ECE 3317
Applied Electromagnetic Waves

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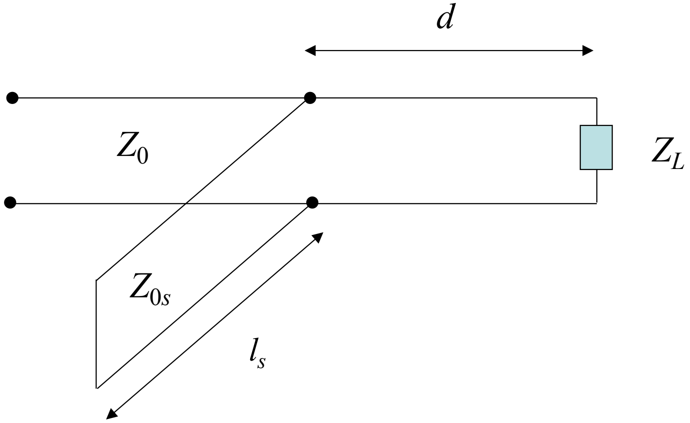
Fall 2023

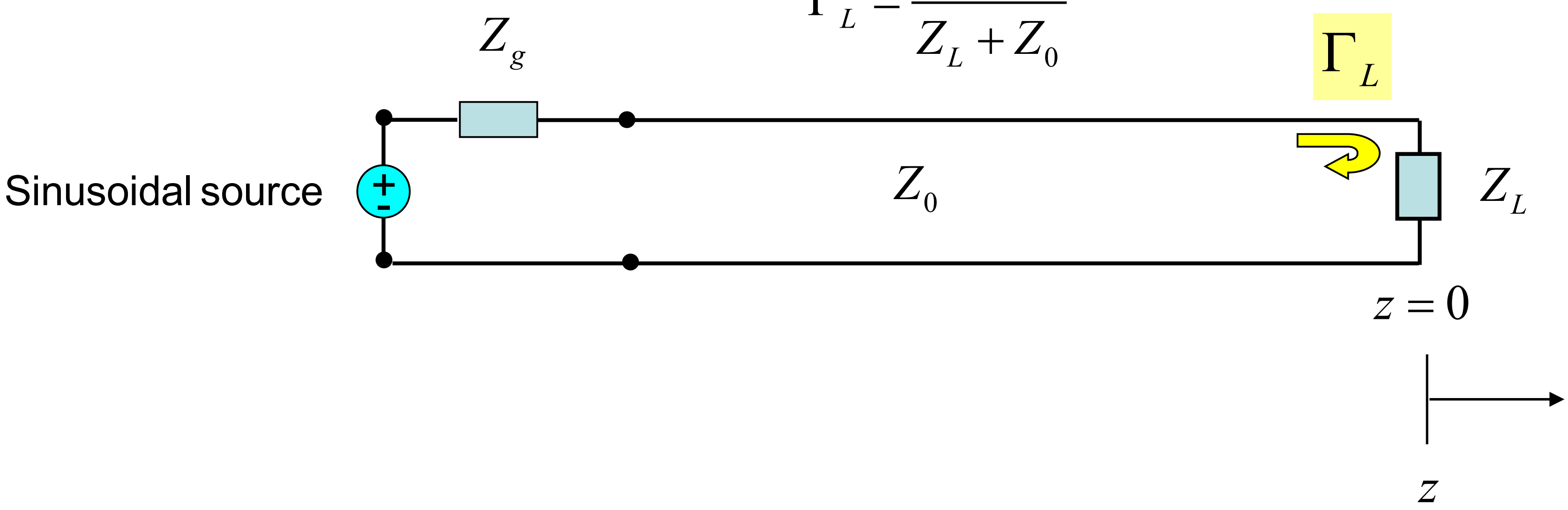
Notes 13
Transmission Lines
(Impedance Matching)



Impedance Matching

Impedance matching is very important to avoid reflected power, which causes a loss of efficiency and interference.

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

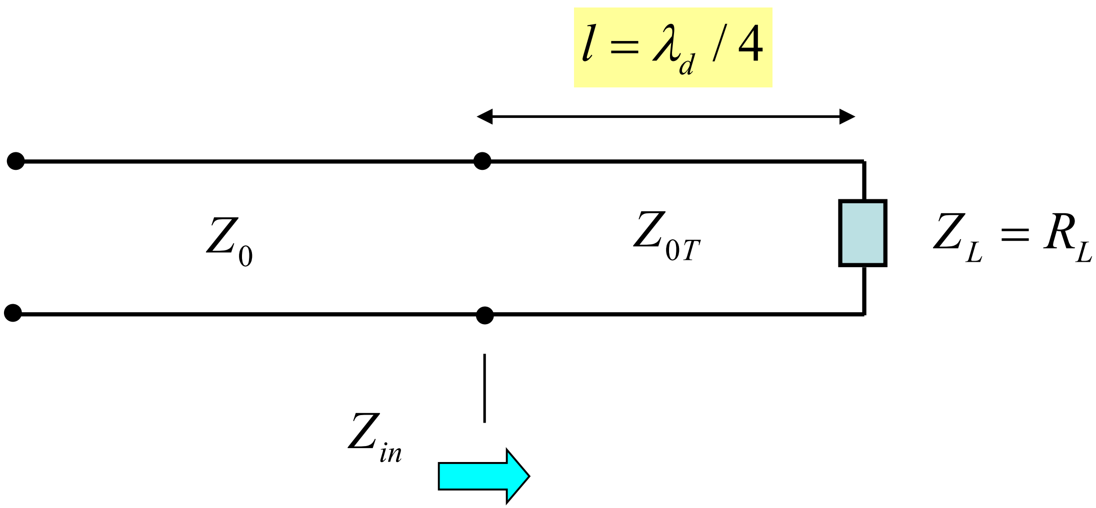


We will discuss two methods:

- ❖ Quarter-wave transformer
- ❖ Single-stub matching

Quarter-Wave Transformer

Quarter-Wave Transformer: First consider a <u>real load</u> on a lossless line.



Note: $\lambda_d = \lambda_0 / \sqrt{\varepsilon_r}$

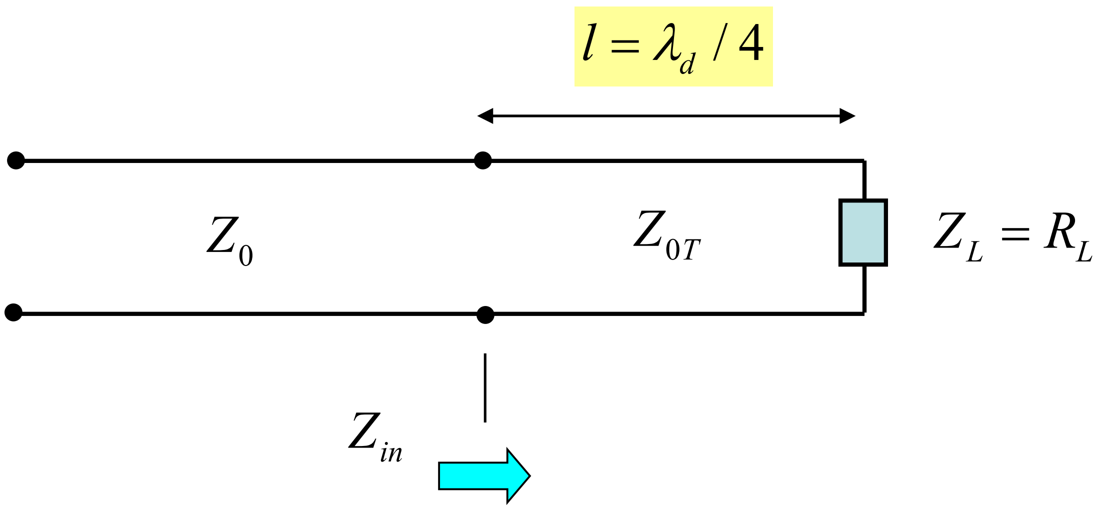
$\lambda_0 = c / f$

$$Z_{in} = Z_{0T}\left(\frac{Z_L + jZ_{0T}\tan(\beta l)}{Z_{0T} + jZ_L\tan(\beta l)}\right) \qquad \tan(\beta l) = \tan\left(\frac{2\pi}{\lambda_d} l\right) = \tan\left(\frac{\pi}{2}\right) = \infty$$

$$\Rightarrow \quad Z_{in} = \frac{Z_{0T}^2}{Z_L}$$

Hence $Z_{in} = \dfrac{Z_{0T}^2}{R_L} = \text{real}$

Quarter-Wave Transformer (cont.)



Set

$$Z_{in} = Z_0$$

Hence

$$\frac{Z_{0T}^2}{R_L} = Z_0$$

This gives us

$$Z_{0T} = \sqrt{Z_0 R_L}$$

Example:

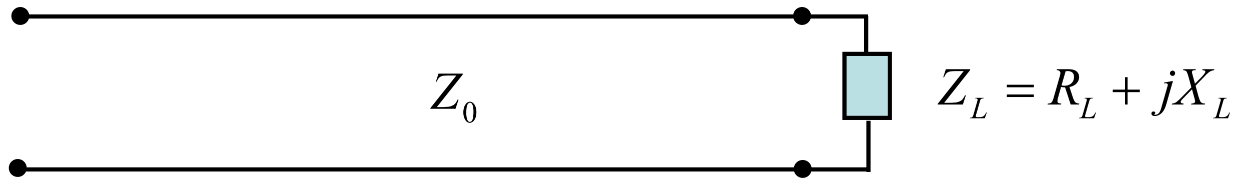
$$Z_0 = 50\ [\Omega]$$

$$Z_L = 100\ [\Omega]$$

$$Z_{0T} = \sqrt{(50)(100)} = 70.71\ [\Omega]$$

Quarter-Wave Transformer with Complex Load

Next, consider a general (complex) load impedance Z_L:

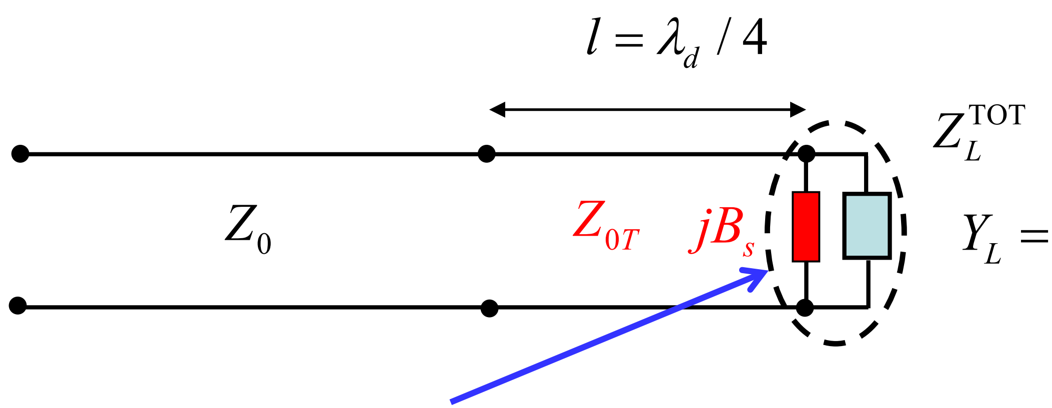


We discuss two methods:

- (a) Shunt element added to load
- (b) Extension line added to load

Quarter-Wave Transformer with Complex Load (cont.)

(a) Shunt element added to load



$$Y_L = 1/Z_L = G_L + jB_L$$

cancels

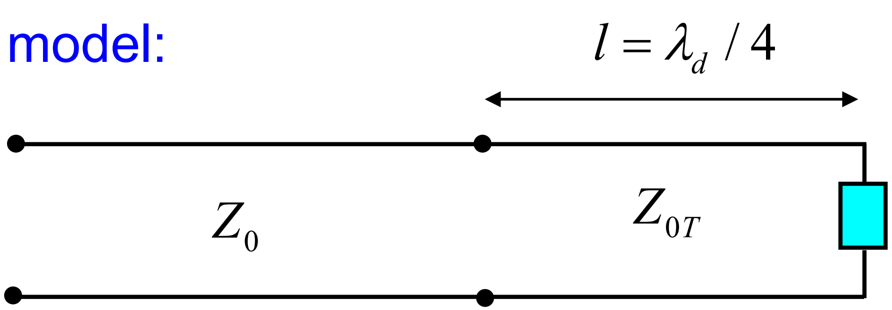
$$Y_s = -jB_L$$

Shunt (parallel) susceptance B_s is added: $Y_s = jB_s$

$$B_s = -B_L$$

The shunt element (susceptance) converts the complex load to a real load.

New model:

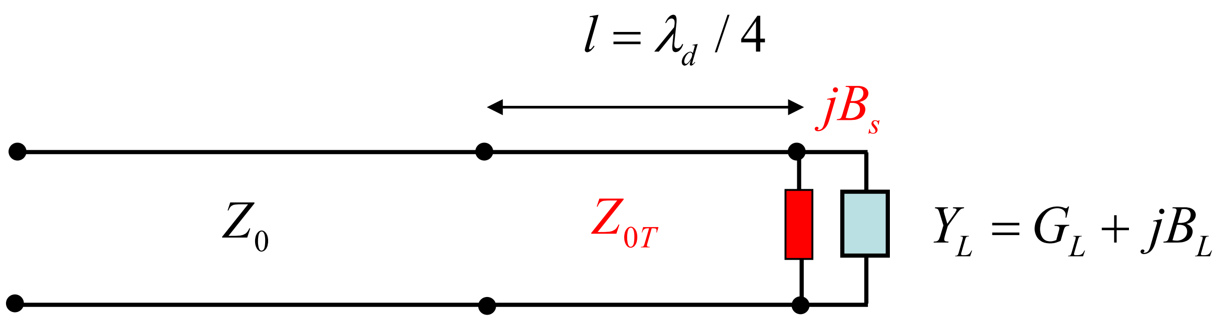


$Z_L^{\text{TOT}} = 1/G_L$ (real)

Quarter-Wave Transformer with Complex Load (cont.)

Summary of quarter-wave transformer matching method

(A shunt susceptance is added to the load.)



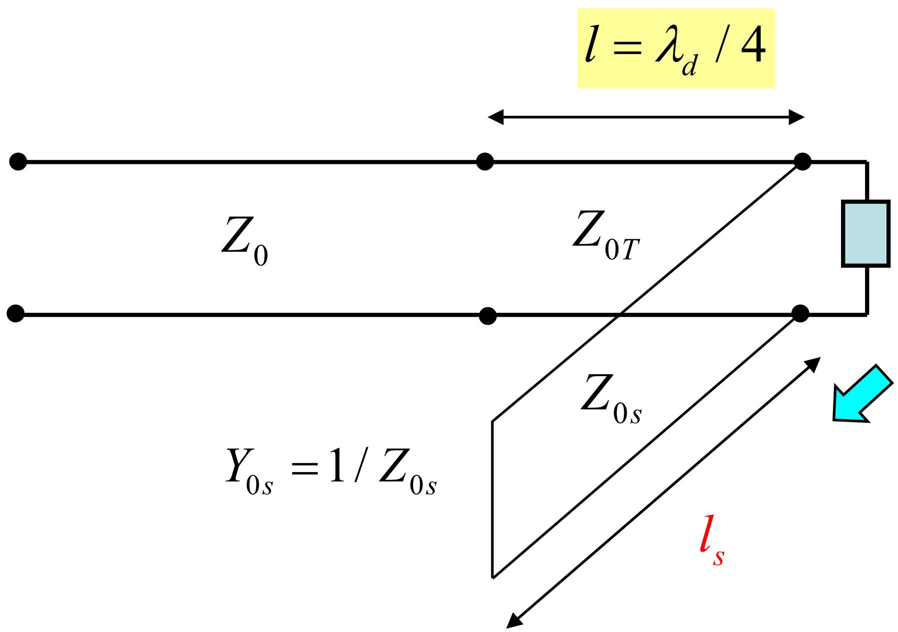
$$Z_{0T} = \sqrt{Z_0 / G_L}$$

$$B_s = -B_L$$

Quarter-Wave Transformer with Complex Load (cont.)

Realization using a shorted stub:

(An open-circuited stub could also be used as the shunt element.)



$Y_L = G_L + jB_L$

cancels

$Y_{in}^{\text{stub}} = -jB_L$

$B_{in}^{\text{stub}} = -B_L$

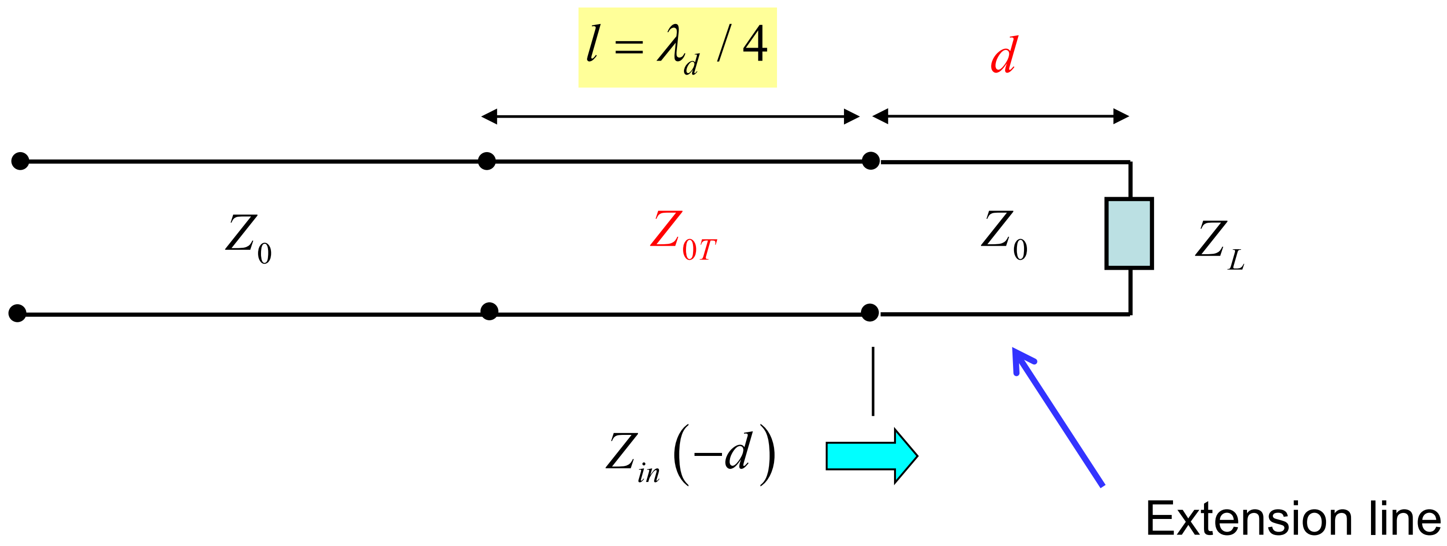
$$Z_{in}^{\text{stub}} = jZ_{0s} \tan\left(\beta_s l_s\right) \Rightarrow Y_{in}^{\text{stub}} = -jY_{0s} \cot\left(\beta_s l_s\right) \Rightarrow B_{in}^{\text{stub}} = -Y_{0s} \cot\left(\beta_s l_s\right)$$

Hence, we have: $-Y_{0s} \cot\left(\beta_s l_s\right) = -B_L \Rightarrow$

$$l_s = \frac{1}{\beta_s} \cot^{-1}\left(B_L / Y_{0s}\right)$$

Quarter-Wave Transformer with Complex Load (cont.)

(b) Extension line added to load



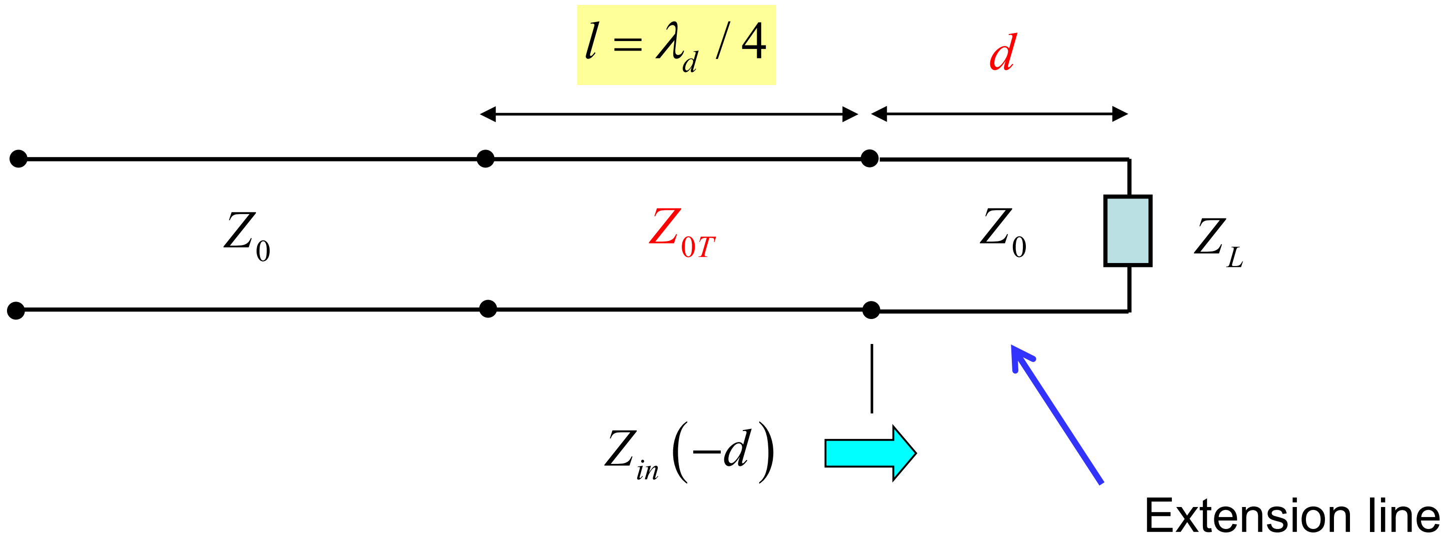
The extension line converts the complex load to a real load.

- We choose the length d to make the input impedance Z_{in} $(-d)$ real.
- We then use a quarter-wave transformer to change the impedance to Z_0.

Quarter-Wave Transformer with Complex Load (cont.)

Example

$$Z_0 = 50\,[\Omega]$$
$$Z_L = 50 + j75\,[\Omega]$$



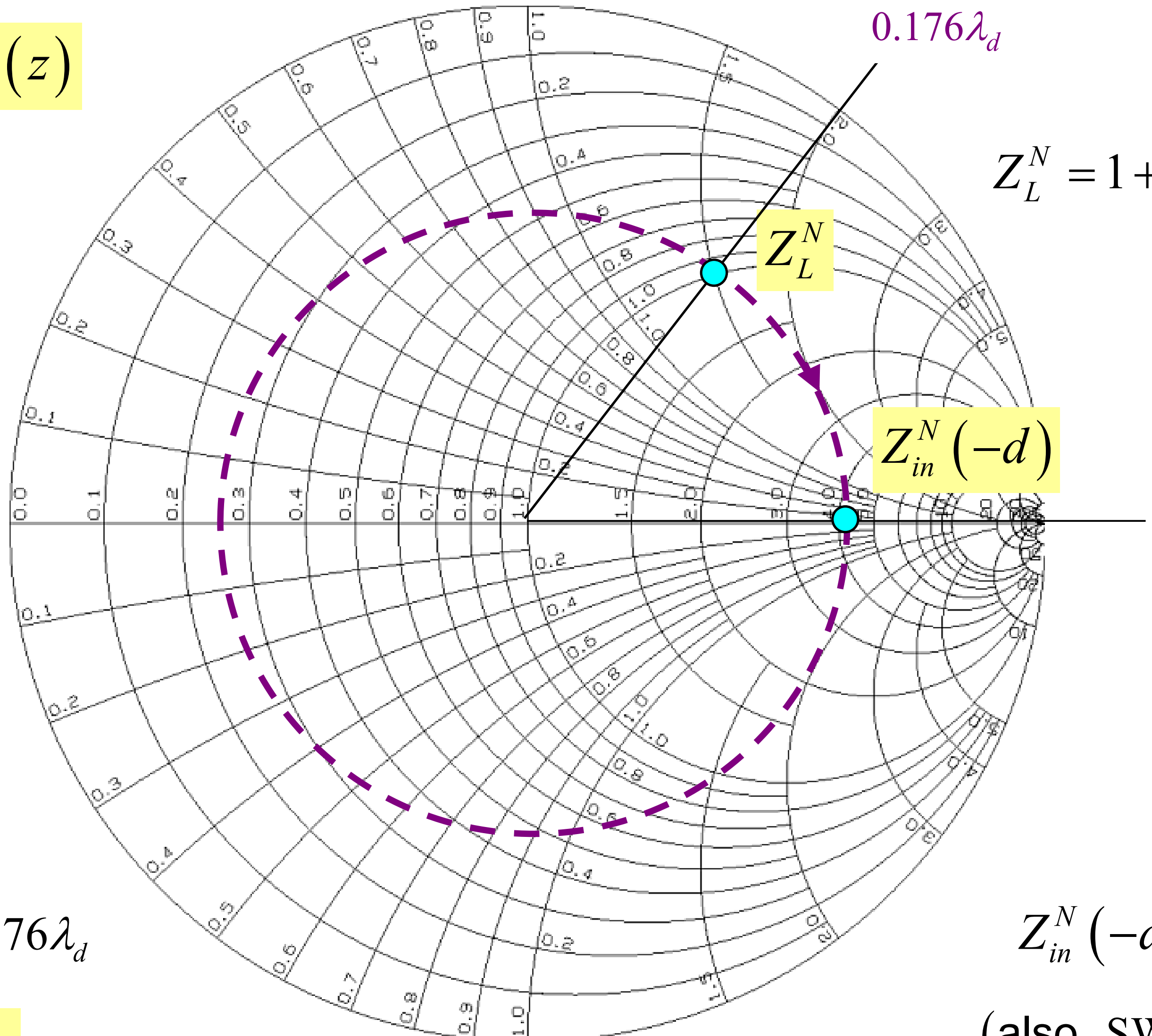
$$Z_L^N = Z_L / Z_0 = 1 + j(1.5)$$

Quarter-Wave Transformer with Complex Load (cont.)

$\Gamma(z)$

Wavelengths towards generator

0λ



$Z_L^N = 1 + j(1.5)$

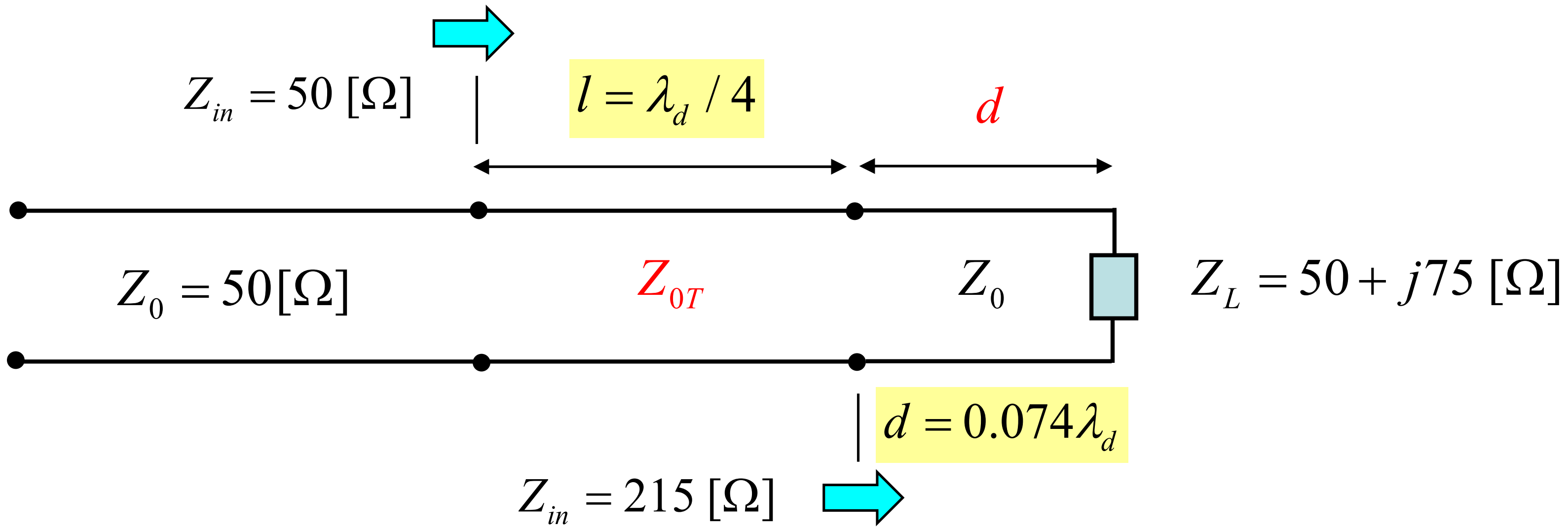
$d = 0.250\lambda_d - 0.176\lambda_d$

$d = 0.074\lambda_d$

$Z_{in}^N(-d) = 4.3$

(also, $\text{SWR} = 4.3$)

Quarter-Wave Transformer with Complex Load (cont.)



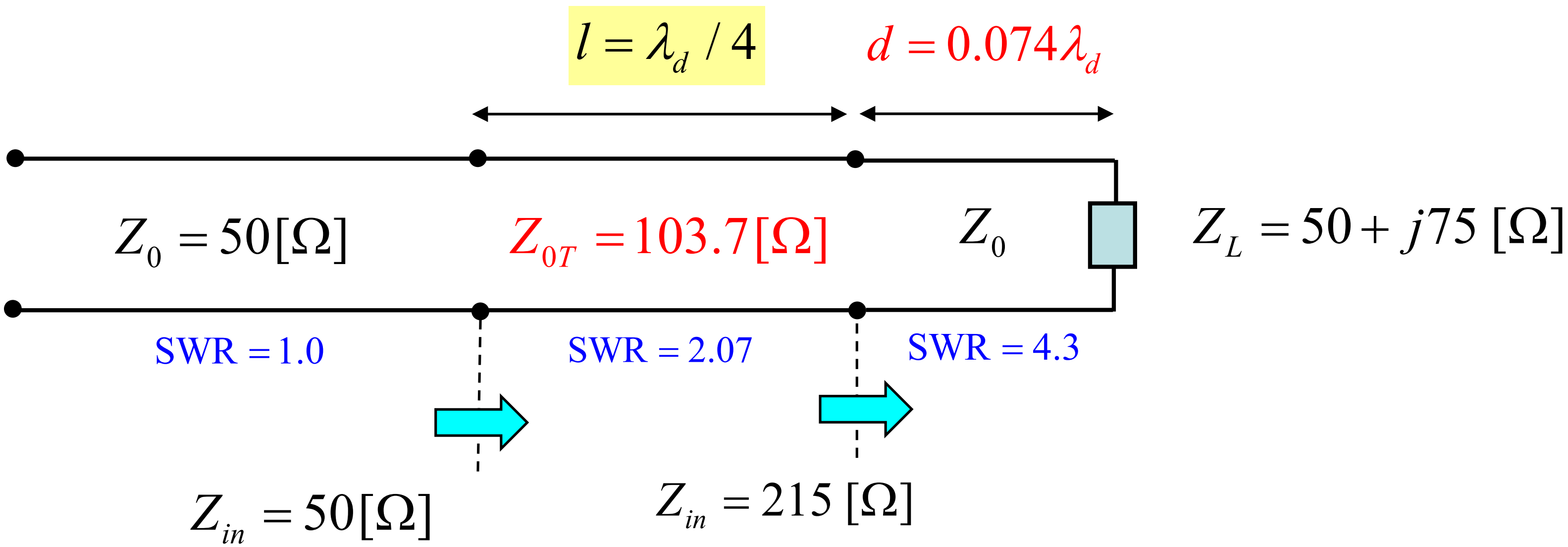
$$Z_{in}^{N}(-d) = 4.3 \quad \Rightarrow \quad Z_{in}(-d) = 50(4.3) = 215\ [\Omega]$$

$$Z_{0T} = \sqrt{(50)(215)}$$

$$Z_{0T} = 103.7[\Omega]$$

Quarter-Wave Transformer with Line Extension (cont.)

Summary of Design

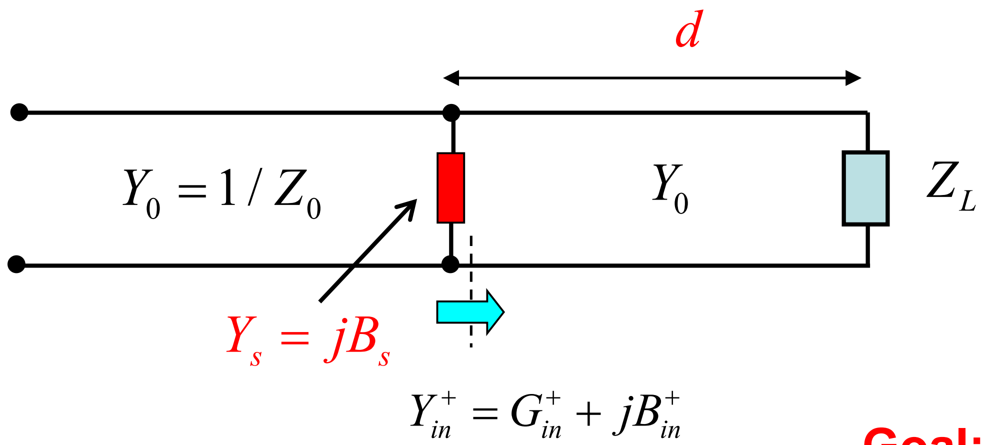


Note: On the transformer line, the line sees a real load R_{LT}, so for the transformer line we have:

$$\text{SWR} = \text{Max}\left(\frac{R_{LT}}{Z_{0T}}, \frac{Z_{0T}}{R_{LT}}\right) = \frac{215\ [\Omega]}{103.7\ [\Omega]} = 2.07$$

Single-Stub Matching

A parallel (shunt) susceptance is added at a distance d from the load.



Note:
The "+" denotes just to the right of the point $z = -d$.

Goal: determine d, B_s

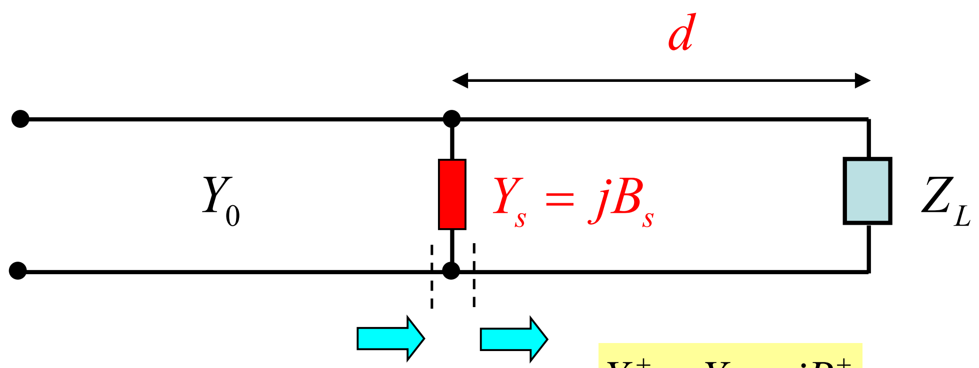
1) We choose the distance d so that at this distance from the load

$$Y_{in}^{+} = Y_0 + jB_{in}^{+} \qquad \left(\text{i.e., } G_{in}^{+} = Y_0, \text{ or } G_{in}^{N+} = 1\right)$$

2) We then choose the shunt susceptance so that

$$B_s = -B_{in}^{+}$$

Single-Stub Matching (cont.)



$$Y_{in}^{+} = Y_0 + jB_{in}^{+}$$

$$Y_{in}^{-} = Y_0$$

$$B_s = -B_{in}^{+}$$

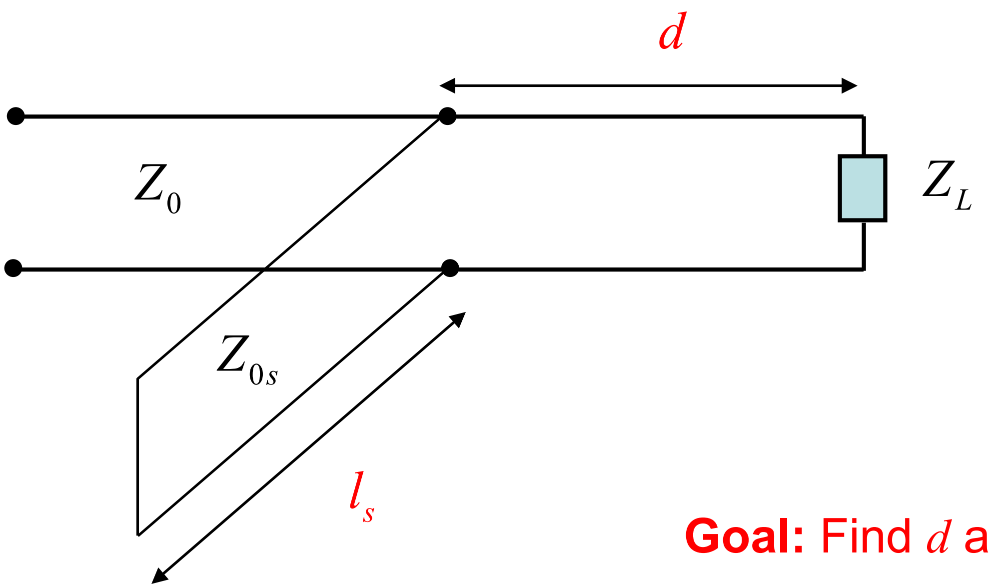
The feeding transmission line on the left sees a perfect match!

$$Y_{in}^{-} = Y_0 \;\Rightarrow\; Z_{in}^{-} = Z_0$$

Single-Stub Matching (cont.)

Realization using a shorted stub

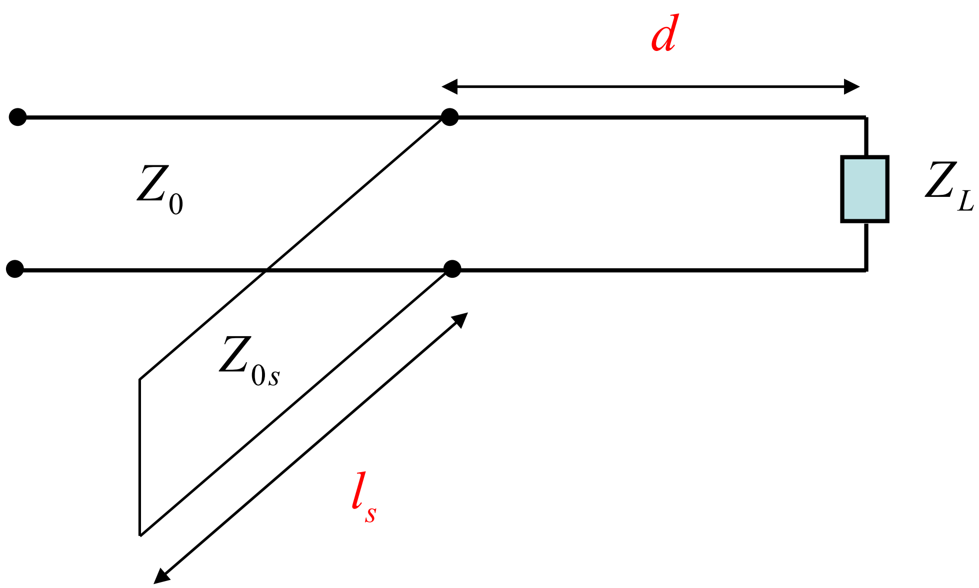
(An open-circuited stub could also be used.)



Goal: Find d and l_s.

Single-Stub Matching (cont.)

We use the Smith chart as an admittance calculator to determine the distance d.



1) Convert the load impedance Z_L to a load admittance Y_L.
2) Determine the distance d to make the normalized input conductance equal to 1.0.
3) Determine the required value of B_s to cancel B_{in}^{+} ($B_s = - B_{in}^{+}$).
4) Determine the stub length l_s from the value of B_s.

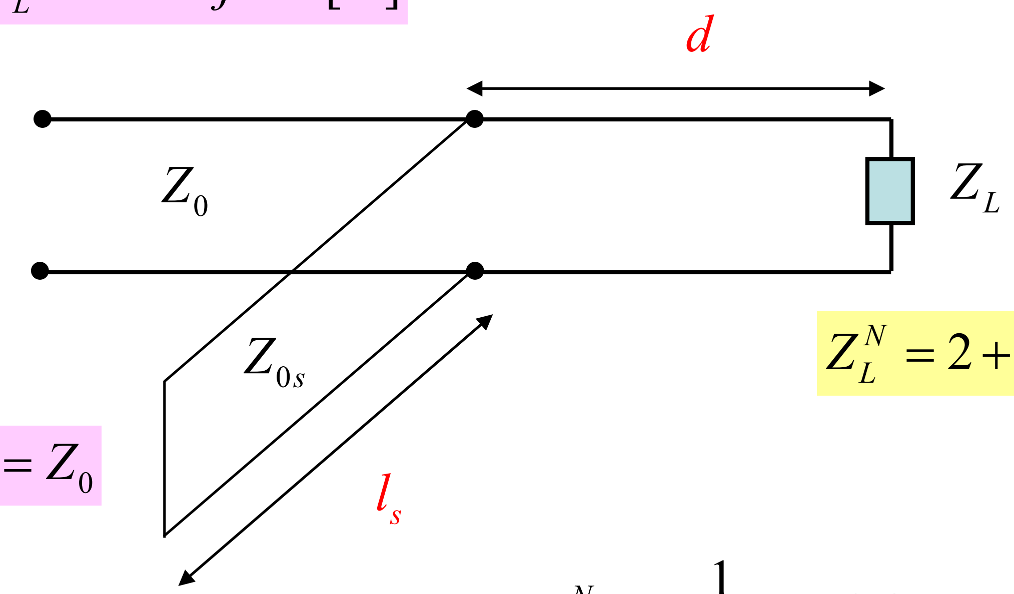
Note: If desired, we can use the Smith chart in step # 4 to find the stub length l_s.

Single-Stub Matching (cont.)

Example

$Z_0 = 50\ [\Omega]$

$Z_L = 100 + j100\ [\Omega]$



$Z_L^N = 2 + j2$

Assume $Z_{0s} = Z_0$

$$Y_L^N = \frac{1}{2 + j2} = 0.25 - j\left(0.25\right)$$

$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{Z_L^N - 1}{Z_L^N + 1}$$

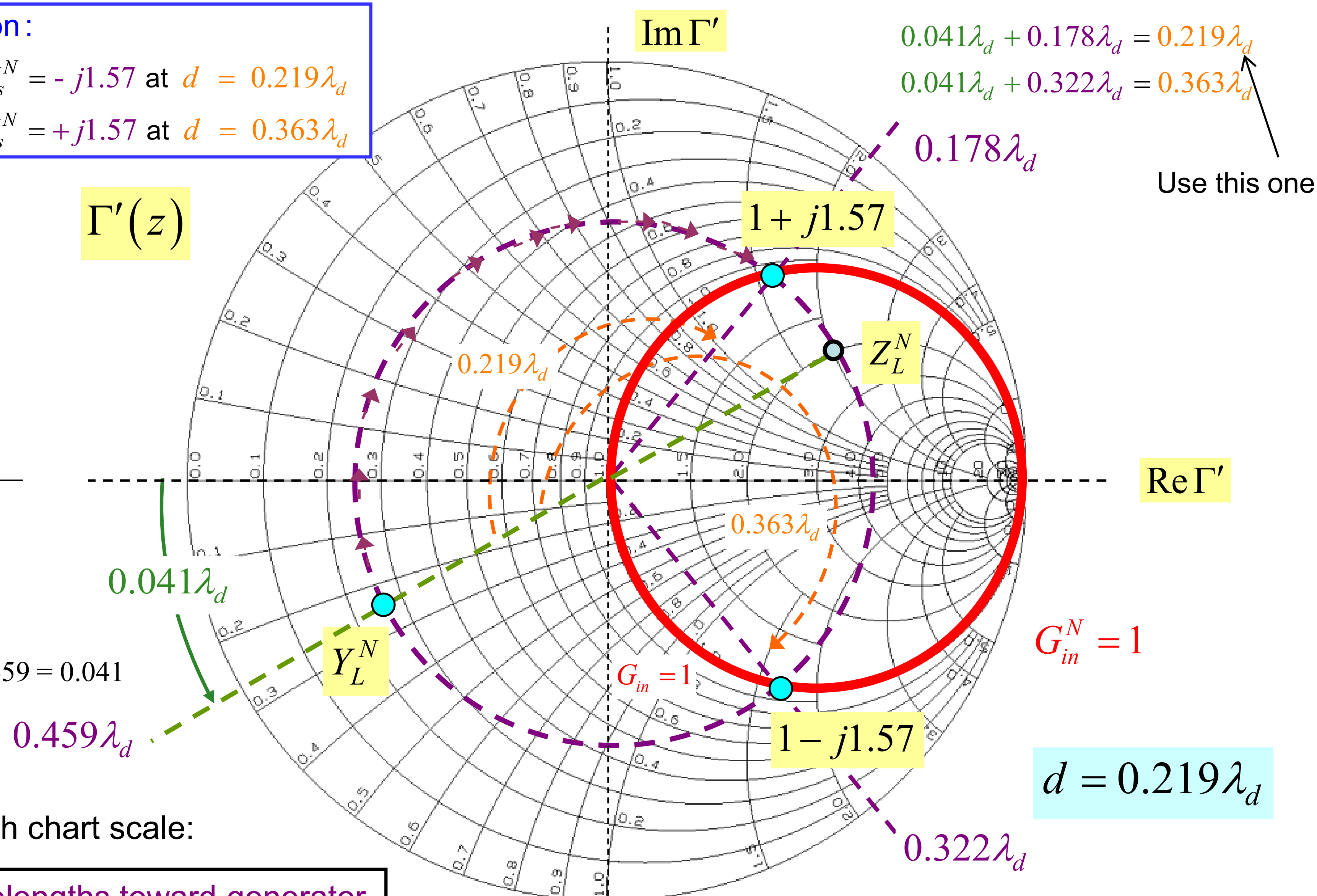
$$\Rightarrow \begin{cases} \Gamma_L = 0.62\, e^{j\pi/6} = 0.62\angle 30^{\circ} \\ \Gamma_L' = -0.62\, e^{j\pi/6} = -0.62\angle 30^{\circ} \end{cases}$$

Single-Stub Matching (cont.)

Solution:

Add $Y_s^N = -j1.57$ at $d = 0.219\lambda_d$

or $Y_s^N = +j1.57$ at $d = 0.363\lambda_d$



Note:
0.5 - 0.459 = 0.041

Smith chart scale:

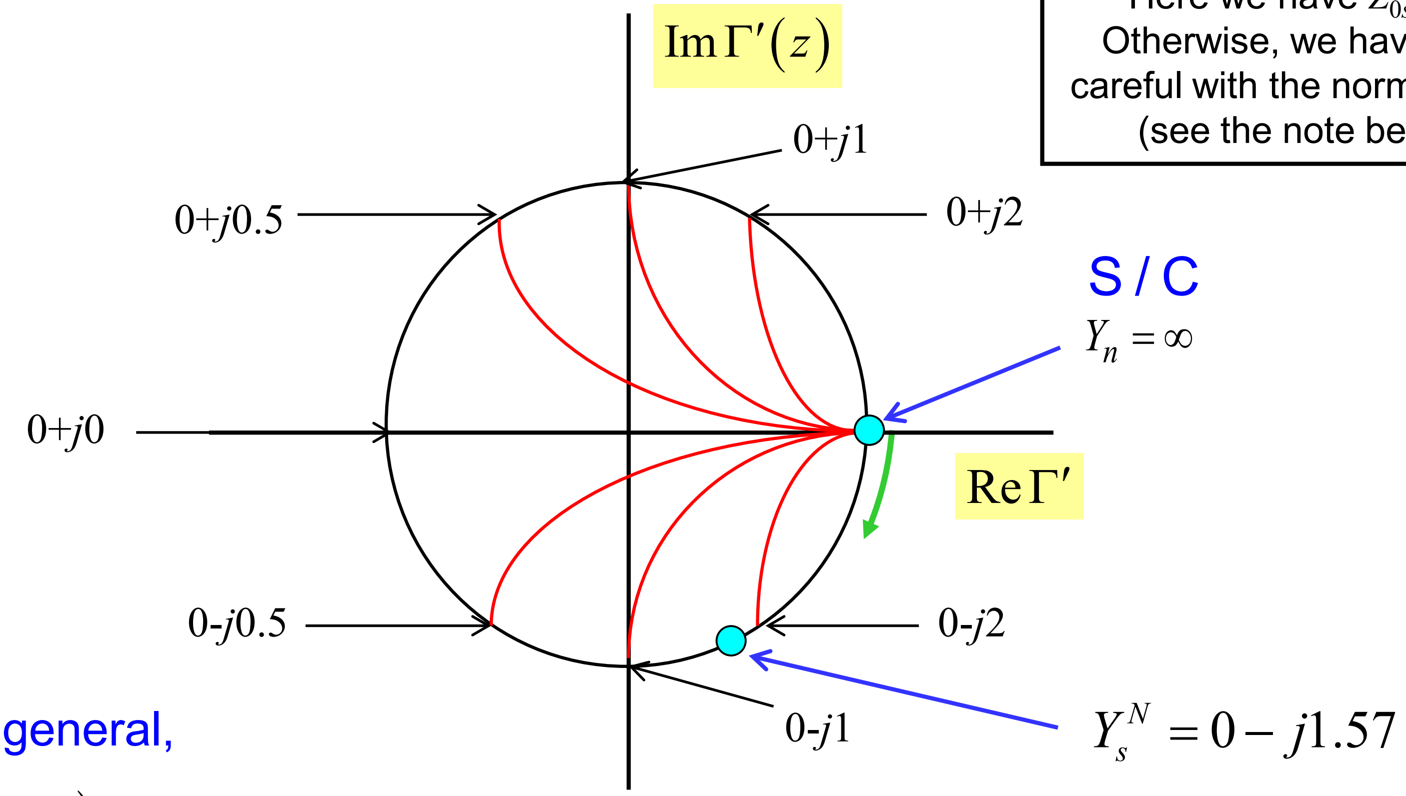
Wavelengths toward generator
Wavelengths toward load

Single-Stub Matching (cont.)

Next, we find the length of the short-circuited stub: $B_s^N = -1.57$

Rotate clockwise from S/C to desired B_s value.

Note:
Here we have $Z_{0s} = Z_0$.
Otherwise, we have to be careful with the normalization (see the note below).



Note: In general,

$$B_s^N = \left(-B_{in}^{N+} Y_0\right) / Y_{0s}$$
$$= -1.57 \left(Y_0 / Y_{0s}\right)$$

Admittance calculator

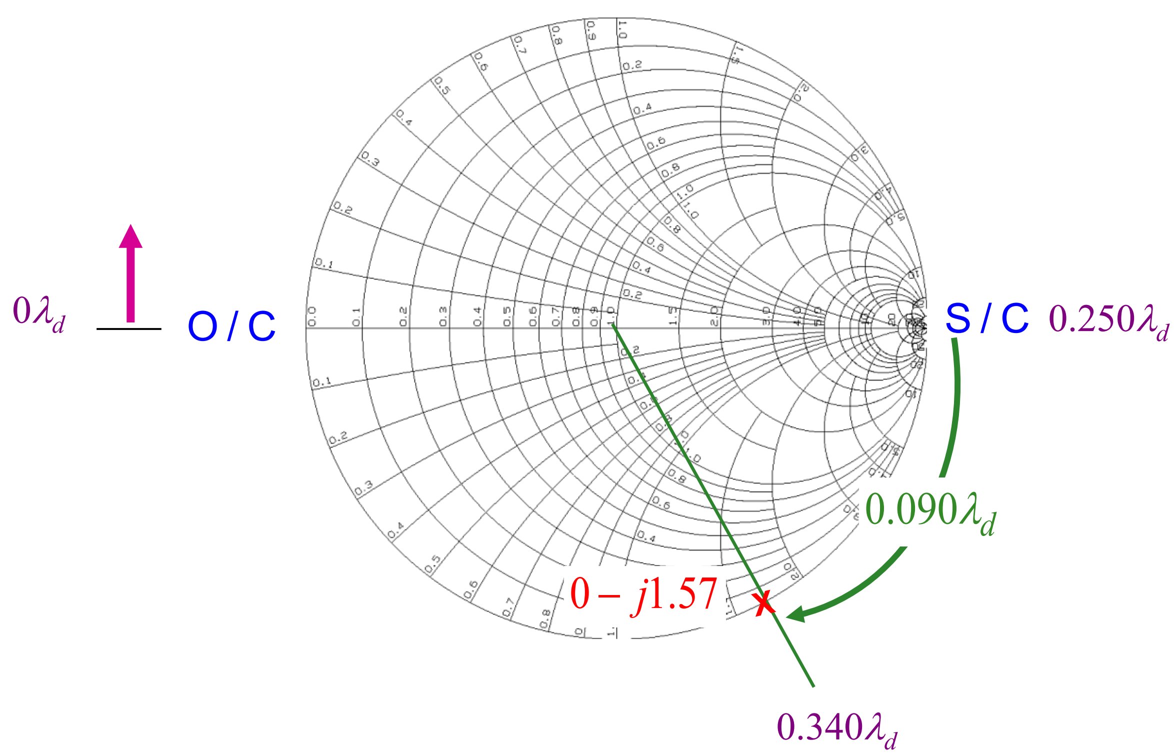
Single-Stub Matching (cont.)

From the Smith chart:

$$l_s = 0.340\lambda_d - 0.250\lambda_d$$

$$l_s = 0.090\lambda_d$$

Admittance chart



Analytically:

$$Z_s^{\text{short}} = jZ_{0s}\tan\left(\beta l_s\right)$$

$$\Rightarrow Y_s^{\text{short}} = -jY_{0s}\cot\left(\beta l_s\right)$$

$$\Rightarrow B_s^N = -\cot\left(\beta l_s\right)$$

Hence:

$$-1.57 = -\cot\beta l_s$$

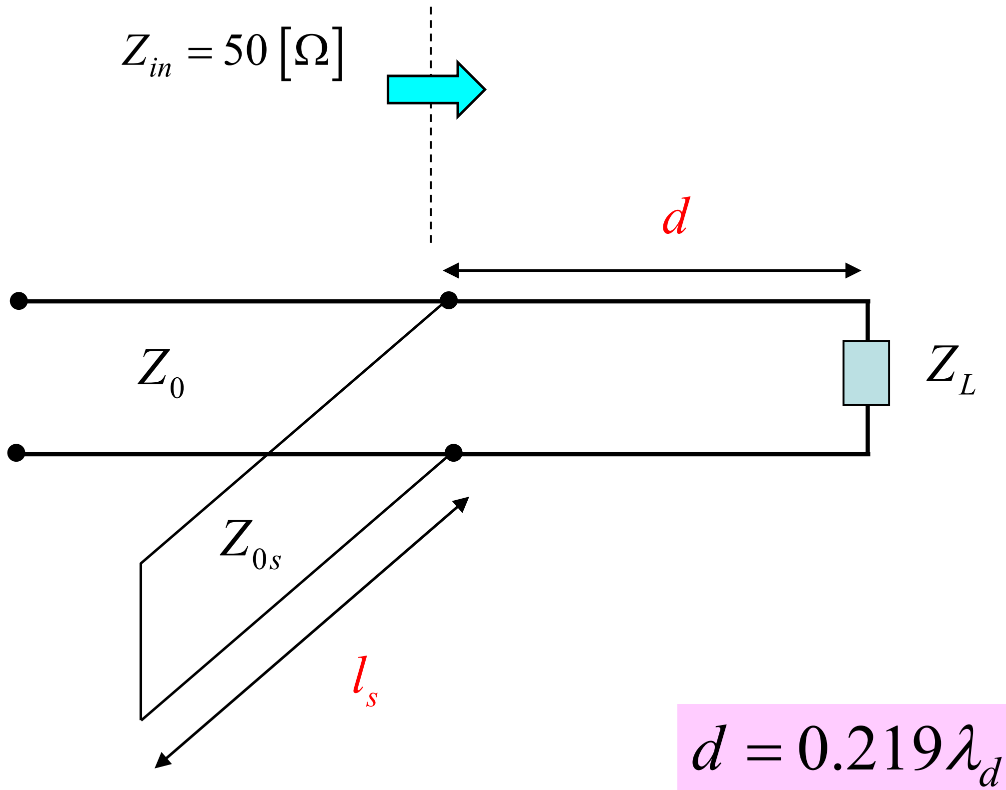
$$\cot\beta l_s = 1.57; \;\; \tan\beta l_s = \frac{1}{1.57} = 0.637$$

$$\beta l_s = \frac{2\pi}{\lambda_d} l_s = \tan^{-1}\left(0.637\right) = 0.567 \text{ [radians]}$$

$$l_s = 0.0903\lambda_d$$

Single-Stub Matching (cont.)

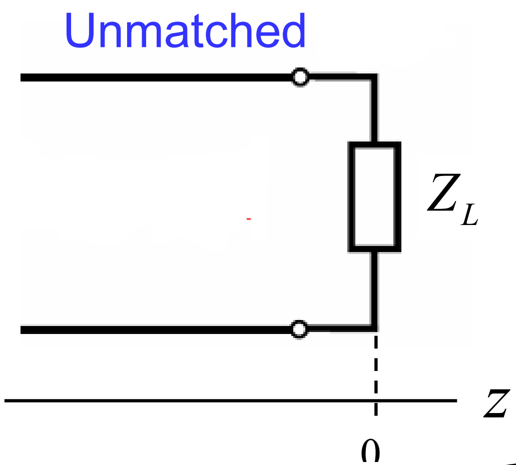
Final Design



$$d = 0.219\lambda_d$$

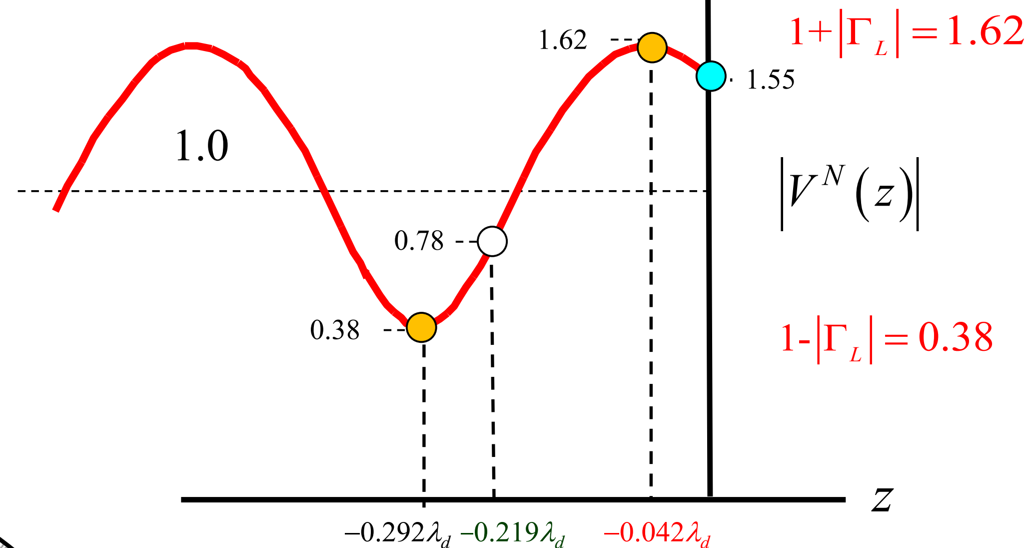
$$l_s = 0.0903\lambda_d$$

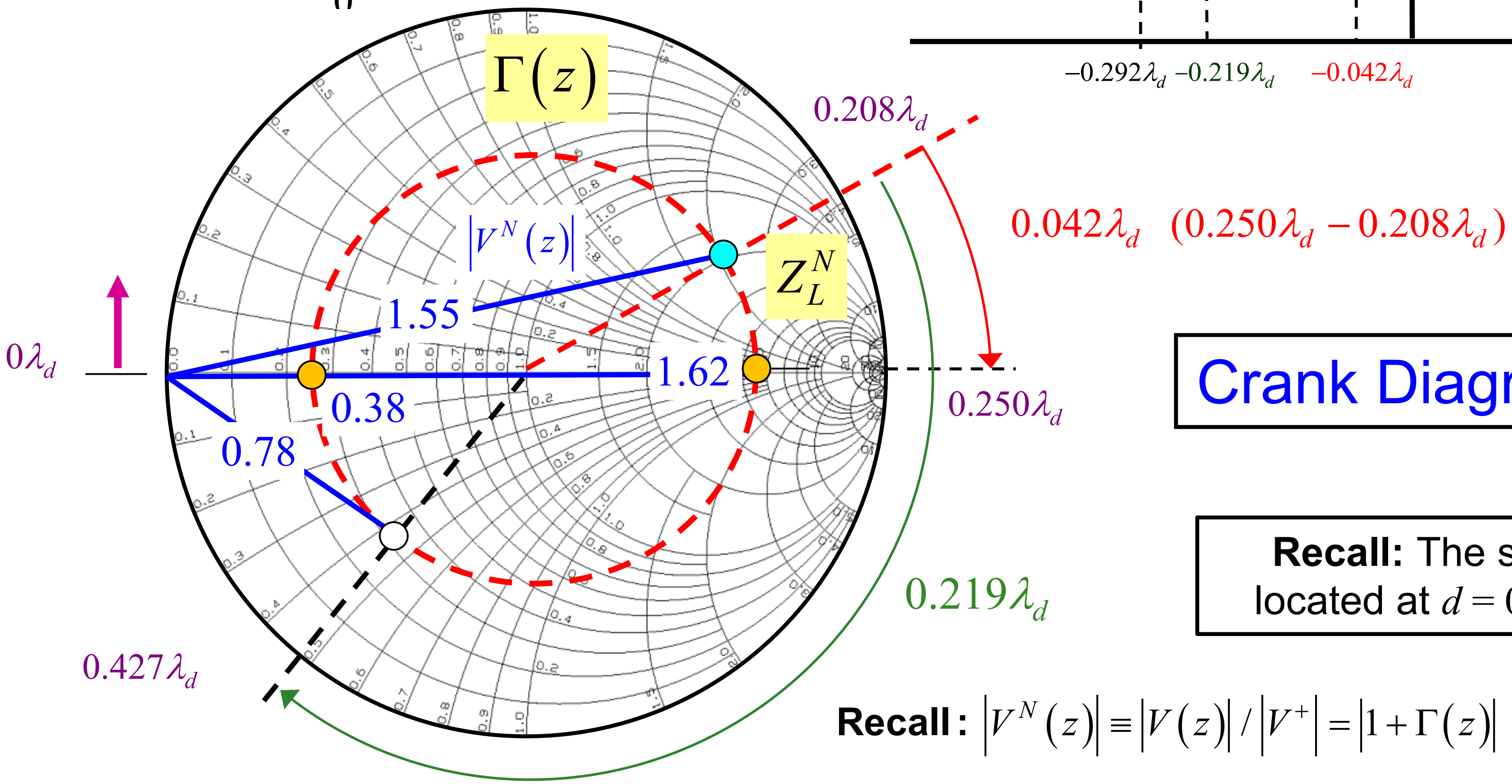
Single-Stub Matching (cont.)



$\Gamma_L = 0.62\, e^{j\pi/6}$

$|\Gamma_L| = 0.62$



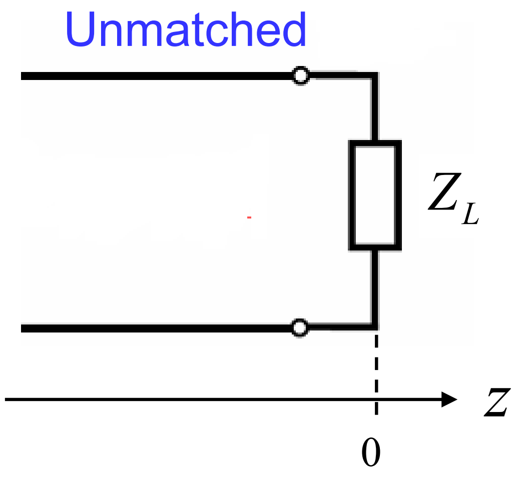


Crank Diagram

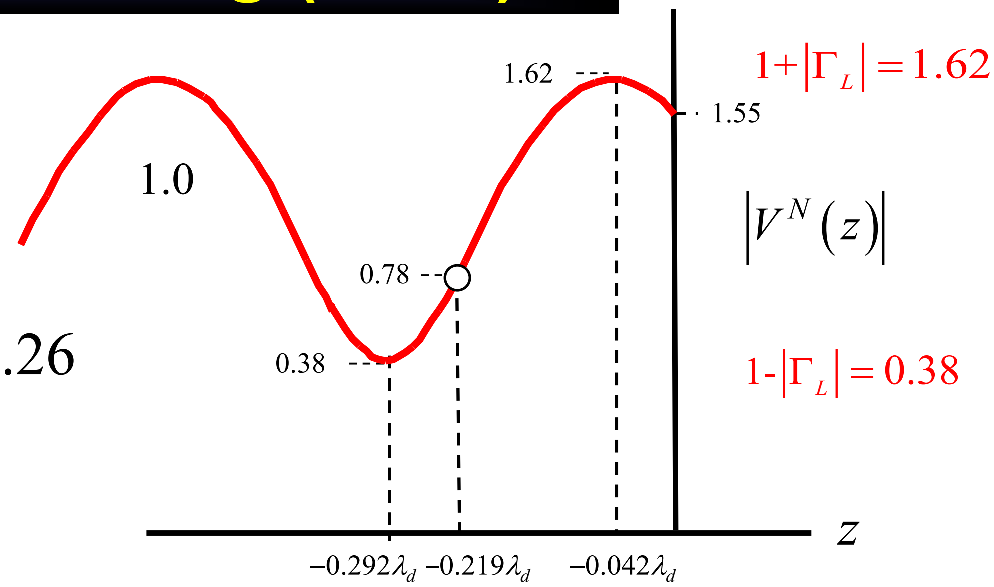
Recall: The stub is located at $d = 0.219\lambda_d$

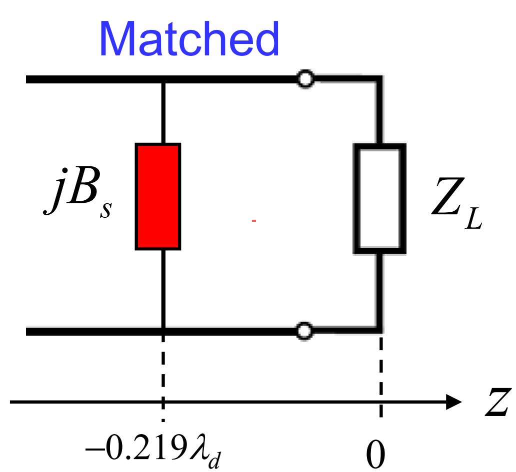
Recall: $|V^N(z)| \equiv |V(z)| / |V^+| = |1 + \Gamma(z)|$

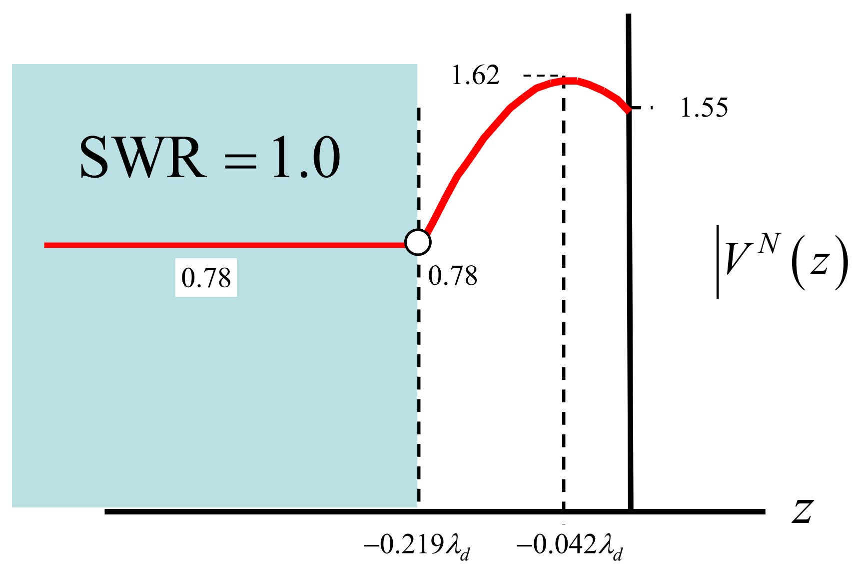
Single-Stub Matching (cont.)



$$\mathrm{SWR} = 4.26$$







Appendix: Summary of Methods

Quarter-wave transformer (real load)

$$Z_{0T} = \sqrt{Z_0 R_L}$$

Quarter-wave transformer (complex load)

(a) Shunt element (susceptance)

$$Z_{0T} = \sqrt{Z_0 / G_L}$$

$$B_s = -B_L$$

(b) Extension line (length d)

$d =$ length of extension line needed to convert load to real value :

$$Z_{in}\left(-d\right) = R_{in}$$

$$Z_{0T} = \sqrt{Z_0 R_{in}}$$

Appendix (cont.)

Single-Stub Matching

$d =$ length of main line needed to convert load admittance to complex value so that :

$$Y_{in}(-d) = Y_0 + jB_{in}(-d) \quad \left(G_{in}^{N} = 1\right)$$

$l_s =$ length of stub line needed to obtain desired stub susceptance :

$$Y_{in}^{stub} = jB_s = -jB_{in}(-d)$$