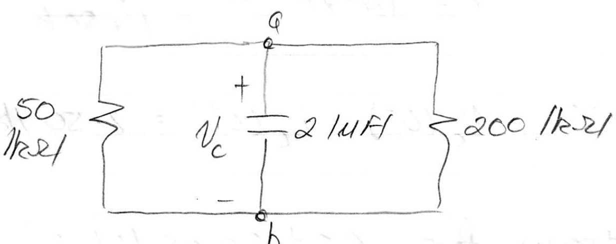
$$v_c(0.05^-) = 200\, e^{-10(0.05)}$$

$$= 121.31\ |V|$$

But there can be no instantaneous change in voltage across C so

$$v_c(0.05^+) = 121.31\ |V|$$

Now for $t \ge 0.05\ |s|$ the circuit is



Now R at the capacitor terminals is

$$R = 200k \parallel 50k = 40\ |k\Omega|$$

$\therefore\ \tau_c = RC = 0.08\ |s|$

(PROBLEM 7.6 con't)

So ... $v_c(t) = 121.31 e^{-(t-0.05)/0.08}$ |V|,

$t \geq 0.05$ |s|

c) Find total energy dissipated in 50 |kΩ|

$$P_{abs} = v_c^2 / 50\times10^3$$

$$W = \int_0^{0.05} \frac{(200)^2}{50\times10^3} e^{-20t}\, dt + \int_{0.05}^{\infty} \frac{(121.31)^2}{50\times10^3} e^{-25(t-0.05)}\, dt$$

$$= 0.8 \int_0^{0.05} e^{-20t}\, dt + 0.294 e^{1.25} \int_{0.05}^{\infty} e^{-25t}\, dt$$

$$= -4\times10^{-2} e^{-20t}\Big|_0^{0.05} - \frac{1.03}{25} e^{-25t}\Big|_{0.05}^{\infty}$$

$= 25.28$ |mJ| $+ 11.75$ |mJ|

$= 37.03$ |mJ|

d) Find total energy dissipated in 200 |kΩ|

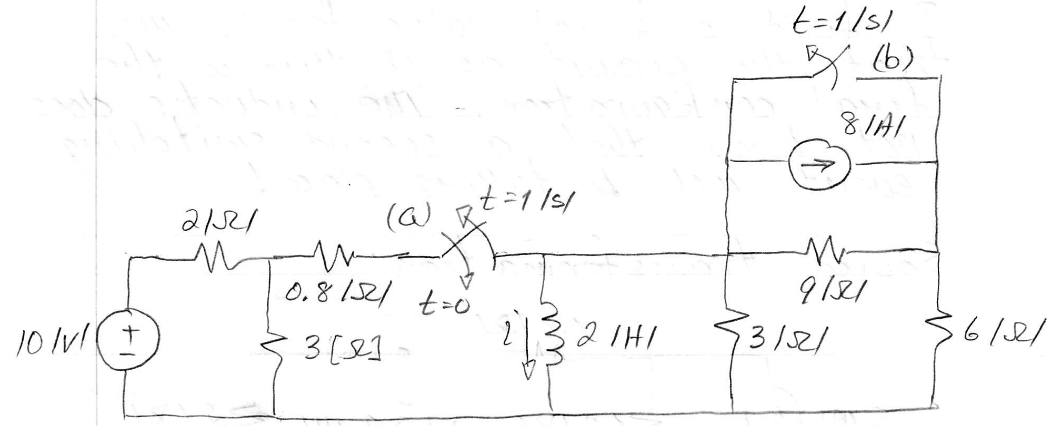
$$W = \int_{0.05}^{\infty} \frac{(121.31)^2}{200\times10^3} e^{-25(t-0.05)}\, dt$$

$$= (7.36\times10^{-2}) e^{1.25} \int_{0.05}^{\infty} e^{-25t}\, dt$$

$$= \frac{-0.257}{25} e^{-25t}\Big|_{0.05}^{\infty}$$

$= 2.94$ |mJ|

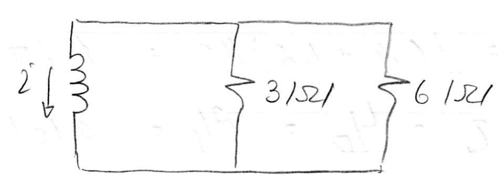
PROBLEM 7.7



Switch a was open and b closed for a long time.

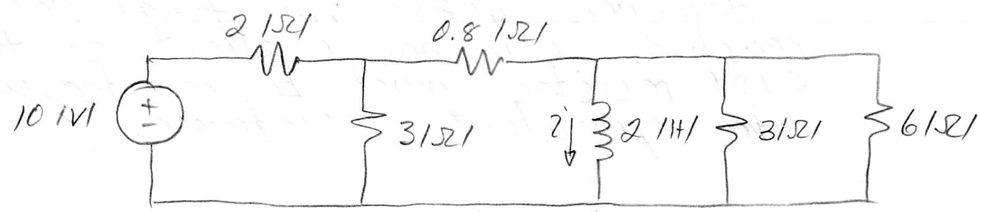
a) Find $i(t)$ for $0 \leq t \leq 1$ /s/

For $t < 0$ we have



So $i(0^-) = i(0^+) = 0$.

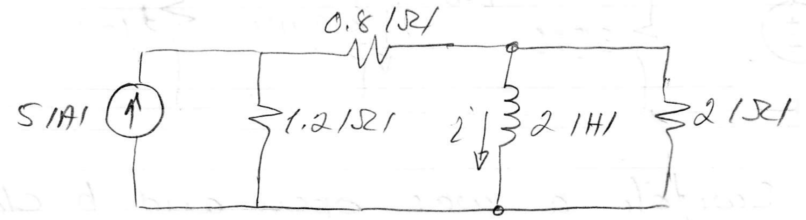
For $0 \leq t \leq 1$ /s/ we re-draw:



Note: This is a step-response problem!

To find a final value for i, we treat the circuit as if this is the final configuration. The inductor does not know that a second switching event will be taking place!

Source transformation:



"Final value" ⇒ L becomes a short.

Then,

$$i_f = 5 \cdot \frac{1.2}{1.2+0.8} = 3\ |A|$$

$$R = (0.8+1.2) \| 2 = 1\ |\Omega|$$

$$\therefore \quad \tau_L = L/R = 2/1 = 2\,|s|$$

<u>Note</u>: In calculating i_f, L is a short so the 2 |Ω| resistor is shorted out.

However, before the final value is reached, L is not a short, so the 2 |Ω| resistor must be counted in the equivalent resistance.

(PROBLEM 7.? con't)

Finally,

$$i(t) = i_f + [\, i(0^+) - i_f \,]\, e^{-t/\tau_L}$$

$$i(t) = 3 + (0-3)\, e^{-t/2}$$

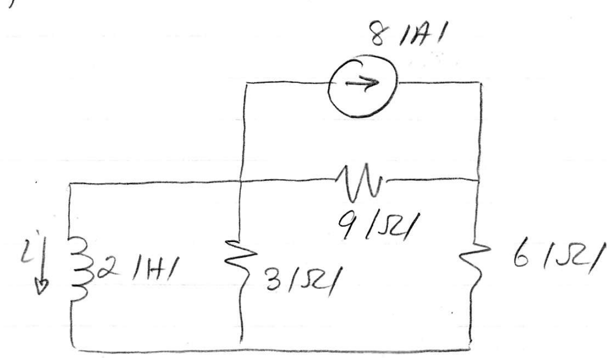
$$i(t) = 3(1-e^{-0.5t})\ [A], \quad 0 \le t \le 1\ [s]$$

b) Find $i(t)$ for $t \ge 1$ [s].

At $t = 1$ [s] we have

$$i(t) = 3(1-e^{-0.5(1)}) = 1.18\ [A] = i(1^+)$$

Re-draw for $t \ge 1$ [s]



Final value ⇒ L becomes a short ⇒
3 [Ω] is shorted out and:

$$i = -8 \cdot \frac{9}{9+6} = \frac{-72}{15} = -4.8\ [A]$$

$$R = (9+6) \parallel 3 = 2.5 \ [\Omega]$$

$$\tau_L = L/R = 0.8 \ [s]$$

$$i(t) = i_f + [i_f - i(1^+)] \, e^{-(t-1)/\tau_L}$$

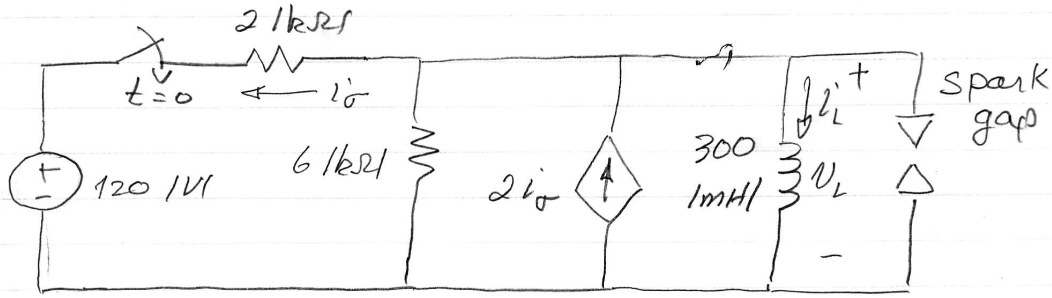
$$i(t) = -4.8 + [1.18 - (-4.8)] \, e^{-(t-1)/0.8}$$

$$i(t) = -4.8 + 5.98 e^{-1.25(t-1)} \quad [A]$$

$$1 < t \quad [s]$$

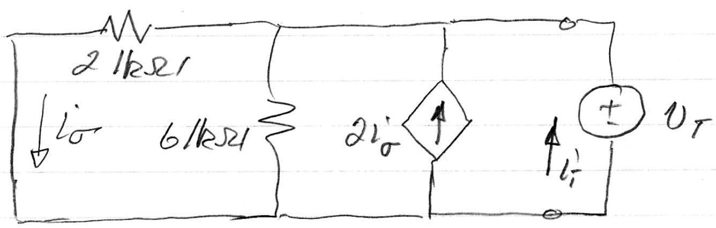
PROBLEM 7.8

The switch in the circuit below was open for a long time and then closed at $t=0$. The spark gap will break down (i.e., arc over and create a short) when the voltage across it reaches 36 [kV]. How long after the switch closes will arcing occur?



$$i_L(0^-) = i_L(0^+) = 0$$

Thevenin resistance at the terminals of the inductor for $t>0$.



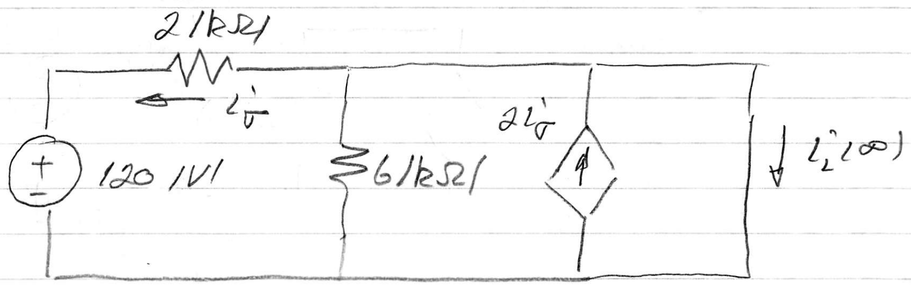
$$i_\sigma = \frac{v_T}{2000} \quad ; \quad i_T = -2i_\sigma + i_\sigma + \frac{v_T}{6000}$$

$$= -\frac{v_T}{2000} + \frac{v_T}{6000}$$

$$= -v_T(-3.333\times10^{-4})$$

$\therefore \quad R_{Th} = \frac{v_T}{i_T} = -3\,[k\Omega] \Rightarrow \tau = -10^{-4}\,[s]$

To find $i_L(\infty)$ we set L to a short:



$$i_\sigma = -\frac{120}{2000} = -60\,[mA]$$

$$i_L(\infty) = 2i_\sigma - i_\sigma = -60\,[mA]$$

$$\therefore \quad i_L(t) = -60 + (0 - (-60))\,e^{+t/100\,[\mu s]} \quad [mA]$$

$$= -60(1 - e^{t/100[\mu s]}) \quad t \geq 0\,[\mu s]$$

Now $\quad v_L(t) = L\frac{di_L}{dt} = L\,\frac{60\times10^{-3}}{100\times10^{-6}}\,e^{t/100[\mu s]} \quad [V]$

$$v_L(t) = 1.8\times10^{2}\,e^{t/100[\mu s]} \quad [V] \quad t > 0\,[\mu s]$$

So the inductor voltage is growing exponentially.
The spark gap will arc at

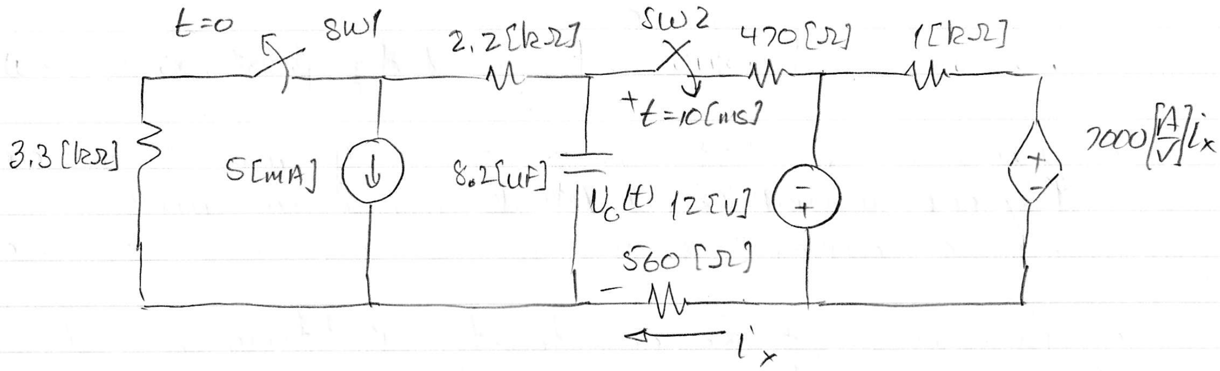
$$1.8\times10^{2}\,e^{t/100\times10^{-6}} = 36000$$

$$e^{t/100\times10^{-6}} = 200$$

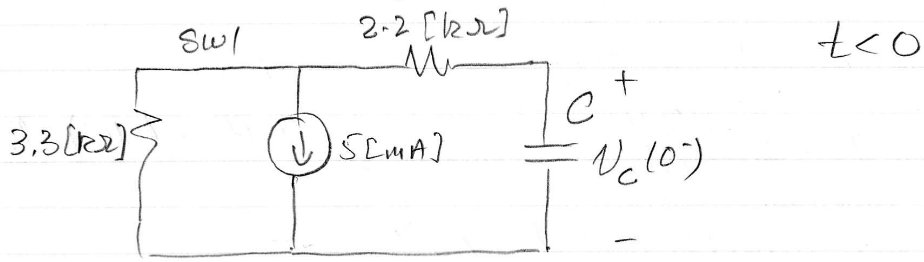
$$t = 100\times10^{-6}\cdot\ln 200 = 529.8\,[\mu s].$$

PROBLEM 7.9

SW1 was closed for a long time and SW2 was open for a long time. At $t=0$, SW1 opened. At $t = 10$ [ms], SW2 closed. Find $i_x(t)$ for $t>0$. *



Start by drawing circuit for $t<0$:



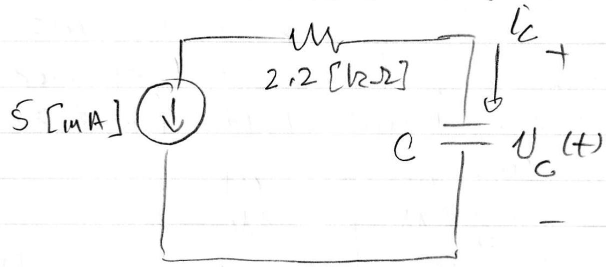
Since SW1 has been closed for a long time, we are in steady state and $C \rightarrow$ open circuit. So...

$$v_c(0^-) = v_c(0^+) = -(0.005)(3300) = -16.5 \text{ [V]}$$

Now SW1 opens...

we will find $v_c(t)$ first, and then worry about i_x

$0 < t < 10$ [ms]



This is <u>not</u> a single-time-constant response!!

Why not?

- There is no "steady state": if we open-circuit the cap, we will violate KCL because we will have a current source going into an open ckt.

- There is no time constant: putting a test source at the capacitor terminals and setting the current source to 0 $\Rightarrow R_{TH} = \infty$!

So this is a job for...

$$i_c = C \frac{dv_c(t)}{dt}$$

$t_0 = 0$

$i_c(t_0) = v_c(0^-) = v_c(0^+)$

$$v_c(t) = \frac{1}{C}\int_{t_0}^{t} i_c(t)\,dt + v_c(t_0)$$

$$= \frac{1}{8.2\times10^{-6}}\int_0^t (0.005)\,dt + (-16.5) \quad [V]$$

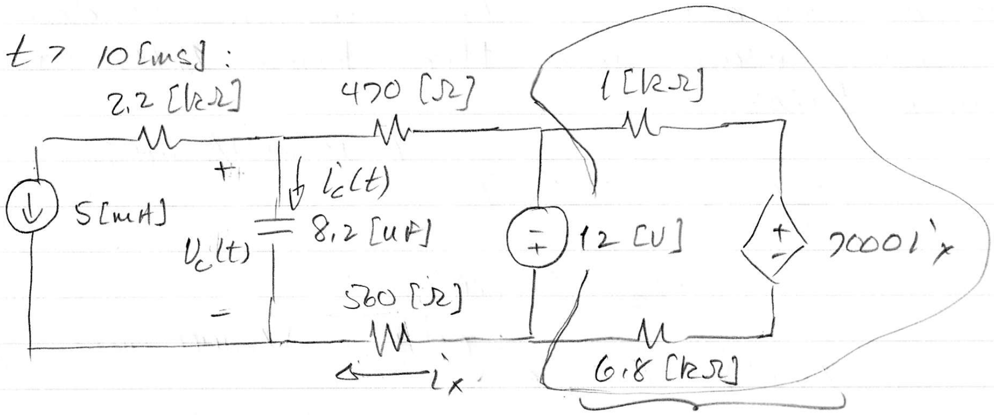
$$v_c(t) = -609.8\,t - 16.5 \quad [V] \qquad 10 \geq t > 0 \text{ [ms]}$$

The second switching event is at $t = 10$ [ms]. At this time,

$$v_c(t = 10\text{[ms]}) = -609.8\,(0.01) - 16.5 = -22.6 \text{ [V]}.$$

(PROBLEM 7.9 con't)

$t > 10$ [ms]:



these components are in parallel with a voltage source and can be ignored.

R_{TH}:

If we remove C and insert a test source, 5[mA] → open and 12 [V] → short, so

$$R_{TH} = 470 + 560 = 1030 \, [\Omega].$$

$v_{c,f}$: Open-circuit C to find $v_{c,f}$, which is the steady-state (final) value of v_c.

$$v_{c,f} + (0.005)560 + 12 + (0.005)470 = 0$$

$$v_{c,f} = -17.15 \, [V]$$

So $$v_c(t) = v_{c,f} + [v_c(t = 10 [ms]) - v_{c,f}] e^{-t/\tau_c}$$

$$\tau_c = R_{TH} C = 8.446 \, (ms]$$

So

$$v_c(t) = -17.5 + [-22.6 - (-17.5)] e^{-(t - 10[ms])/8.446[ms]}$$

[V] $t \geq 10$ [ms]

But we were asked for i_x. This can be done in several ways now that know $v_c(t)$. One way is this:

$$i_c(t) = C\frac{dv_c(t)}{dt} = 5.29\, e^{-(t-10[ms])/8.446[ms]} \ [mA]$$

KCL: $-i_x - i_c - 5[mA] = 0$

$$i_x(t) = 5.29\, e^{-(t-10[ms])/8.446[ms]} - 5 \ [mA] \qquad t > 10[ms]$$

Of course for $0 < t < 10[ms]$, SW 2 is open and...

$$i_x(t) = 0 \qquad 0 < t < 10[ms].$$