Chapter 5
Waveguides and Resonators

ECE 3317

Dr. Stuart Long
What is a “waveguide” (or transmission line)?

Structure that transmits electromagnetic waves in such a way that the wave intensity is limited to a finite cross-sectional area.

In this chapter we will focus on three types of waveguides:

1. Parallel-Plate Waveguides
2. Rectangular Waveguides
3. Coaxial Lines
Parallel Plate Waveguide
Assume both plates to be perfect conductors
Assume waveguide to be very large in $\hat{y}$ direction ($w \gg \lambda$)
Field vectors have no $y$-dependence, $\frac{\partial}{\partial y} = 0$
Neglect any fringing fields at $y = 0$ and $y = w$
Propagation along the $+\hat{z}$ direction
From Maxwell Equations

\[(E_y, H_x, H_z)\) TE (Transverse Electric) \(E_z = 0\)

or

\[(H_y, E_x, E_z)\) TM (Transverse Magnetic) \(H_z = 0\)

(Sect. 5.1)
The wave equation for the TE case derived from Maxwell's Equations is

\[ \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2} + \omega^2 \mu \varepsilon \right) E_y = 0 \]

\( \left( E_y, H_x, H_z \right) \) B.C. at \( x = 0 \) and \( x = a \) \( \Rightarrow \) \( E_y = 0 \)

Remember: \( \frac{\partial}{\partial y} = 0 \)
For ppg. in $+\hat{z}$ direction

$$E_y = E_0 \sin(k_x x) e^{-j k_z z} \quad \text{(satisfies B.C at } x = 0) \quad (5.5)$$

with

$$k_x^2 + k_z^2 = \omega^2 \mu \varepsilon = k^2 \quad (5.6)$$

To satisfy B.C. ($E_y = 0$) at $x = a$

$$k_x a = m\pi \quad (m \text{ is any integer except } 0) \quad (5.7)$$
The Electric field can be expressed in another form by substituting in for $k_x$

$$E_y = E_0 \sin\left(\frac{m\pi}{a} x\right)e^{-jk_z z}$$

with

$$k_z = \left[ \omega^2 \mu \varepsilon - \left(\frac{m\pi}{a}\right)^2 \right]^{\frac{1}{2}} = \omega \sqrt{\mu \varepsilon} \left[ 1 - \left(\frac{m\lambda}{2a}\right)^2 \right]^{\frac{1}{2}}$$

$$\left( \text{where } \lambda = \frac{2\pi}{k} \right)$$
The guided wave propagates with the phase velocity

\[ v_p = \frac{\omega}{k_z} = \frac{1}{\sqrt{\mu \varepsilon}} \left[ 1 - \left( \frac{m \lambda}{2a} \right)^2 \right]^{1/2} \]

Note: \( k_z \) is imaginary for

\[ k < k_x \quad \text{or} \quad \omega \sqrt{\mu \varepsilon} < \frac{m \pi}{a} \quad \text{or} \quad \lambda > \frac{2a}{m} \]

If \( k_z \) becomes imaginary, the wave will attenuate exponentially, and the velocity for the guided wave will also become undefined.
\[ \omega_c = \frac{\pi m}{a \sqrt{\mu \varepsilon}} \]

\[ f_c = \frac{\omega_c}{2\pi} = \frac{m}{2a \sqrt{\mu \varepsilon}} \]

(cutoff frequency of \(TE_m\) mode)

frequency at which \(k_z\) becomes imaginary

exponential attenuation when \(f < f_c\)

\[ \lambda_c = \frac{2a}{m} \]

(cutoff wavelength of \(TE_m\) mode)

wavelength at which \(k_z\) becomes imaginary
Physical Interpretation
TE mode

\[ E_y = E_0 \sin(k_x x) e^{-jk_z z} = E_0 \left[ \frac{j}{2} (e^{-jk_x x} - e^{+jk_x x}) \right] e^{-jk_z z} \]

\[ E_y = \frac{jE_0}{2} \left[ e^{-jk_x x - jk_z z} - e^{jk_x x - jk_z z} \right] \]

(5.8)

wave trav. in +\( \hat{x} \)
and +\( \hat{z} \) dir.
(coeff \( \frac{jE_0}{2} \))

wave trav. in -\( \hat{x} \)
and +\( \hat{z} \) dir.
(coeff \( \frac{-jE_0}{2} \))
For this case $k \equiv \frac{3.5\pi}{a}$
so modes $m = 1, 2, 3$ will ppg. but for $m \geq 4$, $k_z$ is imaginary.

$$k_z = \left[ k^2 - \left( \frac{m\pi}{a} \right)^2 \right]^{\frac{1}{2}}$$  \hspace{1cm} (5.9)
\[ \left( H_y, E_x, E_z \right) \text{ can find } H_y = H_0 \cos k_x x e^{-jk_z z} \]

\[ E_z \sim \frac{\partial}{\partial x} H_y \Rightarrow E_z \sim \sin k_x x \]

For B.C. at \( x = a \) \( (E_z |_{x=0,a} = 0) \)

\[ k_x = \frac{m\pi}{a} \]
For ppg. in $+\hat{z}$ direction

$$H_y = H_0 \cos(k_x x) e^{-j k_z z} \quad \text{(satisfies B.C at } x = 0)$$

with

$$k_x^2 + k_z^2 = \omega^2 \mu \varepsilon = k^2$$

To satisfy B.C. ($E_z = 0$) at $x = a$

$$k_x = \frac{m \pi}{a} \quad \text{(} m \text{ is any integer including } 0 \text{)}$$
The field solutions can be expressed in another form by substituting in for $k_x$

$$H_y = H_0 \cos\left(\frac{m\pi}{a} x\right) e^{-jk_z z}$$

with

$$k_z = \left[ \omega^2 \mu \varepsilon - \left(\frac{m\pi}{a}\right)^2 \right]^{1/2} = \omega \sqrt{\mu \varepsilon} \left[ 1 - \left(\frac{m\lambda}{2a}\right)^2 \right]^{1/2}$$

$$\left(\text{where } \lambda = \frac{2\pi}{k}\right)$$
since \( E_z \sim \frac{\partial}{\partial x} H_y \)

\[
E_z = -\frac{k_x}{j\omega \varepsilon} H_0 \sin\left(\frac{m\pi}{a} x\right) e^{-j k_z z}
\]

\[
E_x = \frac{k_z}{\omega \varepsilon} H_0 \cos\left(\frac{m\pi}{a} x\right) e^{-j k_z z}
\]
Remember that for TM waves we can now have $m = 0$.

For $\mathbf{TM}_0$:

\[
\begin{align*}
  m &= 0, \\
  k_x &= 0 \\
  k_x &= \omega \sqrt{\mu \varepsilon} = k
\end{align*}
\]

For this case $\mathbf{E}$ and $\mathbf{H}$ are both perpendicular to the direction of propagation ($\hat{z}$). We refer to this case as

**TEM mode (Transverse Electromagnetic)**
The field solutions for the TM$_0$ or TEM mode are given by

\[ H_y = H_0 e^{-jkzz} \]  \hspace{1cm} (5.13a)

\[ E_x = \eta H_0 e^{-jkzz} \]  \hspace{1cm} (5.13b)

\[ E_z = 0 \]

or equivalently

\[ E_x = E_0 e^{-jkzz} \]

\[ H_y = \frac{E_0}{\eta} e^{-jkzz} \]
Physical Interpretation

TEM mode

\[ \mathbf{J}_s = \hat{n} \times \mathbf{H} \bigg|_{x=0} = \hat{x} \times \hat{y} \frac{E_0}{\eta} e^{-jkz} = \hat{z} \frac{E_0}{\eta} e^{-jkz} \] (5.13c)

\[ \rho_s = \hat{n} \cdot \mathbf{D} = \hat{x} \cdot \varepsilon \mathbf{E} = \varepsilon E_0 e^{-jkz} \] (5.11d)

(on top plate \( n = -\hat{x} \)) \[ \mathbf{J}_s = -\hat{z} \frac{E_0}{\eta} e^{-jkz} \]

\[ \rho_s = -\varepsilon E_0 e^{-jkz} \]

\( \mathbf{J} \) for TE and TM modes has both \( \hat{y} \) and \( \hat{z} \) components.
Microstrip Lines

- Microstrip line
- W
- t
- D
- Ground plane
- Substrate

http://3.bp.blogspot.com/_QOpBRAPC2rY/TGW9OMdLRnl/AAAAAAAAC8/2y2RGlyuRQ0/s1600/Picture2.jpg

www.amanogawa.com/transmission.html
The time-average Poynting power density can be found by:

\[ \langle S \rangle = \frac{1}{2} \text{Re} \left[ E \times H^* \right] \]

\[ \langle S \rangle = \frac{1}{2} \text{Re} \left[ E_0 e^{-jkz} \frac{E_0}{\eta} e^{+jkz} (\hat{x} \times \hat{y}) \right] \]

\[ \langle S \rangle = \hat{z} \frac{E_0^2}{2\eta} \left[ \frac{W}{m^2} \right] \]

Total Power = \[ P = \frac{E_0^2}{2\eta} wD \left[ W \right] \]
Microstrip Lines

$$\varepsilon = \varepsilon_0$$

quasi TEM-fields

$$k = \omega \sqrt{\mu \varepsilon}$$

$$\varepsilon > \varepsilon_0$$
Rectangular Waveguide
MAX equations $\Rightarrow$ HELMHOLTZ equation

(wave equation)

$$\nabla^2 E + \omega^2 \mu \varepsilon E = 0$$

$$\nabla^2 H + \omega^2 \mu \varepsilon H = 0$$

6 equation like

$$\frac{\partial^2 H_z}{\partial x^2} + \frac{\partial^2 H_z}{\partial y^2} + \frac{\partial^2 H_z}{\partial z^2} + \omega^2 \mu \varepsilon H_z = 0$$
For a separable solution assume
\[ H_z(x, y, z) = X(x)Y(y)Z(z) \]
\[ \Rightarrow \frac{1}{X} \frac{\partial^2 X}{\partial x^2} + \frac{1}{Y} \frac{\partial^2 Y}{\partial y^2} + \frac{1}{Z} \frac{\partial^2 Z}{\partial z^2} = -\omega^2 \mu \varepsilon \]

each term is a constant
\[ -k_x^2 - k_y^2 - k_z^2 = -\omega^2 \mu \varepsilon \]
\[ \frac{1}{X} \frac{\partial^2 X}{\partial x^2} = -k_x^2 \]
\[ \Rightarrow \frac{\partial^2 X}{\partial x^2} + k_x^2 X = 0 \]
Using Maxwell's equations we can solve for the transverse fields \((E_x, E_y, H_x, H_y)\) in terms of the longitudinal fields \((E_z, H_z)\).
Rectangular Waveguide

\[
\begin{align*}
E_x &= -\frac{jk_z}{k_c^2} \frac{\partial E_z}{\partial x} - \frac{j\omega \mu}{k_c^2} \frac{\partial H_z}{\partial y} ; & E_y &= -\frac{jk_z}{k_c^2} \frac{\partial E_z}{\partial y} + \frac{j\omega \mu}{k_c^2} \frac{\partial H_z}{\partial x} \\
H_x &= -\frac{jk_z}{k_c^2} \frac{\partial H_z}{\partial x} + \frac{j\omega \varepsilon}{k_c^2} \frac{\partial E_z}{\partial y} ; & H_y &= -\frac{jk_z}{k_c^2} \frac{\partial H_z}{\partial y} - \frac{j\omega \varepsilon}{k_c^2} \frac{\partial E_z}{\partial x}
\end{align*}
\]

where \( k_c^2 = k_x^2 + k_y^2 = \omega^2 \mu \varepsilon - k_z^2 \)
Special cases:

1) when $E_z = 0$ ; $H_z \neq 0 \Rightarrow \text{Transverse Electric mode (TE)}$

2) when $H_z = 0$ ; $E_z \neq 0 \Rightarrow \text{Transverse Magnetic mode (TM)}$

If $E_z = H_z = 0 \Rightarrow \text{all comp.} = 0$

Transverse ElectricMagnetic mode (TEM) can not exist
Rectangular Waveguide TE: Case \((E_z=0)\)

B.C. tangential \(E \rightarrow 0\) at \(x = 0, a\) and \(y = 0, b\)

Side walls \(E_y \sim \frac{\partial H_z}{\partial x} = 0\) (at \(x = 0, a\))

\[
E_y \sim k_x c_1 \cos k_x x - k_x c_2 \sin k_x x \Rightarrow c_1 = 0 \text{ and } k_x = \frac{m\pi}{a}
\]

Top and bottom walls \(E_x \sim \frac{\partial H_z}{\partial y} = 0\) (at \(y = 0, b\))

\[
E_x \sim k_y c_3 \cos k_y y - k_y c_4 \sin k_y y \Rightarrow c_3 = 0 \text{ and } k_y = \frac{n\pi}{b}
\]

Let \(c_5c_2c_4 = H_0\) \(\Rightarrow\) \(H_z = X Y Z\)

\[
H_z = H_0 \cos \frac{m\pi x}{a} \cos \frac{n\pi y}{b} e^{-jk_z z}
\]
Rectangular Waveguide TE:  
**Case (E_z=0)**

Propagation constant  
\[ k_z = \sqrt{k^2 - k_c^2} \]

where  
\[ k_c^2 = \left( \frac{m\pi}{a} \right)^2 + \left( \frac{n\pi}{b} \right)^2 \]

substituing in for \( k_c^2 \)

\[
k_z = \sqrt{\omega^2 \mu\varepsilon - \left( \frac{m\pi}{a} \right)^2 - \left( \frac{n\pi}{b} \right)^2}
\]

(5.19)

**Note:** propagation (\( k_z \) real) for \( k > k_c \)

exponential attenuation (\( k_z \) imaginary) for \( k < k_c \)
Rectangular Waveguide TE: Case \((E_z=0)\)

\[
f_c = \frac{\omega_c}{2\pi} = \frac{1}{2\pi\sqrt{\mu\varepsilon}} \left[ \left( \frac{m\pi}{a} \right)^2 + \left( \frac{n\pi}{b} \right)^2 \right]^{\frac{1}{2}}
\]

(cutoff frequency of TE\(_{mn}\) mode)

frequency at which \(k_z\) becomes imaginary

exponential attenuation when \(f < f_c\)

\[
\lambda_g = \frac{2\pi}{k_z} = \frac{2\pi}{\sqrt{k^2 - k_c^2}}
\]

(guide wavelength of TE\(_{mn}\) mode)

wavelength at which \(k_z\) becomes imaginary

defines a doubly infinite set of modes TE\(_{mn}\)
Rectangular Waveguide TM: Case ($H_z=0$)

$$E_z = E_0 \sin \frac{m \pi x}{a} \sin \frac{n \pi y}{b} e^{-jk_z z}$$  \hspace{1cm} (5.18)

Same $k_z, k_c, f_c, \lambda_g$ as TE

For TM $m \neq 0$; and $n \neq 0$ lowest order (smallest $f_c$) is $TM_{11}$

The lowest order of all is $TE_{10}$ called the DOMINANT MODE for rectangular waveguides (no TEM possible)

(there exists a frequency range where only it can propagate)
Rectangular Waveguide $\text{TE}_{10}$ mode ($m=1$, $n=0$)

\[ H_z = H_0 \cos \frac{\pi x}{a} e^{-jk_z z} \]

\[ E_x = -\frac{j\omega\mu}{k_c^2} \frac{\partial H_z}{\partial y} = 0 \quad ; \quad E_y = \frac{j\omega\mu}{k_c^2} \frac{\partial H_z}{\partial x} = -\frac{j\omega\mu}{k_c} H_0 \sin \frac{\pi x}{a} e^{-jk_z z} \]

\[ H_x = -\frac{jk_z}{k_c^2} \frac{\partial H_z}{\partial x} = \frac{jk_z}{k_c} H_0 \sin \frac{\pi x}{a} e^{-jk_z z} \quad ; \quad H_y = -\frac{jk_z}{k_c^2} \frac{\partial H_z}{\partial y} = 0 \]
Rectangular Waveguide TE_{10} mode (m=1, n=0)

\[ k_z = \sqrt{k_z^2 - \left(\frac{\pi}{a}\right)^2} \]

\[ \lambda_c = 2a \quad ; \quad f_c = \frac{1}{2a\sqrt{\mu\varepsilon}} \quad ; \quad k_c = \frac{\pi}{a} \]

\[ \lambda_g = \frac{2\pi}{k_z} = \frac{\lambda}{\sqrt{1 - \left(\frac{\lambda}{2a}\right)^2}} \]

Note: these are the eqns. found in slide 5-32 and 5-33 with \( m=1 \) and \( n=0 \)
Physical Interpretation TE_{10} mode (m=1, n=0)
\[ k_z = \omega \sqrt{\mu \varepsilon} \]

\[ \nu_p = \frac{\omega}{k_z} = \tan \xi \quad \text{(function of frequency)} \]

where \[ k_z = \sqrt{k^2 - k_c^2} \] and \[ \nu_g = \frac{\partial \omega}{\partial k_z} \]
Example 5.4

\[ f_c = \frac{1}{2\pi\sqrt{\mu \varepsilon}} \left[ \left( \frac{m\pi}{a} \right)^2 + \left( \frac{n\pi}{b} \right)^2 \right]^{1/2} \]

\[ f_{c_{TE_{10}}} = \frac{1}{2\sqrt{\mu \varepsilon}} \left( \frac{1}{a} \right) \]

\[ f_{c_{TE_{01}}} = \frac{1}{2\sqrt{\mu \varepsilon}} \left( \frac{1}{b} \right) \]

\[ f_{c_{TE_{20}}} = \frac{1}{2\sqrt{\mu \varepsilon}} \left( \frac{2}{a} \right) \]
Design of Practical Rectangular Waveguide

(Cont. e.g 5.4)

for \( b > \frac{a}{2} \)

for \( b < \frac{a}{2} \)

for \( b = \frac{a}{2} \)
Transmitting Power in waveguide \([\text{TE}_{10} \text{ mode}]\)

\[
\langle S \rangle = \frac{1}{2} \text{Re} \left[ E \times H^* \right]
\]

\[
\langle S \rangle = \hat{z} \frac{E_0^2 k_z}{2 \omega \mu} \sin^2 \left( \frac{\pi x}{a} \right)
\]

\[
P = \int_{0}^{a} \int_{0}^{b} \langle S \rangle \hat{z} \, dx \, dy
\]

\[
P = \frac{E_0^2 a b k_z}{4 \omega \mu} \text{ W}
\]

Time-average Poynting vector

Total transmitted power
Note:

- Do not want $b > \frac{a}{2}$ for bandwidth

- Do not want $b$ small for power handling

Usual case

$$b = \frac{a}{2}$$
X-Band Waveguide (8-12 GHz)

\[ f_{c_{\text{TE10}}} = 6.55 \text{ GHz} \]

\[ f_{c_{\text{TE20}}} = 13.1 \text{ GHz} \]

\[ f_{c_{\text{TE01}}} = 14.7 \text{ GHz} \]
Rectangular Waveguide TE_{10} mode

\[ H_z = H_0 \cos \left( \frac{\pi x}{a} \right) e^{-j k_z z} \]

\[ E_y = -j \frac{\omega \mu}{\pi / a} H_0 \sin \left( \frac{\pi x}{a} \right) e^{-j k_z z} \]

\[ H_x = \frac{j k_z}{\pi / a} H_0 \sin \left( \frac{\pi x}{a} \right) e^{-j k_z z} \]
Rectangular Waveguide TE\textsubscript{10} mode

or

\[ E_y = E_0 \sin \frac{\pi x}{a} e^{-jk_z z} \]

where

\[ E_0 = -\frac{j \omega \mu}{\pi/a} H_0 \]

\[ H_x = \frac{-k_z}{\omega \mu} E_0 \sin \frac{\pi x}{a} e^{-jk_z z} \]

\[ H_z = \frac{\pi}{a} \frac{E_0}{-j \omega \mu} \cos \frac{\pi x}{a} e^{-jk_z z} \]

(5.23)
Example 5.5 & 5.6

Design a Rectangular waveguide with 10 GHz ($\lambda=3\text{cm}$) at Mid-Band and $b = \frac{a}{2}$

$$\text{BW} \Rightarrow \frac{3 \times 10^8}{2a} < f < \frac{3 \times 10^8}{a}$$

let $10 \times 10^9 = \frac{1}{2} \left[ \frac{3 \times 10^8}{2a} + \frac{3 \times 10^8}{a} \right] \Rightarrow a = 2.25 \text{ cm}$

$$b = \frac{a}{2} \Rightarrow b = 1.125 \text{ cm}$$
Max power handling \( \Rightarrow P = \frac{E_0^2 ab}{4Z_{TE}} \)

where \( Z_{TE} = \frac{\omega \mu}{k_z} \) [\( \Omega \)]

\( E_{BD_{air}} = 2 \times 10^6 \left[ \frac{V}{m} \right] \)

take \( E_{\text{max}} = 2 \times 10^5 \left[ \frac{V}{m} \right] \) (safety factor of 10)

\[
Z_{TE} = \frac{(2\pi \times 10^{10})(4\pi \times 10^{-7})}{\sqrt{k^2 - (\pi/a)^2}} = 505.8 \ [\Omega]
\]
Example

(Cont. e.g 5.5 & 5.6)

note:

\[ k_z = \frac{2\pi\sqrt{1-(\lambda/2a)^2}}{\lambda} = \frac{2\pi\sqrt{1-(3/4.5)^2}}{3} \]

\[ k_z = 1.561 \, \text{cm}^{-1} = 156.1 \, \text{m}^{-1} \]

\[ P_{\text{max}} = \frac{(2 \times 10^5)^2(0.0225)(0.01125)}{4(505.8)} = 5.004 \, \text{KW} \]
Cylindrical Coordinates

(\(\mathbf{\rho} \times \hat{\phi} = \hat{z}\))

(fig. 5.16)
Coaxial Transmission Lines

Maxwell Eqns. ⇒

\[ E = \frac{V_0}{\rho} e^{-j k z} \hat{\rho} \]  \hspace{0.5cm} (5.48a)

\[ H = \frac{V_0}{\eta \rho} e^{-j k z} \hat{\phi} \]  \hspace{0.5cm} (5.48b)

\[ k = \omega \sqrt{\mu \varepsilon} \]  \hspace{0.5cm} (5.49a)

\[ \eta = \sqrt{\frac{\mu}{\varepsilon}} \]  \hspace{0.5cm} (5.49b)
The “Del” Operator in Cylindrical Coordinates

\[ \nabla \cdot \mathbf{A} = \frac{1}{\rho} \frac{\partial (\rho A_\rho)}{\partial \rho} + \frac{1}{\rho} \frac{\partial (A_\phi)}{\partial \phi} + \frac{\partial A_z}{\partial z} \]  

\[ \nabla \times \mathbf{A} = \left( \frac{1}{\rho} \frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z} \right) \hat{\rho} + \left( \frac{\partial A_\rho}{\partial z} - \frac{\partial A_z}{\partial \rho} \right) \hat{\phi} + \frac{1}{\rho} \left( \frac{\partial (\rho A_\phi)}{\partial \rho} - \frac{\partial A_\rho}{\partial \phi} \right) \hat{z} \]
\[ J_s = \hat{n} \times H \bigg|_{\rho=a} = \hat{\rho} \times \hat{\phi} \frac{V_0}{\eta \rho} e^{-j\kappa z} \bigg|_{\rho=a} = \hat{\zeta} \frac{V_0}{\eta a} e^{-j\kappa z} \quad (5.50a) \]

\[ I = \int_0^{2\pi} J_s \bigg|_{\rho=a} a \, d\phi = \frac{2\pi V_0}{\eta} e^{-j\kappa z} \quad (5.50b) \]
for \( z < 0 \)

both Incident Fields and Reflected Fields exist

\[
E = \hat{\rho} \left( \frac{V_0}{\rho} e^{-j k z} + \frac{V_1}{\rho} e^{+j k z} \right)
\]

\[
H = \hat{\phi} \left( \frac{V_0}{\rho \eta} e^{-j k z} - \frac{V_1}{\rho \eta} e^{+j k z} \right)
\]

Negative sign comes from same place as in reflected H-field in Chapter 4

(5.52a)

B.C at \( z = 0 \) \( \Rightarrow \) tangential \( E = 0 \) \( \Rightarrow \) \( E_\rho = 0 \) \( \Rightarrow \) \( V_1 = -V_0 \)

(5.52b)
Similarly, \( \mathbf{H} = \hat{\phi} \frac{2V_0}{\eta \rho} \cos kz \) \hspace{1cm} (5.53b)

To measure standing wave pattern cut longitudinal slot in outer conductor for moveable probe.

**Note:** \( \mathbf{H} \) totally in \( \hat{\phi} \) direction so \( \mathbf{J}_s \) totally in \( \hat{z} \) direction
Calculate the transmitted power along a coaxial line.

\[ E = \frac{V_0}{\rho} e^{-jkz} \hat{\rho} \]

\[ H = \frac{V_0}{\eta \rho} e^{-jkz} \hat{\phi} \]

\[ \langle S \rangle = \frac{1}{2} \text{Re} \left[ E \times H^* \right] = \frac{1}{2} \text{Re} \left\{ \frac{V_0}{\rho} e^{-jkz} \hat{\rho} \times \frac{V_0}{\eta \rho} e^{jkz} \hat{\phi} \right\} = \hat{z} \frac{V_0^2}{\eta \rho^2} \left[ \frac{W}{m^2} \right] \]

\[ P = \frac{\pi V_0^2}{\eta} \ln \left( \frac{b}{a} \right) \] [W]
Transmission Lines

2 or more conductors

✓ TEM possible
✓ along with TE and TM

Coaxial

three-wire (shielded)

two-wire (shielded)
Transmission Lines

2 or more conductors

✓ TEM possible
✓ along with TE and TM

Parallel Plate

Microstrip

Stripline

Conductor

Dielectric
Waveguides:

- Rectangular
- Elliptical
- Ridged
- Circular
- Dielectric Slab
- Optical Fiber

http://optics.org
Rectangular

Circular

Elliptical

Bends