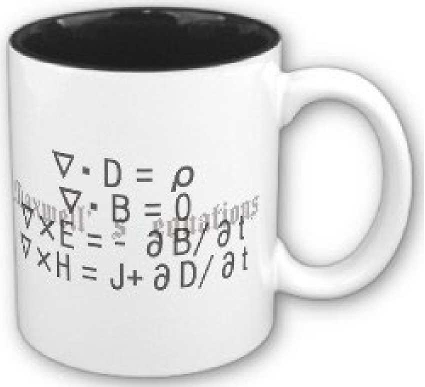
ECE 3317
Applied Electromagnetic Waves

Prof. David R. Jackson

Fall 2023



Notes 4
Maxwell's Equations

Adapted from notes by Prof. Stuart A. Long

Overview

Here we present an overview of Maxwell's equations. A much more thorough discussion of Maxwell's equations may be found in the text and class notes for ECE 3318:

http://courses.egr.uh.edu/ECE/ECE3318

Notes 10: Electric Gauss's law
Notes 18: Faraday's law
Notes 28: Ampere's law
Notes 28: Magnetic Gauss law

Extra reference: D. Fleisch, *A Student's Guide to Maxwell's Equations*, Cambridge University Press, 2008. (This is on reserve in the Library.)

Electromagnetic Fields

Four vector quantities

$\mathscr{E}$	electric field	[Volt/meter]
$\mathscr{D}$	electric flux density	[Coulomb/meter^2]
$\mathscr{H}$	magnetic field	[Amp/meter]
$\mathscr{B}$	magnetic flux density	[Weber/meter^2] or [Tesla]

Each are functions of space and time
e.g. $\mathscr{E}(x,y,z,t)$

Reminder:
The Handscript SF font is used to denote time-varying vectors.

$\mathscr{J}$	electric current density	[Amp/meter^2]
ρ_v	electric charge density	[Coulomb/meter^3]

MKS units

Length – meter [m]

Mass – kilogram [kg]

Time – second [s]

Some common prefixes and the power of ten each represent are listed below

femto - f - 10^{-15}	centi - c - 10^{-2}	mega - M - 10^{6}
pico - p - 10^{-12}	deci - d - 10^{-1}	giga - G - 10^{9}
nano - n - 10^{-9}	deka - da - 10^{1}	tera - T - 10^{12}
micro - μ - 10^{-6}	hecto - h - 10^{2}	peta - P - 10^{15}
milli - m - 10^{-3}	kilo - k - 10^{3}	

Maxwell's Equations

(Time-varying, differential form)

$$\nabla \times \underline{\mathscr{E}} = -\frac{\partial \underline{\mathscr{B}}}{\partial t}$$

$$\nabla \times \underline{\mathscr{H}} = \underline{\mathscr{J}} + \frac{\partial \underline{\mathscr{D}}}{\partial t}$$

$$\nabla \cdot \underline{\mathscr{B}} = 0$$

$$\nabla \cdot \underline{\mathscr{D}} = \rho_v$$

Maxwell

James Clerk Maxwell (1831–1879)

James Clerk Maxwell was a Scottish mathematician and theoretical physicist. His most significant achievement was the development of the classical electromagnetic theory, synthesizing all previous unrelated observations, experiments and equations of electricity, magnetism and even optics into a consistent theory. His set of equations—Maxwell's equations—demonstrated that electricity, magnetism and even light are all manifestations of the same phenomenon: the electromagnetic field. From that moment on, all other classical laws or equations of these disciplines became simplified cases of Maxwell's equations. Maxwell's work in electromagnetism has been called the "*second great unification in physics*", after the first one carried out by Isaac Newton.

Maxwell demonstrated that electric and magnetic fields travel through space in the form of waves, and at the constant speed of light. Finally, in 1864 Maxwell wrote *A Dynamical Theory of the Electromagnetic Field* where he first proposed that light was in fact undulations in the same medium that is the cause of electric and magnetic phenomena. His work in producing a unified model of electromagnetism is considered to be one of the greatest advances in physics.

(Wikipedia)

Maxwell's Equations (cont.)

$$\nabla \times \underline{\mathcal{E}} = -\frac{\partial \underline{\mathcal{B}}}{\partial t}$$ Faraday's law

$$\nabla \times \underline{\mathcal{H}} = \underline{\mathcal{J}} + \frac{\partial \underline{\mathcal{D}}}{\partial t}$$ Ampere's law

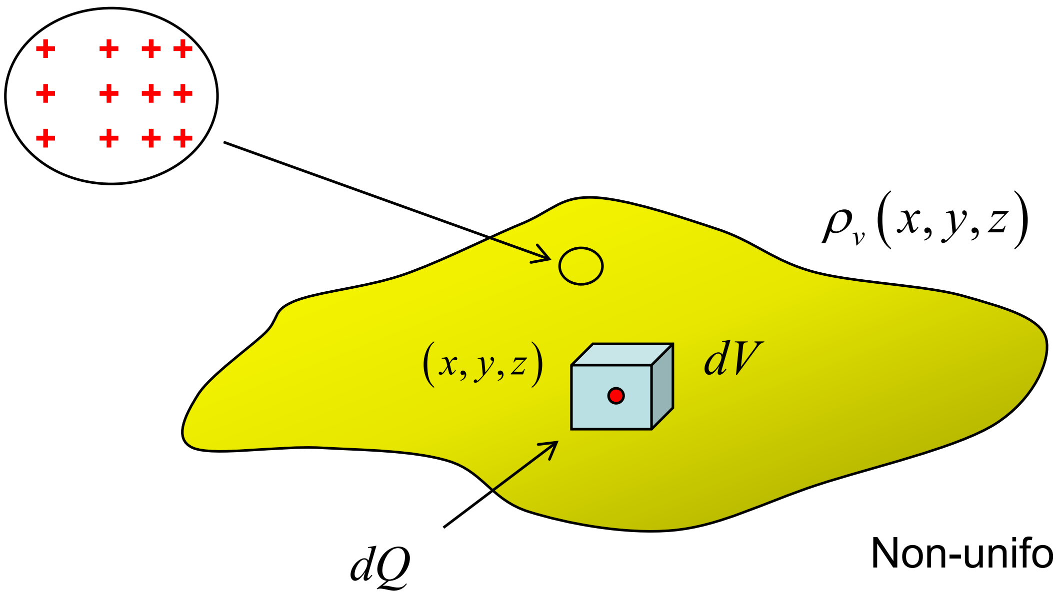
$$\nabla \cdot \underline{\mathcal{B}} = 0$$ Magnetic Gauss law

$$\nabla \cdot \underline{\mathcal{D}} = \rho_v$$ Electric Gauss law

Questions: When does a magnetic field produce an electric field? When does an electric field produce a magnetic field? When does a current flow produce a magnetic field? When does a charge density produce an electric field?

Charge Density

$$\rho_v(x,y,z) = \lim_{\Delta V \to 0} \frac{\Delta Q}{\Delta V} = \frac{dQ}{dV}$$

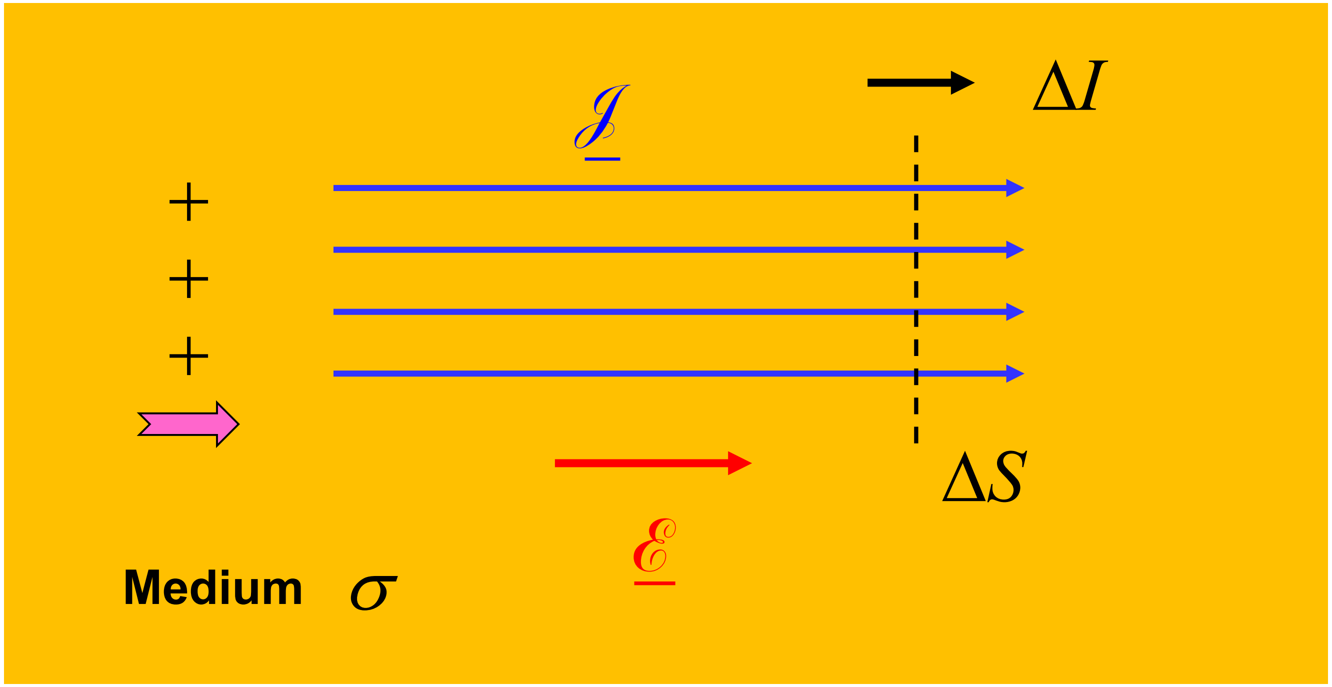


Non-uniform cloud of charge density

Example: Protons are closer together as we move to the right.

Current Density Vector

$\underline{\mathscr{J}}$ = current density vector $\left[\mathrm{A/m^2}\right]$



$$\Delta I = \left|\underline{\mathscr{J}}\right| \Delta S$$

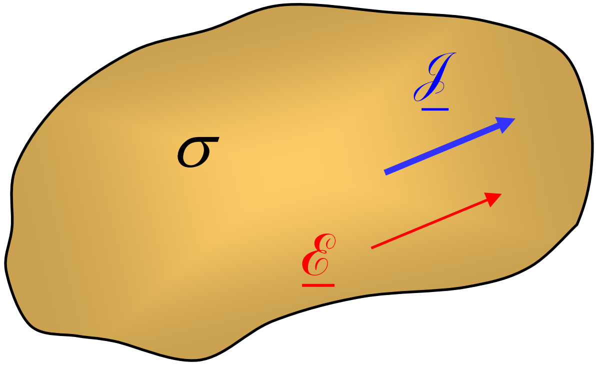
Current flow is defined to be in the direction that <u>positive</u> charges move in.

Note: If negative charges are moving, we can pretend that positive charges are moving in the opposite direction.

Current Density Vector (cont.)

Ohm's law

$$\underline{\mathscr{J}} = \sigma \underline{\mathscr{E}}$$

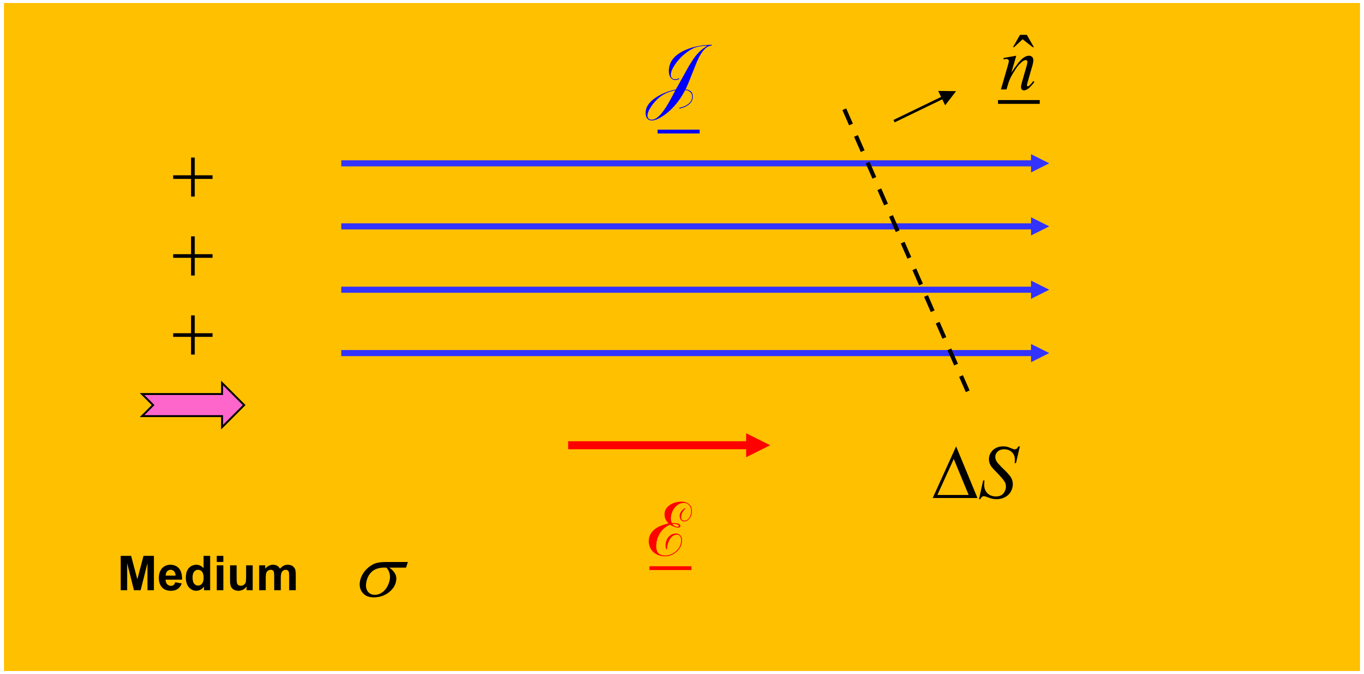


Material	σ [S/m]
Silver	6.3×10^{7}
Copper	6.0×10^{7}
Copper (annealed)	5.8×10^{7}
Gold	4.1×10^{7}
Aluminum	3.5×10^{7}
Zinc	1.7×10^{7}
Brass	1.6×10^{7}
Nickel	1.4×10^{7}
Iron	1.0×10^{7}
Tin	9.2×10^{6}
Steel (carbon)	7.0×10^{6}
Steel (stainless)	1.5×10^{6}

http://en.wikipedia.org/wiki/Electrical_resistivity_and_conductivity

Current Density Vector (cont.)

Current through a tilted surface:



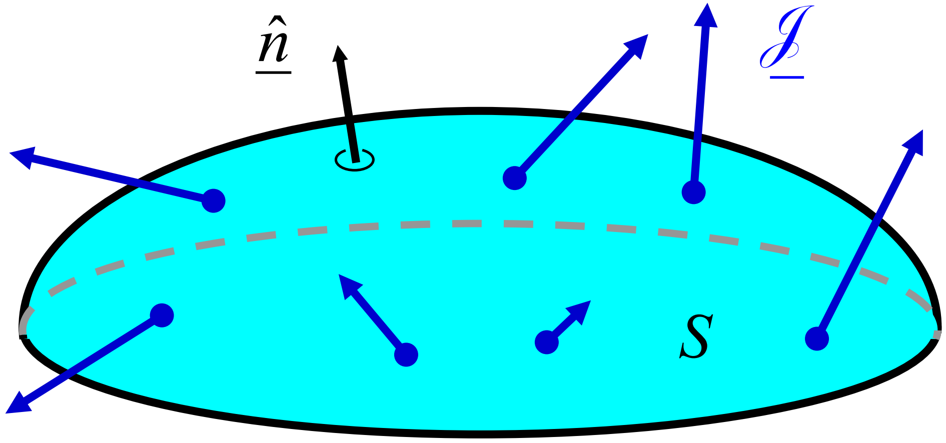
$$\Delta I = \left(\underline{\mathscr{J}} \cdot \underline{\hat{n}} \right) \Delta S$$

Current Density Vector (cont.)

$$\Delta I = \left(\underline{\mathcal{J}} \cdot \underline{\hat{n}}\right) \Delta S$$

$$I = \int_S \underline{\mathcal{J}} \cdot \underline{\hat{n}} \; dS$$

Note:
The direction of the unit normal vector determines whether the current is measured going up or down through the surface.



Law of Conservation of Electric Charge (Continuity Equation)

$$\nabla \times \underline{\mathcal{H}} = \underline{\mathcal{J}} + \frac{\partial \underline{\mathcal{D}}}{\partial t}$$

("zero identity")

$$\nabla \cdot \left(\nabla \times \underline{\mathcal{H}} \right) = \nabla \cdot \underline{\mathcal{J}} + \nabla \cdot \left(\frac{\partial \underline{\mathcal{D}}}{\partial t} \right)$$

$$0 = \nabla \cdot \underline{\mathcal{J}} + \frac{\partial}{\partial t} \left(\nabla \cdot \underline{\mathcal{D}} \right) \qquad \left(\text{Recall}: \nabla \cdot \underline{\mathcal{D}} = \rho_v \right)$$

Flow of electric current out of volume (per unit volume) ⟶

$$\nabla \cdot \underline{\mathcal{J}} = -\frac{\partial \rho_v}{\partial t}$$

⟵ Rate of decrease of electric charge (per unit volume)

This is the continuity equation in point or differential form.

Continuity Equation (cont.)

$$\nabla \cdot \underline{\mathcal{J}} = -\frac{\partial \rho_v}{\partial t}$$

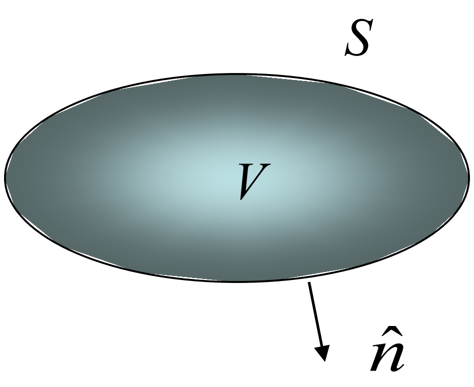
Integrate both sides over an arbitrary volume V:

$$\int_V \nabla \cdot \underline{\mathcal{J}} \, dV = \int_V -\frac{\partial \rho_v}{\partial t} \, dV$$

Apply the divergence theorem:

$$\int_V \nabla \cdot \underline{\mathcal{J}} \, dV = \oint_S \underline{\mathcal{J}} \cdot \hat{n} \, dS = i_{\text{out}}$$

(current flowing out of V)



(outward normal)

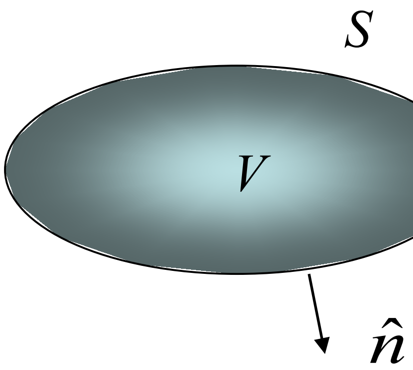
Hence:

$$i_{\text{out}} = \int_V -\frac{\partial \rho_v}{\partial t} \, dV$$

Continuity Equation (cont.)

$$i_{\text{out}} = \int_V -\frac{\partial \rho_v}{\partial t}\, dV$$

Physical interpretation:



Right-hand side:

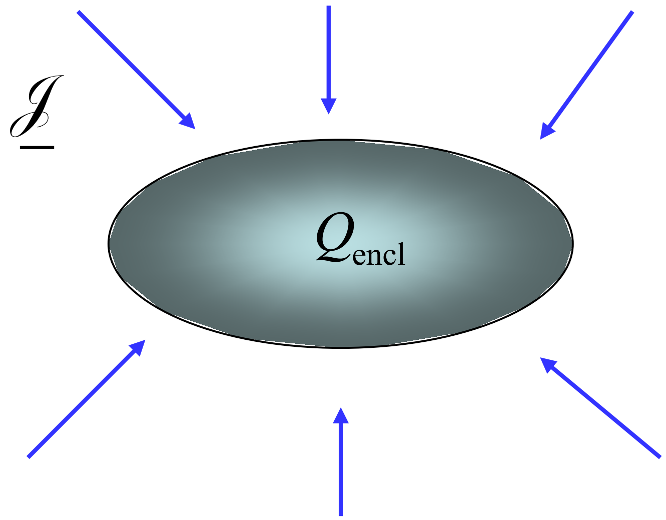
$$\int_V -\frac{\partial \rho_v}{\partial t}\, dV = -\frac{\partial}{\partial t}\int_V \rho_v\, dV = -\frac{\partial Q_{\text{encl}}}{\partial t}$$

(This assumes that the surface is stationary.)

Hence $$i_{\text{out}} = -\frac{\partial Q_{\text{encl}}}{\partial t}$$ or $$i_{\text{in}} = \frac{\partial Q_{\text{encl}}}{\partial t}$$

Continuity Equation (cont.)

$$i_{\text{in}} = \frac{\partial Q_{\text{encl}}}{\partial t}$$



This implies that charge is never created or destroyed.
It only moves from one place to another!

Maxwell's Equations (cont.)

Time - Dependent

$$\nabla \times \underline{\mathcal{E}} = -\frac{\partial \underline{\mathcal{B}}}{\partial t} \qquad \nabla \times \underline{\mathcal{H}} = \underline{\mathcal{J}} + \frac{\partial \underline{\mathcal{D}}}{\partial t} \qquad \nabla \cdot \underline{\mathcal{B}} = 0 \qquad \nabla \cdot \underline{\mathcal{D}} = \rho_v$$

Time - Independent (Statics)

$$\nabla \times \underline{E} = \underline{0} \qquad \nabla \cdot \underline{D} = \rho_v$$

$$\nabla \times \underline{H} = \underline{J} \qquad \nabla \cdot \underline{B} = 0$$

Statics decouples $\underline{E}$ and $\underline{H}$ $\Rightarrow$ $\underline{E}$ comes from ρ_v and $\underline{H}$ comes from $\underline{J}$

Note: Regular (not script) font is used for statics, just as it is for phasors.

Maxwell's Equations (cont.)

Time-harmonic (phasor) domain

$$\frac{\partial}{\partial t} \rightarrow j\omega$$

$$\nabla \times \underline{E} = -j\omega \underline{B}$$

$$\nabla \times \underline{H} = \underline{J} + j\omega \underline{D}$$

$$\nabla \cdot \underline{B} = 0$$

$$\nabla \cdot \underline{D} = \rho_v$$

Constitutive Relations

The characteristics of the media relate $\underline{\mathscr{D}}$ to $\underline{\mathscr{E}}$ and $\underline{\mathscr{H}}$ to $\underline{\mathscr{B}}$

Free Space

$$\underline{\mathscr{D}} = \varepsilon_0 \underline{\mathscr{E}} \quad (\varepsilon_0 = \text{permittivity})$$

$$\underline{\mathscr{B}} = \mu_0 \underline{\mathscr{H}} \quad (\mu_0 = \text{permeability})$$

$$\varepsilon_0 \doteq 8.8541878 \times 10^{-12} \text{ [F/m]}$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ [H/m] (exact*)}$$

*Prior to 2019

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

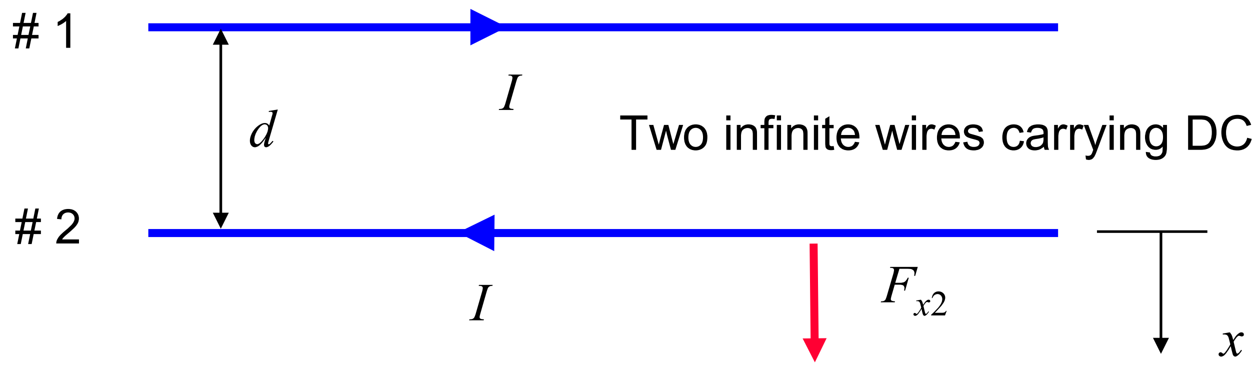
$$c \equiv 2.99792458 \times 10^{8} \text{ [m/s]}$$

(exact value that is defined)

(since 1983)

Constitutive Relations (cont.)

Definition of the Amp*: *Prior to 2019



Two infinite wires carrying DC currents

From ECE 3318:

$$F_{x2} = \frac{I^2 \mu_0}{2\pi d}$$

$$\mu_0 = 4\pi \times 10^{-7} \quad [\text{H/m}]$$

Definition of $I = 1$ Amp:

$$F_{x2} = 2 \times 10^{-7} \; [\text{N/m}] \;\; \text{when} \;\; d = 1 \;\; [\text{m}]$$

Constitutive Relations (cont.)

Free space, in the phasor domain:

$$\underline{D} = \varepsilon_0 \underline{E} \quad (\varepsilon_0 = \text{permittivity})$$

$$\underline{B} = \mu_0 \underline{H} \quad (\mu_0 = \text{permeability})$$

This follows from the fact that

$$a\underline{\mathcal{V}}(t) \Leftrightarrow a\underline{V}$$

(where a is a real number)

Example

Given the following electric field $\underline{\mathscr{E}}$ in free space:

$$\underline{\mathscr{E}}(t) = \underline{\hat{\theta}}\left(E_0 \cos\left(\omega t - k_0 r + \phi_0\right)\right)\frac{1}{r}\sin\theta \quad [\mathrm{V/m}]$$

Find the magnetic field $\underline{\mathscr{H}}$.

$$k_0 = \omega\sqrt{\mu_0 \varepsilon_0}$$

$$\nabla \times \underline{E} = -j\omega \underline{B}$$
$$\nabla \times \underline{H} = \underline{J} + j\omega \underline{D}$$
$$\nabla \cdot \underline{B} = 0$$
$$\nabla \cdot \underline{D} = \rho_v$$

In the phasor domain:

$$\underline{E} = \underline{\hat{\theta}}\left(E_0 \frac{1}{r} e^{-jk_0 r} e^{j\phi_0}\right)\sin\theta$$

$$\nabla \times \underline{E} = -j\omega \underline{B}$$

⇩

$$\nabla \times \underline{E} = -j\omega\mu_0 \underline{H}$$

⇩

$$\underline{H} = \frac{1}{-j\omega\mu_0}\nabla \times \underline{E}$$

Hence

$$\underline{H} = \frac{1}{-j\omega\mu_0}\nabla \times \underline{E}$$

$$\nabla \times \underline{E} = \underline{\hat{r}}\frac{1}{r\sin\theta}\left[\frac{\partial\left(E_\phi \sin\theta\right)}{\partial\theta} - \frac{\partial E_\theta}{\partial\phi}\right] + \underline{\hat{\theta}}\frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial E_r}{\partial\phi} - \frac{\partial\left(rE_\phi\right)}{\partial r}\right] + \underline{\hat{\phi}}\frac{1}{r}\left[\frac{\partial\left(rE_\theta\right)}{\partial r} - \frac{\partial E_r}{\partial\theta}\right]$$

(no ϕ variation)

Example (cont.)

$$\underline{E} = \underline{\hat{\theta}}\left(E_0\, e^{j\phi_0}\right)\frac{1}{r}e^{-jk_0 r}\sin\theta$$

$$\underline{H} = \frac{1}{-j\omega\mu_0}\nabla\times\underline{E}$$

$$\nabla\times\underline{E} = \underline{\hat{\phi}}\frac{1}{r}\left[\frac{\partial\left(rE_\theta\right)}{\partial r}\right]$$

$$= \underline{\hat{\phi}}\frac{1}{r}\left[\frac{\partial\left(r\left(E_0 e^{j\phi_0}\frac{1}{r}e^{-jk_0 r}\right)\sin\theta\right)}{\partial r}\right]$$

$$= \underline{\hat{\phi}}\frac{1}{r}e^{j\phi_0}E_0\sin\theta\left[\frac{\partial\left(e^{-jk_0 r}\right)}{\partial r}\right]$$

$$= \underline{\hat{\phi}}\frac{1}{r}e^{j\phi_0}E_0\sin\theta\left(-jk_0\right)e^{-jk_0 r}$$

$$\underline{H} = \frac{1}{-j\omega\mu_0}\left(\underline{\hat{\phi}}\frac{1}{r}e^{j\phi_0}E_0\sin\theta\left(-jk_0\right)e^{-jk_0 r}\right)$$

$$\underline{H} = \underline{\hat{\phi}}\left[E_0\left(\frac{k_0}{\omega\mu_0}\right)\frac{1}{r}\sin\theta\right]e^{-jk_0 r}e^{j\phi_0}$$

$$\underline{\mathscr{H}} = \underline{\hat{\phi}}\left[E_0\left(\frac{k_0}{\omega\mu_0}\right)\right]\frac{1}{r}\sin\theta\cos\left(\omega t - k_0 r + \phi_0\right)\ \left[\text{A/m}\right]$$

Example (cont.)

Alternative approach (in the time domain directly):

$$\underline{\mathscr{E}}(t) = \hat{\underline{\theta}}\left(E_0 \cos\left(\omega t - k_0 r + \phi_0\right)\right)\frac{1}{r}\sin\theta \quad [\text{V/m}]$$

$$\nabla \times \underline{\mathscr{E}} = -\frac{\partial \underline{\mathscr{B}}}{\partial t}$$

$$\nabla \times \underline{\mathscr{H}} = \underline{\mathscr{J}} + \frac{\partial \underline{\mathscr{D}}}{\partial t}$$

$$\nabla \cdot \underline{\mathscr{B}} = 0$$

$$\nabla \cdot \underline{\mathscr{D}} = \rho_v$$

$$\nabla \times \underline{\mathscr{E}} = -\frac{\partial \underline{\mathscr{B}}}{\partial t}$$

$$\Downarrow$$

$$\frac{\partial \underline{\mathscr{B}}}{\partial t} = -\nabla \times \underline{\mathscr{E}}$$

(no ϕ variation)

$$\nabla \times \underline{\mathscr{E}} = \hat{\underline{r}}\frac{1}{r\sin\theta}\left[\frac{\partial\left(\mathscr{E}_\phi \sin\theta\right)}{\partial\theta} - \frac{\partial \mathscr{E}_\theta}{\partial\phi}\right] + \hat{\underline{\theta}}\frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial \mathscr{E}_r}{\partial\phi} - \frac{\partial\left(r\mathscr{E}_\phi\right)}{\partial r}\right] + \hat{\underline{\phi}}\frac{1}{r}\left[\frac{\partial\left(r\mathscr{E}_\theta\right)}{\partial r} - \frac{\partial \mathscr{E}_r}{\partial\theta}\right]$$

$$\nabla \times \underline{\mathscr{E}} = \hat{\underline{\phi}}\frac{1}{r}\left[\frac{\partial\left(r\mathscr{E}_\theta\right)}{\partial r}\right]$$

$$= \hat{\underline{\phi}}\frac{1}{r}\left[\frac{\partial\left(r\left(E_0 \frac{1}{r}\cos\left(\omega t - k_0 r + \phi_0\right)\right)\sin\theta\right)}{\partial r}\right]$$

$$= \hat{\underline{\phi}}\frac{1}{r}E_0 \sin\theta\left[\frac{\partial\left(\cos\left(\omega t - k_0 r + \phi_0\right)\right)}{\partial r}\right]$$

$$= \hat{\underline{\phi}}\frac{1}{r}E_0 \sin\theta\left(-k_0\right)\left(-\sin\left(\omega t - k_0 r + \phi_0\right)\right)$$

Example (cont.)

$$\frac{\partial \underline{\mathscr{B}}}{\partial t} = -\nabla \times \underline{\mathscr{E}}$$

$$\nabla \times \underline{\mathscr{E}} = \underline{\hat{\phi}} \frac{1}{r} E_0 \sin\theta (k_0)\left(\sin\left(\omega t - k_0 r + \phi_0\right)\right)$$

So

$$\frac{\partial \underline{\mathscr{B}}}{\partial t} = -\underline{\hat{\phi}}\, k_0 E_0 \frac{1}{r} \sin\theta \left(\sin\left(\omega t - k_0 r + \phi_0\right)\right)$$

$$\Downarrow$$

$$\underline{\mathscr{B}} = -\underline{\hat{\phi}}\, k_0 E_0 \frac{1}{r} \sin\theta \frac{1}{\omega}\left(-\cos\left(\omega t - k_0 r + \phi_0\right)\right) + C(r,\theta,\phi)$$

All fields must be pure sinusoidal waves in the time-harmonic steady state.

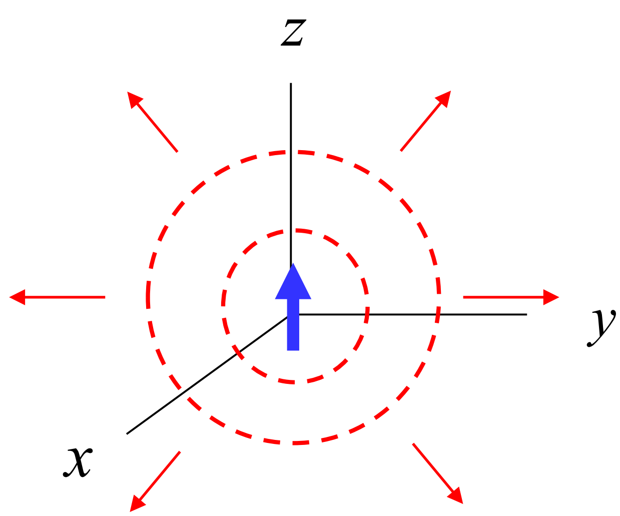
$$\Downarrow \quad \underline{\mathscr{B}} = \mu_0 \underline{\mathscr{H}}$$

$$\underline{\mathscr{H}} = \underline{\hat{\phi}} \left[E_0 \frac{k_0}{\omega \mu_0} \right] \frac{1}{r} \sin\theta \left(\cos\left(\omega t - k_0 r + \phi_0\right)\right) \quad [\text{A/m}]$$

Example (cont.)

$$\underline{\mathscr{E}}(t) = \underline{\hat{\theta}}\left(E_0 \cos\left(\omega t - k_0 r + \phi_0\right)\right)\frac{1}{r}\sin\theta \quad [\text{V/m}]$$

$$\underline{\mathscr{H}} = \underline{\hat{\phi}}\left[E_0 \frac{k_0}{\omega\mu_0}\right]\frac{1}{r}\sin\theta\left(\cos\left(\omega t - k_0 r + \phi_0\right)\right) \quad [\text{A/m}]$$



This describes the far-field radiation from a small vertical dipole antenna.

Material Properties

In a material medium:

$$\underline{D} = \varepsilon \underline{E} \quad (\varepsilon = \text{permittivity})$$

$$\underline{B} = \mu \underline{H} \quad (\mu = \text{permeability})$$

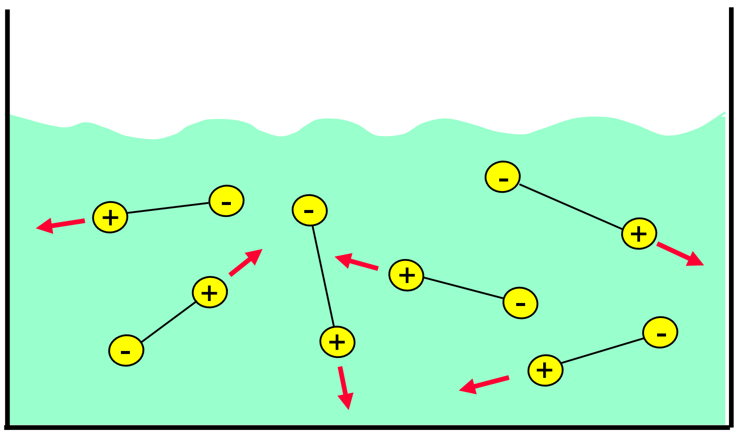
$$\varepsilon = \varepsilon_0 \varepsilon_r \qquad \varepsilon_r = \text{relative permittivity}$$

$$\mu = \mu_0 \mu_r \qquad \mu_r = \text{relative permeability}$$

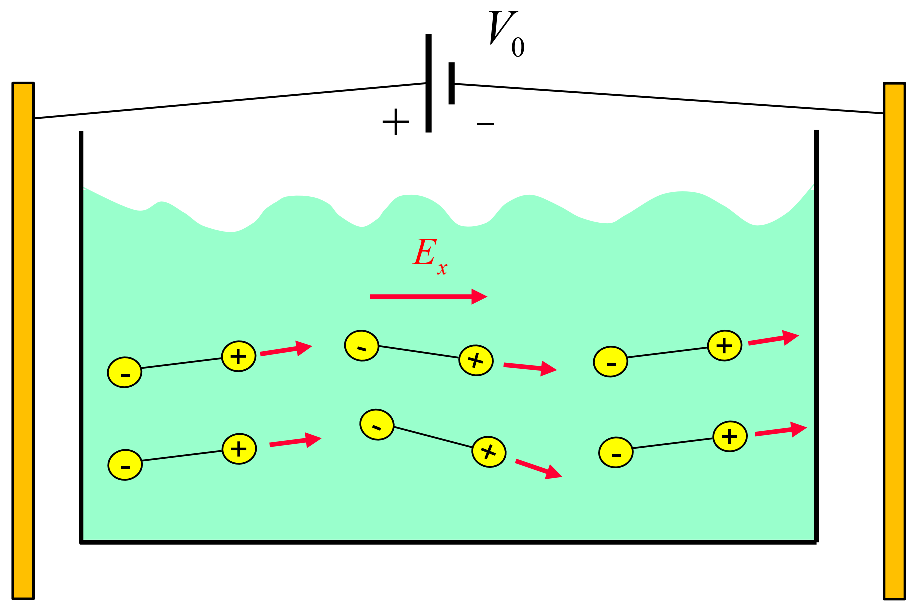
Note: The fields $\underline{E}$ and $\underline{B}$ are the *physical* fields, meaning they exert a force on a charged particle that can be measured. The other two fields are defined.

Material Properties (cont.)

Where does permittivity come from?

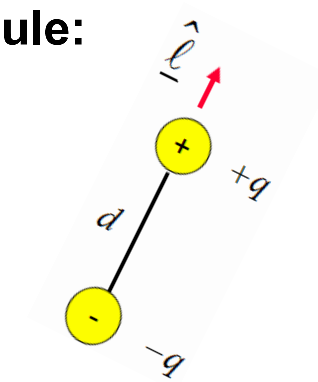


Water



$$\underline{D} \equiv \varepsilon_0 \underline{E} + \underline{P}$$

Molecule:



$$\underline{P} \equiv \frac{1}{\Delta V} \sum_{\Delta V} \underline{p}_i$$

$$\underline{p}_i = p\hat{\underline{\ell}}_i$$

$$p = qd$$

Material Properties (cont.)

$$\underline{D} \equiv \varepsilon_0 \underline{E} + \underline{P}$$

Linear material: $\underline{P} = \varepsilon_0 \chi_e \underline{E}$

The term χ_e is called the "electric susceptibility."

Note: $\chi_e > 0$ for most materials

so

$$\underline{D} = \varepsilon_0 \underline{E} + \varepsilon_0 \chi_e \underline{E}$$
$$= \varepsilon_0 \left(1 + \chi_e\right) \underline{E}$$

Define: $\varepsilon_r \equiv 1 + \chi_e$

Then $\underline{D} = \varepsilon_0 \varepsilon_r \underline{E}$

Material Properties (cont.)

Teflon $\varepsilon_r = 2.2$

Water $\varepsilon_r = 81$ (a very polar molecule, fairly free to rotate)

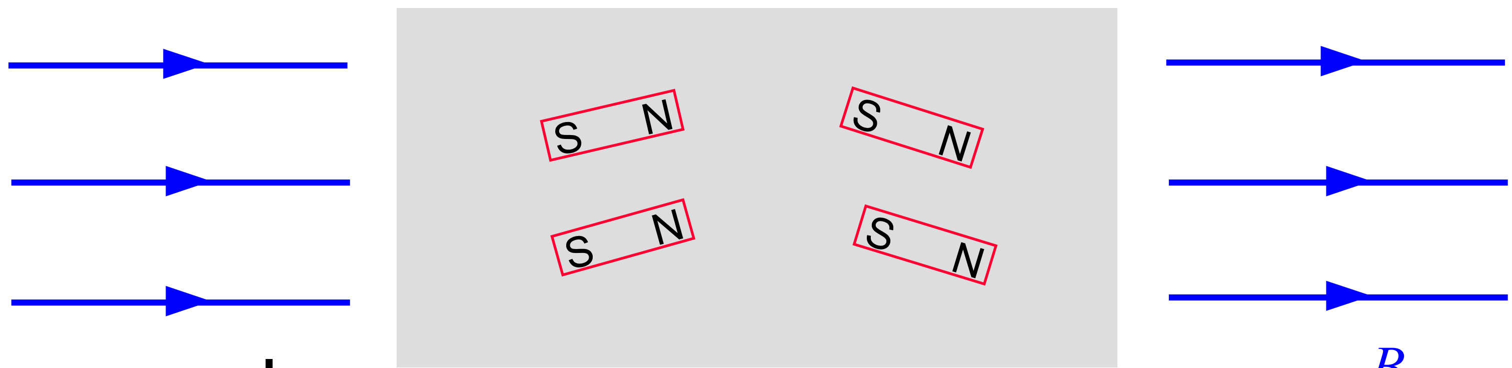
Styrofoam $\varepsilon_r = 1.03$

Quartz $\varepsilon_r = 5$

Note: $\varepsilon_r > 1$ for most materials: $\varepsilon_r \equiv 1 + \chi_e, \quad \chi_e > 0$

Material Properties (cont.)

Where does permeability come from?



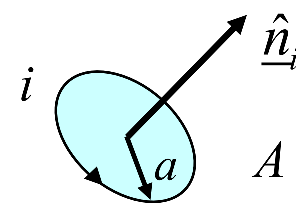
Iron

$\underline{B}$

Because of *electron spin*, atoms tend to acts as little current loops, and hence as electromagnetics, or bar magnets. When a magnetic field is applied, the little atomic magnets tend to line up.

$$\underline{\mathscr{H}} \equiv \frac{1}{\mu_0}\underline{\mathscr{B}} - \underline{\mathscr{M}}$$

Electron:



$$A = \pi a^2$$

$$\underline{\mathscr{M}} \equiv \frac{1}{\Delta V}\sum_{\Delta V} \underline{\boldsymbol{m}}_i$$

$$\underline{\boldsymbol{m}}_i = \underline{\hat{n}}_i \left(iA\right)$$

Material Properties (cont.)

$$\underline{\mathscr{B}} = \mu_0 \underline{\mathscr{H}} + \mu_0 \underline{\mathscr{M}}$$

Linear material: $\underline{\mathscr{M}} = \chi_m \underline{\mathscr{H}}$

The term χ_m is called the "magnetic susceptibility."

Note: $\chi_m > 0$ for most materials

so

$$\underline{\mathscr{B}} = \mu_0 \underline{\mathscr{H}} + \mu_0 \chi_m \underline{\mathscr{H}}$$
$$= \mu_0 \left(1 + \chi_m\right) \underline{\mathscr{H}}$$

Define: $\mu_r = \left(1 + \chi_m\right)$

Then $\underline{\mathscr{B}} = \mu_0 \mu_r \underline{\mathscr{H}}$

Material Properties (cont.)

Material	Relative Permeability μ_r
Vacuum	1
Air	1.0000004
Water	0.999992
Copper	0.999994
Aluminum	1.00002
Silver	0.99998
Nickel	600
Iron	5000
Carbon Steel	100
Transformer Steel	2000
Mumetal	50,000
Supermalloy	1,000,000

Note: Values can often vary depending on purity and processing.

http://en.wikipedia.org/wiki/Permeability_(electromagnetism)

Lorenz Force Law

The fields $\underline{\mathscr{E}}$ and $\underline{\mathscr{B}}$ are the two physical fields, since they exert a force on a particle (the Lorenz force law). The $\underline{\mathscr{D}}$ and $\underline{\mathscr{H}}$ fields are the defined fields.

Lorenz force law:

$$\underline{\mathscr{F}} = q\left(\underline{\mathscr{E}} + \underline{v} \times \underline{\mathscr{B}}\right)$$

This experimental law gives us the force on a particle with charge q moving with a velocity vector $\underline{v}$.

Terminology

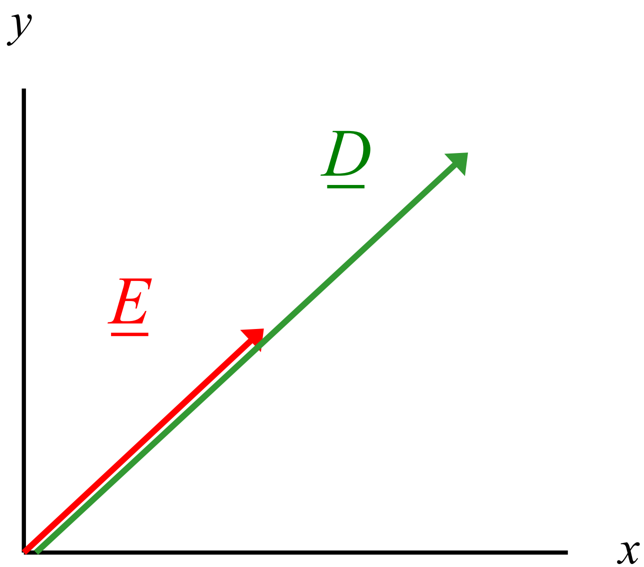
Properties of ε or μ

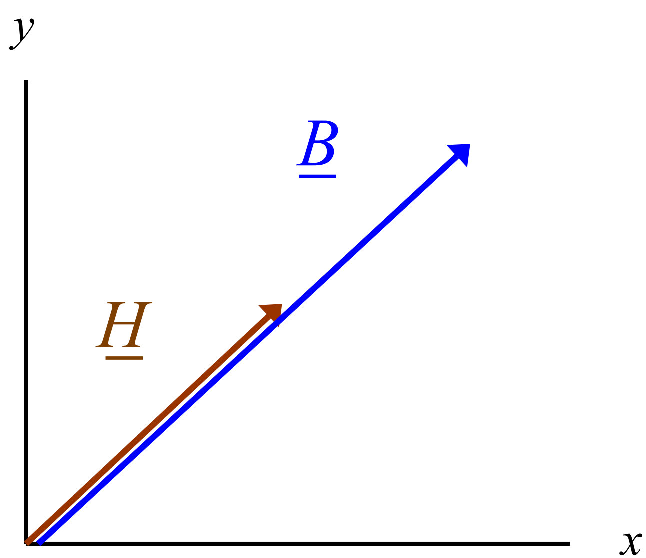
Variation	Independent of	Dependent on
Space	Homogenous	Inhomogeneous
Frequency	Non-dispersive	Dispersive
Time	Stationary	Time-varying
Field strength	Linear	Non-linear
Direction of $\underline{E}$ or $\underline{H}$	Isotropic	Anisotropic

Isotropic Materials

Isotropic: This means that ε and μ are <u>scalar</u> quantities, which means that $\underline{D} \parallel \underline{E}$ (and $\underline{B} \parallel \underline{H}$)

$$\underline{D} = \varepsilon \underline{E}$$
$$\underline{B} = \mu \underline{H}$$





Anisotropic Materials

Here ε (or μ) is a tensor (can be written as a matrix)

Example:

"biaxial medium"

$$\left.\begin{aligned} D_x &= \varepsilon_x E_x \\ D_y &= \varepsilon_y E_y \\ D_z &= \varepsilon_z E_z \end{aligned}\right\}$$

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \begin{bmatrix} \varepsilon_x & 0 & 0 \\ 0 & \varepsilon_y & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$

or

$$\underline{D} = \underline{\underline{\varepsilon}} \cdot \underline{E}$$

This results in $\underline{E}$ and $\underline{D}$ **NOT** being in the same direction.

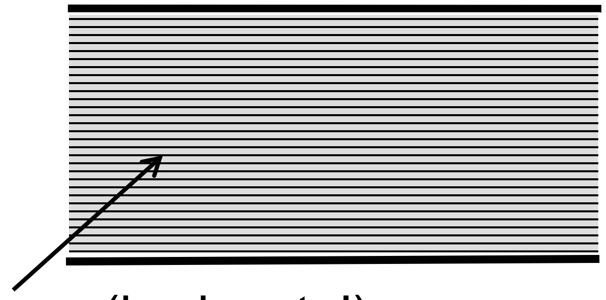
Anisotropic Materials (cont.)

Practical example: <u>uniaxial</u> substrate material

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \begin{bmatrix} \varepsilon_h & 0 & 0 \\ 0 & \varepsilon_h & 0 \\ 0 & 0 & \varepsilon_v \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$

There are <u>two</u> different permittivity values, a <u>horizontal</u> one and a <u>vertical</u> one.

Teflon substrate



Anisotropic Materials (cont.)

RT/duroid® 5870/5880/5880LZ High Frequency Laminates

This column indicates that ε_v is being measured.

↓

RT/duroid 5870/5880 Laminates

PROPERTY	TYPICAL VALUE [2]		DIRECTION	UNITS[3]	CONDITION	TEST METHOD
	RT/duroid 5870	RT/duroid 5880				
[1]**Dielectric Constant, ε_r** *Process*	2.33 2.33 ± 0.02 spec.	2.20 2.20 ± 0.02 spec.	Z Z		C24/23/50 C24/23/50	1 MHz IPC-TM-650 2.5.5.3 10 GHz IPC-TM 2.5.5.5
[5] **Dielectric Constant, ε_r** *Design*	2.33	2.20	Z		8 GHz - 40 GHz	Differential Phase Length Method
Dissipation Factor, tan δ	0.0005 0.0012	0.0004 0.0009	Z Z		C24/23/50 C24/23/50	1 MHz IPC-TM-650, 2.5.5.3 10 GHz IPC-TM-2.5.5.5
Thermal Coefficient of ε_r	-115	-125		ppm/°C	-50 - 150°C	IPC-TM-650, 2.5.5.5
Volume Resistivity	2 X 10^7	2 X 10^7	Z	Mohm cm	C96/35/90	ASTM D257
Surface Resistivity	2 X 10^7	3 X 10^7	Z	Mohm	C/96/35/90	ASTM D257

https://www.rogerscorp.com/advanced-electronics-solutions/rt-duroid-laminates/rt-duroid-5870-laminates