ECE 3317
Applied Electromagnetic Waves

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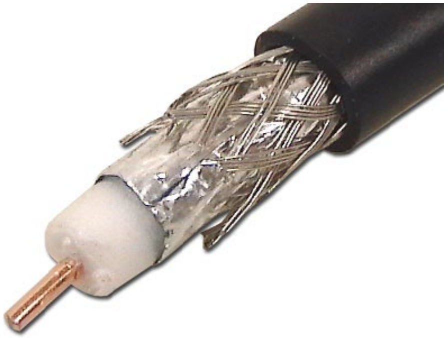
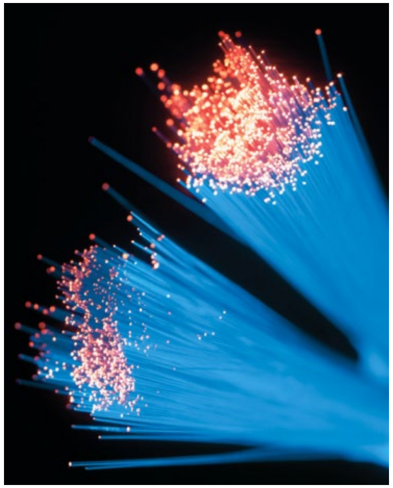
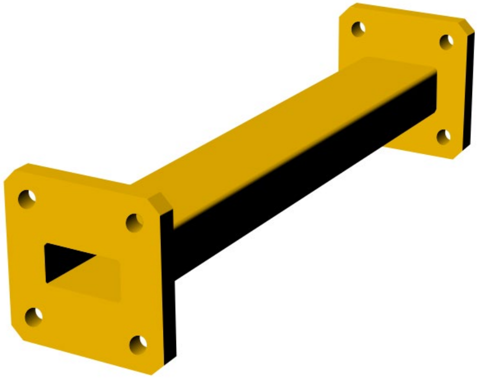
Notes 19

Waveguiding Structures

Waveguiding Structures

A waveguiding structure is one that carries a signal (or power) from one point to another without having energy escape.

There are three common types:

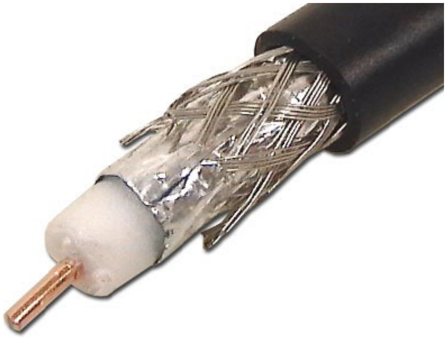
- Transmission lines
- Fiber-optic guides
- Waveguides (hollow pipes)

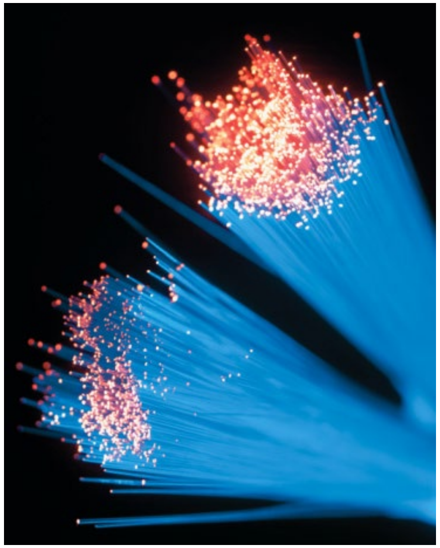
An alternative to a waveguiding system is a wireless system using antennas.

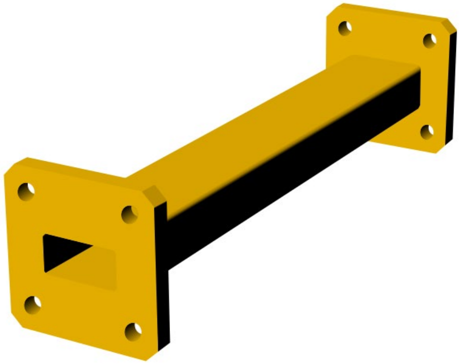
Waveguiding Structures (cont.)

Three common types of waveguiding structures:

- Transmission lines
- Fiber-optic guides
- Waveguides (hollow pipes)



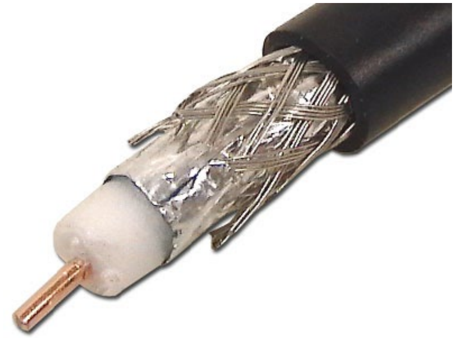




Transmission lines

Properties

- Has two conductors running parallel
- Can propagate a signal at any frequency (in theory)
- Becomes lossy at high frequency
- Can handle low or moderate amounts of power
- Has signal distortion due to loss
- May or may not be immune to interference
- Does not have E_z or H_z components of the fields (TEM_z)



$$\gamma = \alpha + j\beta = \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$k_z = -j\gamma = \beta - j\alpha$$

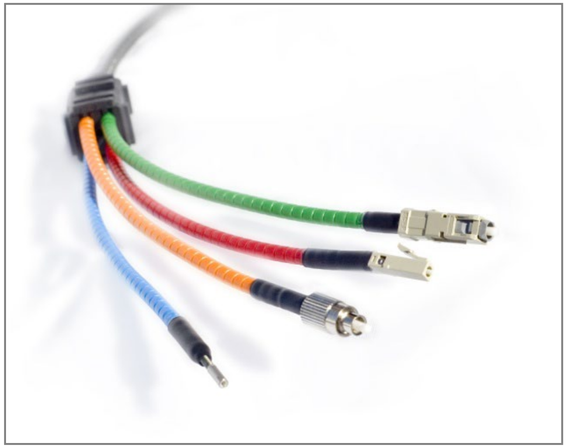
Lossless:

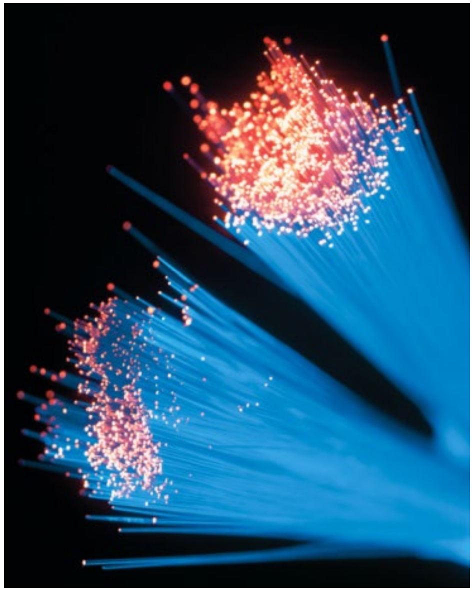
$$k_z = \omega\sqrt{LC} = \omega\sqrt{\mu\varepsilon} = k = k_0\sqrt{\varepsilon_r} \quad \text{(nonmagnetic filling)}$$

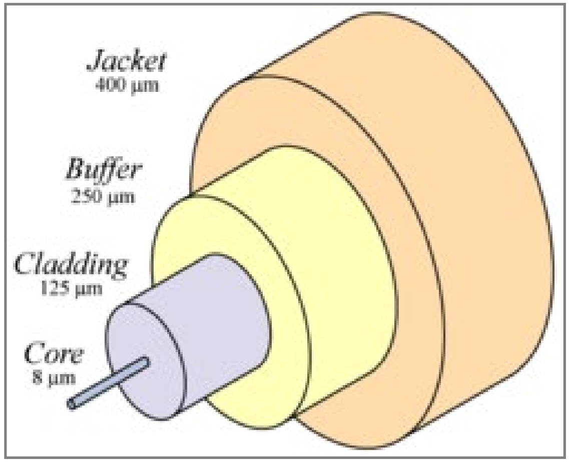
Fiber-Optic Guide

Properties

- Has a single dielectric rod
- Can propagate a signal at any frequency (in theory)
- Can be made very low loss (no metal + low-loss glass)
- Has minimal signal distortion
- Very immune to interference
- Not suitable for high power
- Has both E_z and H_z components of the fields ("hybrid mode")







Fiber-Optic Guide (cont.)

Two types of fiber-optic guides:

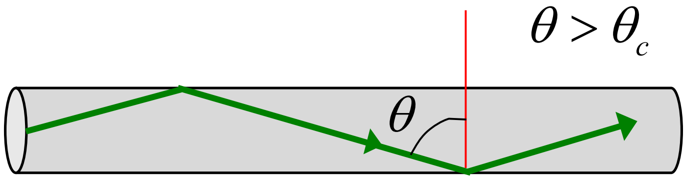
1) Single-mode fiber

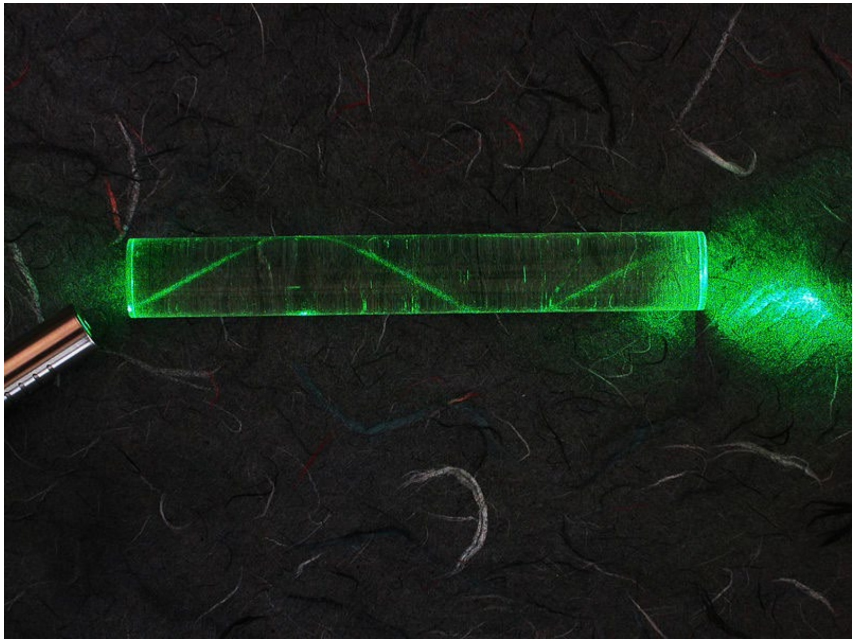
Carries a single mode. Requires the fiber diameter to be small relative to a wavelength. This is the lowest loss type of fiber.

2) Multi-mode fiber

Has a fiber diameter that is large relative to a wavelength. It operates on the principle of total internal reflection (critical angle effect).

Multi-Mode Fiber

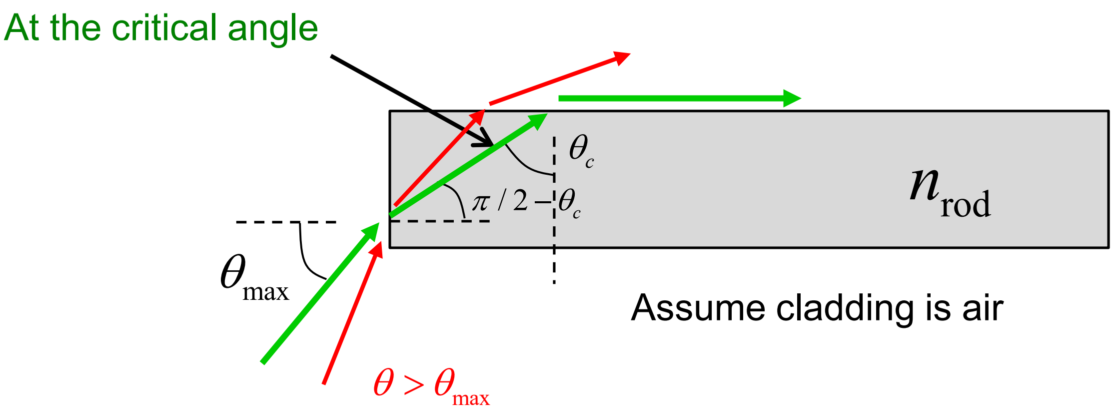


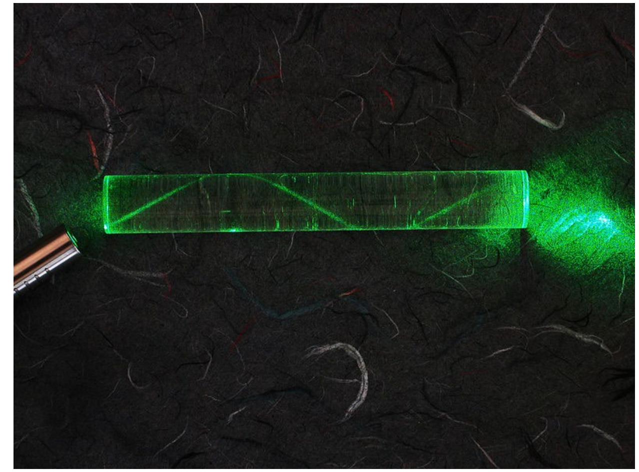


http://en.wikipedia.org/wiki/Optical_fiber

Multi-Mode Fiber (cont.)

There is a maximum angle θ_{max} at which the beam can be incident from outside the rod.





$$n_{\text{rod}} \sin\theta_c = n_{\text{air}} \sin(90^{\circ}) = 1$$

$$\Rightarrow \theta_c = \sin^{-1}(1/n_{\text{rod}})$$

$$\Rightarrow \cos\theta_c = \sqrt{1-\sin^2\theta_c} = \sqrt{1-(1/n_{\text{rod}}^2)}$$

At left end of rod (input), Snell's law gives us:

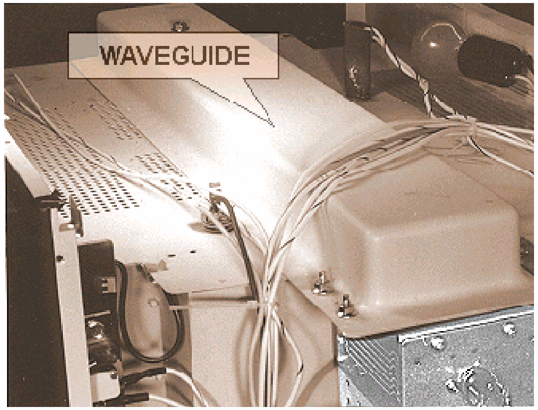
$$n_{\text{air}} \sin\theta_{\text{max}} = n_{\text{rod}} \sin\left(\frac{\pi}{2}-\theta_c\right) \Rightarrow \sin\theta_{\text{max}} = n_{\text{rod}} \cos\theta_c$$

$$\Rightarrow \sin\theta_{\text{max}} = n_{\text{rod}} \sqrt{1-(1/n_{\text{rod}}^2)}$$

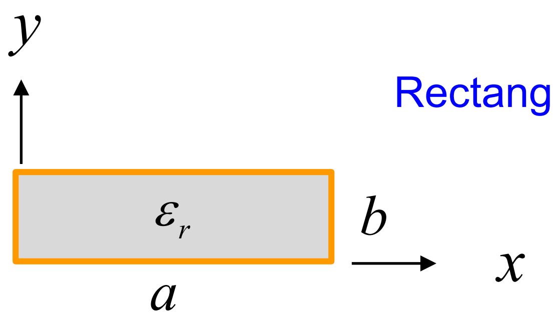
Waveguide

Properties

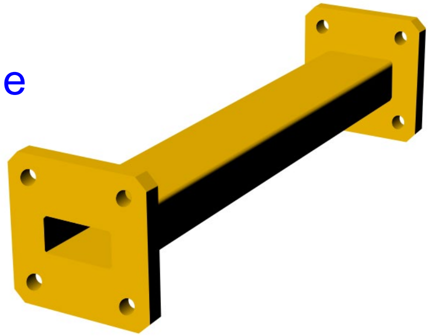
- Has a single hollow metal pipe
- Can propagate a signal only at high frequency: $f > f_c$
- The width a must be at least one-half of a wavelength λ_d
- Has signal distortion, even in the lossless case
- Immune to interference
- Can handle large amounts of power
- Has low loss (compared with a transmission line)
- Has either E_z or H_z component of the fields (TM_z or TE_z)



Inside microwave oven



Rectangular waveguide



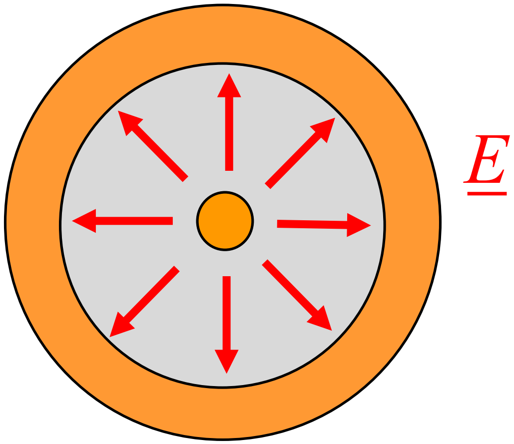
Note: The pipe may be filled with a material having ε_r. $\left(\lambda_d = \lambda_0 / \sqrt{\varepsilon_r}\right)$

https://en.wikipedia.org/wiki/Waveguide_(radio_frequency)

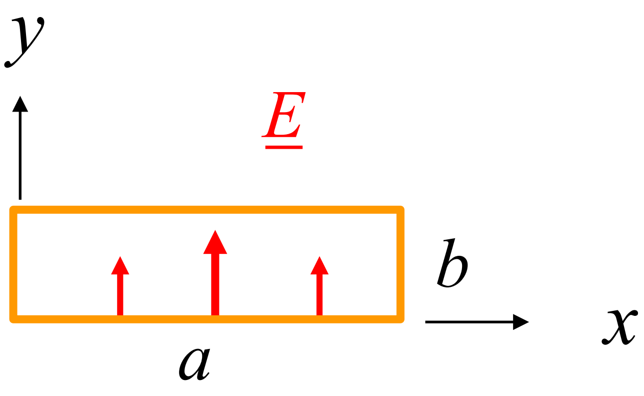
Waveguide (cont.)

Waveguide vs. Coax

- The coax has higher loss and is more susceptible to dielectric breakdown (can handle less power).
  - The electric field and surface current of the coax are concentrated near the narrow inner conductor (very strong electric field and current density there). This gives more loss and also more susceptibility to dielectric breakdown.
  - The waveguide is usually hollow, and thus has no dielectric loss.



Coax



Waveguide

Waveguide (cont.)

Wavenumber inside a waveguide (derived later):

$$k_z = \left(k^2 - k_c^2\right)^{1/2}$$

where

$$k = \omega\sqrt{\mu\varepsilon}$$ (wavenumber of material inside waveguide)

$$k_c = \pi / a$$ for dominant TE_{10} mode

Cutoff frequency f_c for a lossless waveguide (k is real):

frequency for which $k = k_c$

$f > f_c$: $k_z = \sqrt{k^2 - k_c^2} =$ real (propagation)

$f < f_c$: $k_z = -j\sqrt{k_c^2 - k^2} =$ imaginary (evanescent decay)

Field Expressions for a Guided Wave

Statement:

All six field components of a guided wave can be expressed in terms of the two fundamental field components E_z and H_z.

"Guided-wave theorem"

Assumption:

$$\underline{E}(x,y,z) = \underline{E}_0(x,y)\, e^{-jk_z z}$$
$$\underline{H}(x,y,z) = \underline{H}_0(x,y)\, e^{-jk_z z}$$

(This is the definition of a guided wave.)

A proof of this "statement" is given in Appendix A.

See the table on the next slide for the results.

Field Expressions of a Guided Wave (cont.)

Summary of Fields

$$E_x = \left(\frac{-j\omega\mu}{k^2 - k_z^2}\right)\frac{\partial H_z}{\partial y} - \left(\frac{jk_z}{k^2 - k_z^2}\right)\frac{\partial E_z}{\partial x}$$

$$E_y = \left(\frac{j\omega\mu}{k^2 - k_z^2}\right)\frac{\partial H_z}{\partial x} - \left(\frac{jk_z}{k^2 - k_z^2}\right)\frac{\partial E_z}{\partial y}$$

$$H_x = \left(\frac{j\omega\varepsilon}{k^2 - k_z^2}\right)\frac{\partial E_z}{\partial y} - \left(\frac{jk_z}{k^2 - k_z^2}\right)\frac{\partial H_z}{\partial x}$$

$$H_y = \left(\frac{-j\omega\varepsilon}{k^2 - k_z^2}\right)\frac{\partial E_z}{\partial x} - \left(\frac{jk_z}{k^2 - k_z^2}\right)\frac{\partial H_z}{\partial y}$$

TEM_z Wave

Assume a TEM_z wave:

$$E_z = 0$$
$$H_z = 0$$

To avoid having a completely zero field (see table on previous slide):

$$k^2 - k_z^2 = 0$$

Hence,

$$\text{TEM}_z \iff k_z = k$$

TEM$_z$ Wave (cont.)

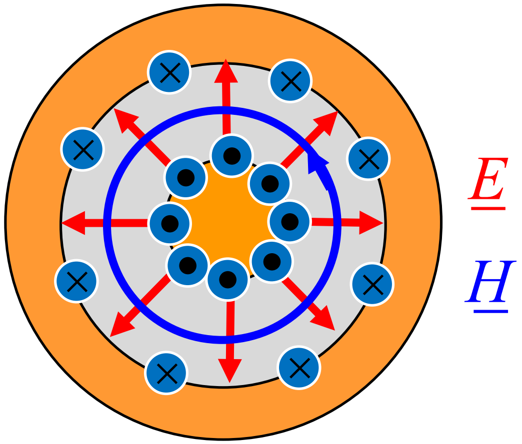
Examples of TEM$_z$ waves:

- A wave on a transmission line*
- A plane wave

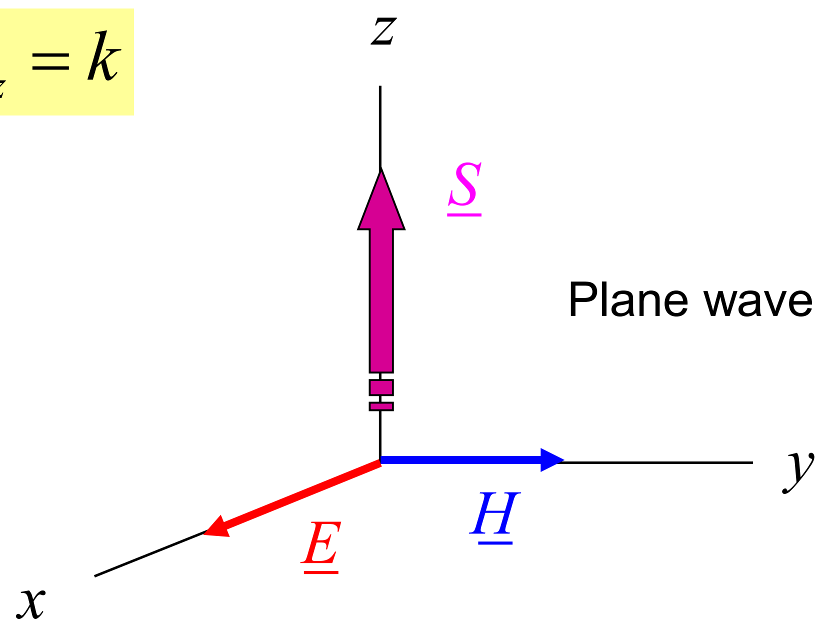
* Exactly true if there is no conductor loss.

In each case the fields do not have a z component !

$$k_z = k$$



Coax



Plane wave

TEM$_z$ Wave (cont.)

Relation Between $\underline{E}$ and $\underline{H}$ for any TEM$_z$ Mode:

$$\underline{E} = -\eta\left(\underline{\hat{z}} \times \underline{H}\right)$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \eta_0\sqrt{\frac{\mu_r}{\varepsilon_r}} \quad \left(\eta_0 = 376.7303\ [\Omega]\right)$$

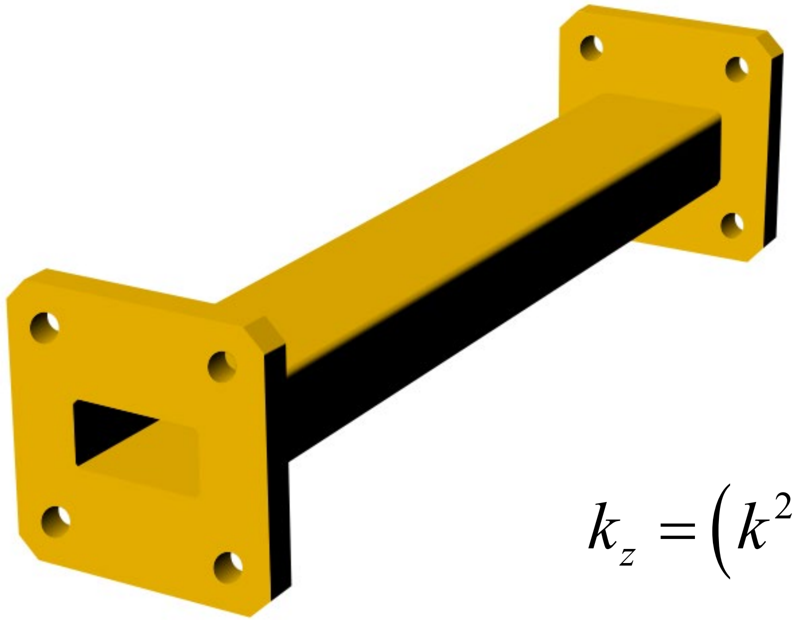
(intrinsic impedance of filling material)

The electric and magnetic fields of a TEM$_z$ wave are perpendicular to each other, and the amplitudes of them are related by η.

A proof is given in Appendix B.

Waveguide Modes

In a waveguide, the fields cannot be TEM$_z$.



$$k_z = \left(k^2 - k_c^2\right)^{1/2} \neq k$$

Waveguide Modes (cont.)

In a waveguide (hollow pipe of metal), there are <u>two</u> types of modes:

$$\text{TM}_z: \quad H_z = 0, \quad E_z \neq 0$$

$$\text{TE}_z: \quad E_z = 0, \quad H_z \neq 0$$

Each type of mode can exist independently.

Appendix A

Proof of Guided Wave Theorem

(illustrated for E_y)

$$\nabla \times \underline{H} = j\omega\varepsilon \underline{E}$$

$$\Rightarrow \quad E_y = \frac{1}{j\omega\varepsilon}\left(-\frac{\partial H_z}{\partial x} + \frac{\partial H_x}{\partial z}\right)$$

or

$$E_y = \frac{1}{j\omega\varepsilon}\left(-\frac{\partial H_z}{\partial x} - jk_z H_x\right)$$

Now solve for H_x:

$$\nabla \times \underline{E} = -j\omega\mu \underline{H}$$

Appendix A (cont.)

$$\nabla \times \underline{E} = -j\,\omega\mu\,\underline{H}$$

$$\Rightarrow \quad H_x = -\frac{1}{j\omega\mu}\left(\frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z}\right)$$

$$= -\frac{1}{j\omega\mu}\left(\frac{\partial E_z}{\partial y} + jk_z E_y\right)$$

Substituting this into the equation for E_y yields the result

$$E_y = \frac{1}{j\omega\varepsilon}\left[-\frac{\partial H_z}{\partial x} - jk_z\left[\left(-\frac{1}{j\omega\mu}\right)\left(\frac{\partial E_z}{\partial y} + jk_z E_y\right)\right]\right]$$

Next, multiply by $-j\omega\mu\left(j\omega\varepsilon\right) = k^2$

Appendix A (cont.)

This gives us

$$k^2 E_y = j\omega\mu \frac{\partial H_z}{\partial x} - jk_z \frac{\partial E_z}{\partial y} + k_z^2 E_y$$

Combining terms,

$$\left(k^2 - k_z^2\right) E_y = j\omega\mu \frac{\partial H_z}{\partial x} - jk_z \frac{\partial E_z}{\partial y}$$

Solving for E_y, we have:

$$E_y = \left(\frac{j\omega\mu}{k^2 - k_z^2}\right) \frac{\partial H_z}{\partial x} - \left(\frac{jk_z}{k^2 - k_z^2}\right) \frac{\partial E_z}{\partial y}$$

The other three components E_x, H_x, H_y may be found similarly.

Appendix B

Relation Between $\underline{E}$ and $\underline{H}$ for any TEM$_z$ Mode

Faraday's Law: $\nabla \times \underline{E} = -j\omega\mu\underline{H}$

Take the x component of both sides: $\dfrac{\partial E_z}{\partial y} - \dfrac{\partial E_y}{\partial z} = -j\omega\mu H_x$

Assume that the field varies as $E_y(x,y,z) = E_{y0}(x,y)\, e^{-jkz}$

Hence, $-(-jk)E_y = -j\omega\mu H_x$

Therefore, we have $\dfrac{E_y}{H_x} = -\dfrac{\omega\mu}{k} = -\dfrac{\omega\mu}{\omega\sqrt{\mu\varepsilon}} = -\sqrt{\dfrac{\mu}{\varepsilon}} = -\eta$

Appendix B (cont.)

$$\nabla \times \underline{E} = -j\omega\mu \underline{H}$$

Now take the y component of both sides: $-\frac{\partial E_z}{\partial x} + \frac{\partial E_x}{\partial z} = -j\omega\mu H_y$

Hence, $(-jk) E_x = -j\omega\mu H_y$

Therefore, we have $\frac{E_x}{H_y} = \frac{\omega\mu}{k} = \frac{\omega\mu}{\omega\sqrt{\mu\varepsilon}} = \sqrt{\frac{\mu}{\varepsilon}} = \eta$

Hence, $\frac{E_x}{H_y} = \eta$

Appendix B (cont.)

Summary: $\dfrac{E_y}{H_x} = -\eta \qquad \dfrac{E_x}{H_y} = \eta$

These two equations may be written as a single vector equation:

$$\underline{E} = -\eta\left(\underline{\hat{z}} \times \underline{H}\right)$$

The electric and magnetic fields of a TEM_z wave are perpendicular to each other, and the amplitudes of them are related by η.