# ECE 3317
# Applied Electromagnetic Waves

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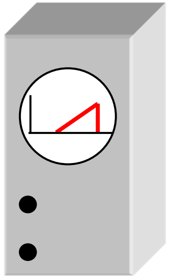
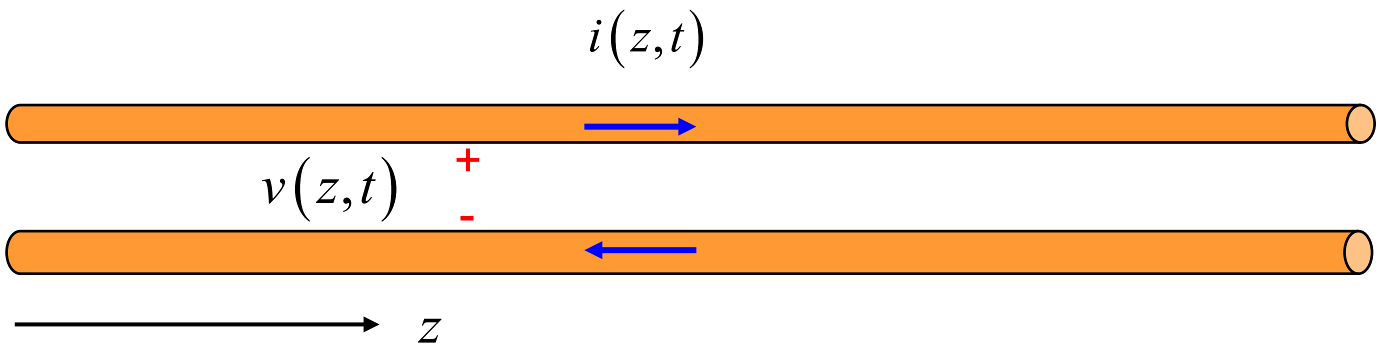
Fall 2023

# Notes 7
# Transmission Lines
# (Pulse Propagation and Reflection)



# Pulse on Transmission Line (cont.)




Convenient for visualizing a wave traveling on the line.

$$v(z,t) = F(z - c_d t) + G(z + c_d t)$$

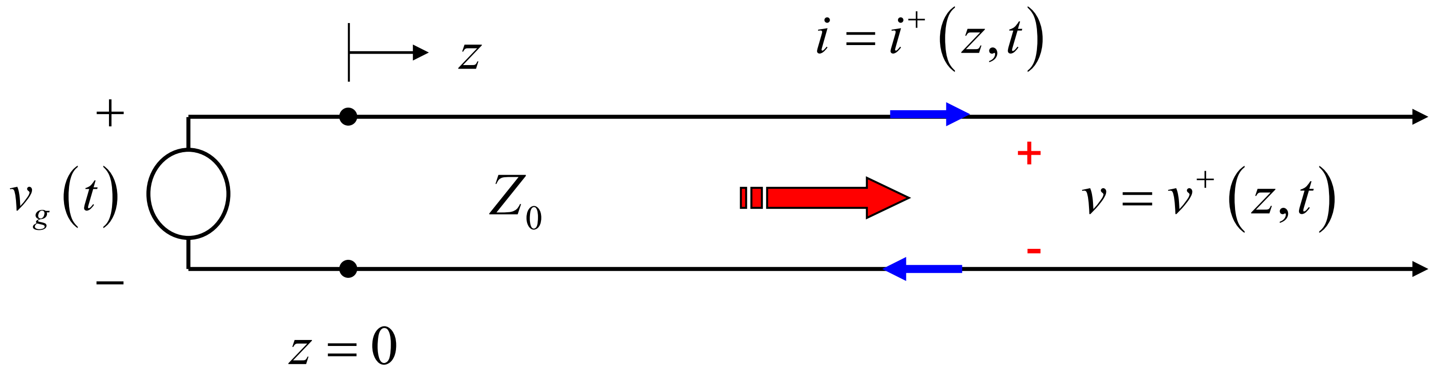
Alternative notation: $v^+(z,t)$ $v^-(z,t)$

$$v(z,t) = f(t - z / c_d) + g(t + z / c_d)$$

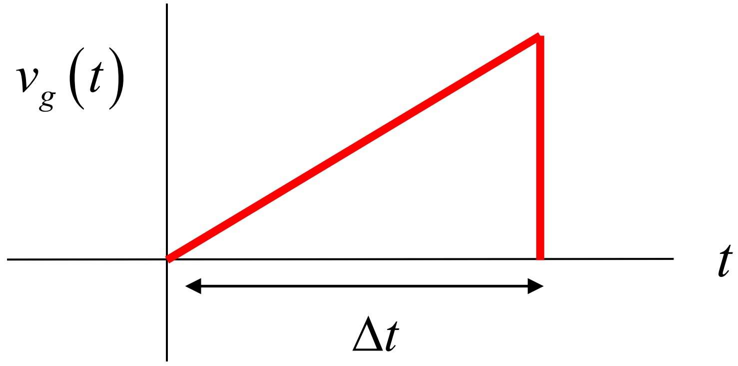
Convenient for seeing what an oscilloscope will display.

# Pulse on Transmission Line

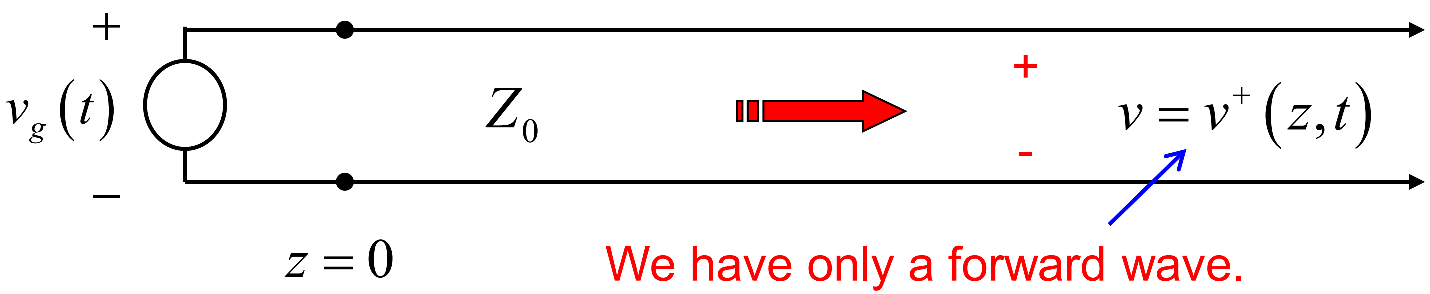
A voltage signal is applied at the input of a semi-infinite transmission line.




**Example:** Sawtooth wave



# Pulse on Transmission Line (cont.)




$$v^+(z,t) = f(t - z/c_d)$$

Set $z = 0$: $\quad v^+(0,t) = f(t)$

Also, we have: $\quad v^+(0,t) = v_g(t)$

$$\Rightarrow \quad f(t) = v_g(t)$$

Hence

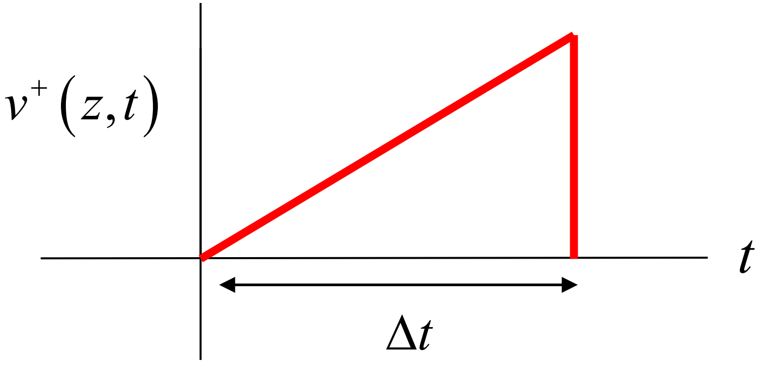
$$v^+(z,t) = v_g(t - z/c_d)$$

# Pulse on Transmission Line (cont.)

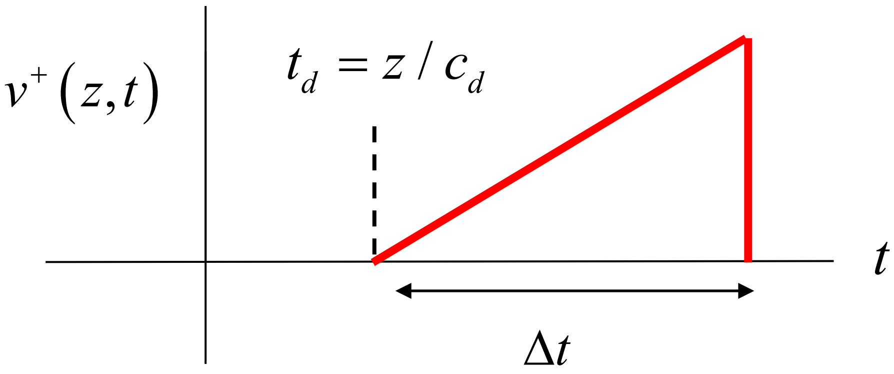
$$v^+(z,t) = v_g\left(t - z / c_d\right)$$

At any position $z$, the pulse (oscilloscope trace) that is measured is the same as the input pulse, except that it is delayed by a time $t_d = z / c_d$.

$z = 0$

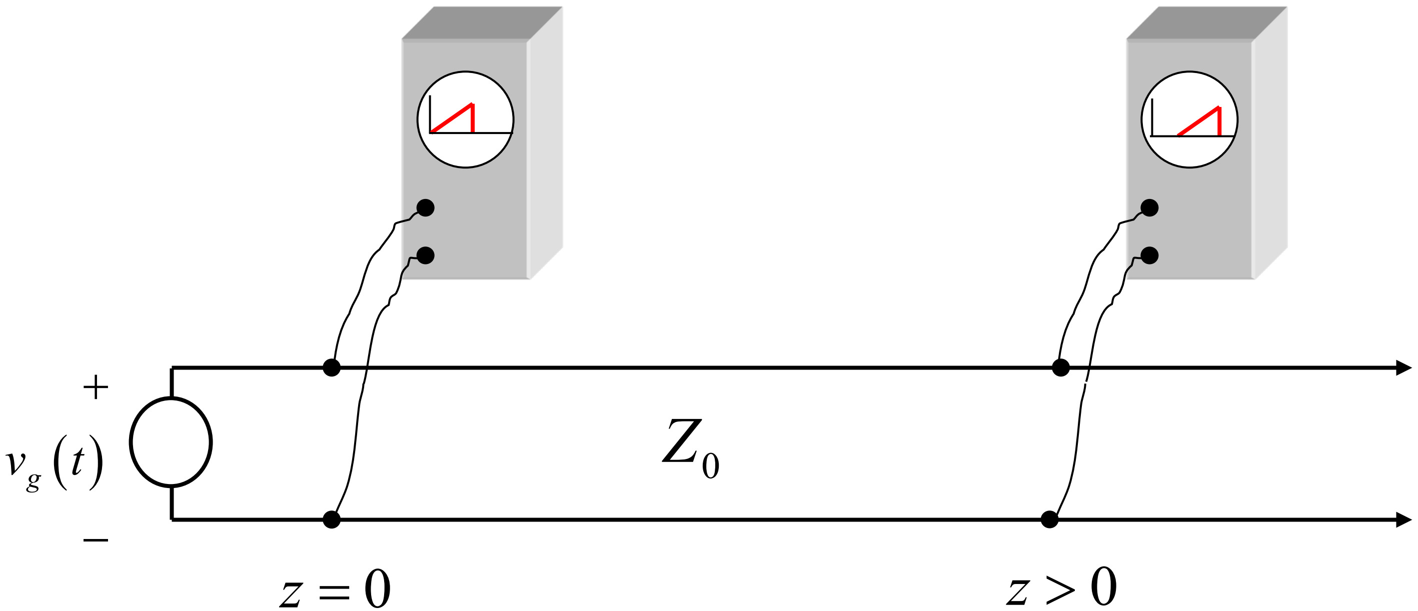



$z > 0$



# Pulse on Transmission Line (cont.)

Note the delay in the trace on this oscilloscope.

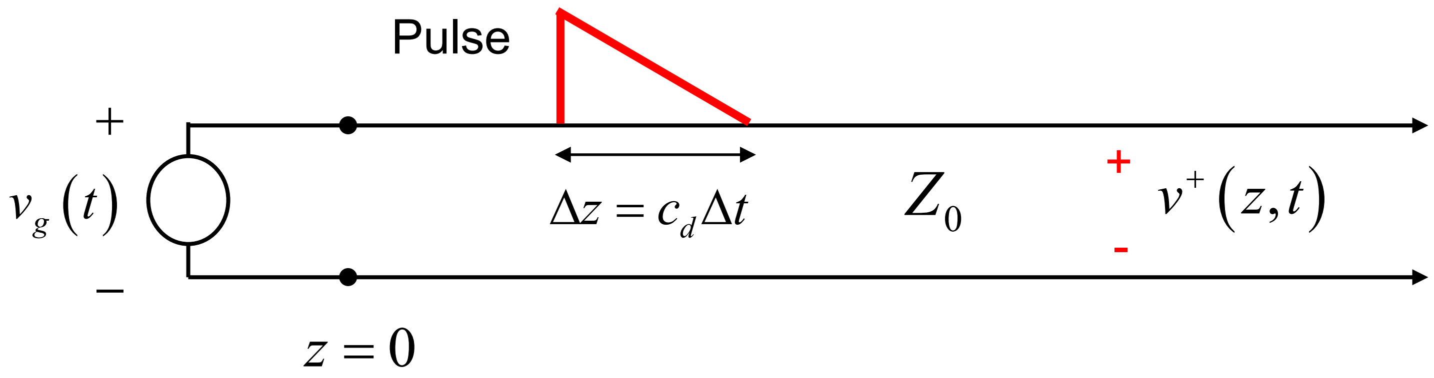



Here is what oscilloscopes will show.

$$v^+(z,t) = v_g(t - z / c_d)$$

# Pulse on Transmission Line (cont.)

"**Snapshot**" of voltage wave at one fixed time.



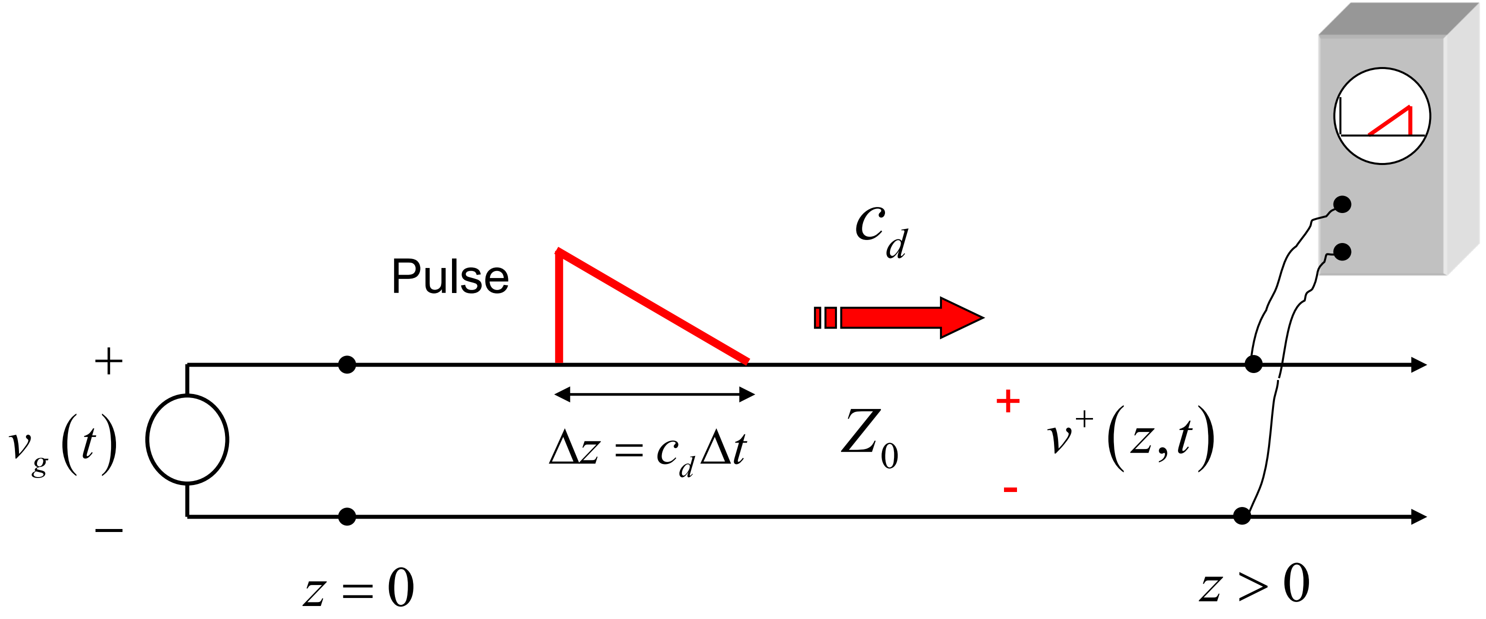

Note that the shape of the pulse as a function of $z$ is a scaled mirror image of the pulse shape as a function of $t$.

$$v^+(z,t) = v_g(t - z/c_d)$$

# Pulse on Transmission Line (cont.)

This voltage waveform moves past the oscilloscope to create the trace.

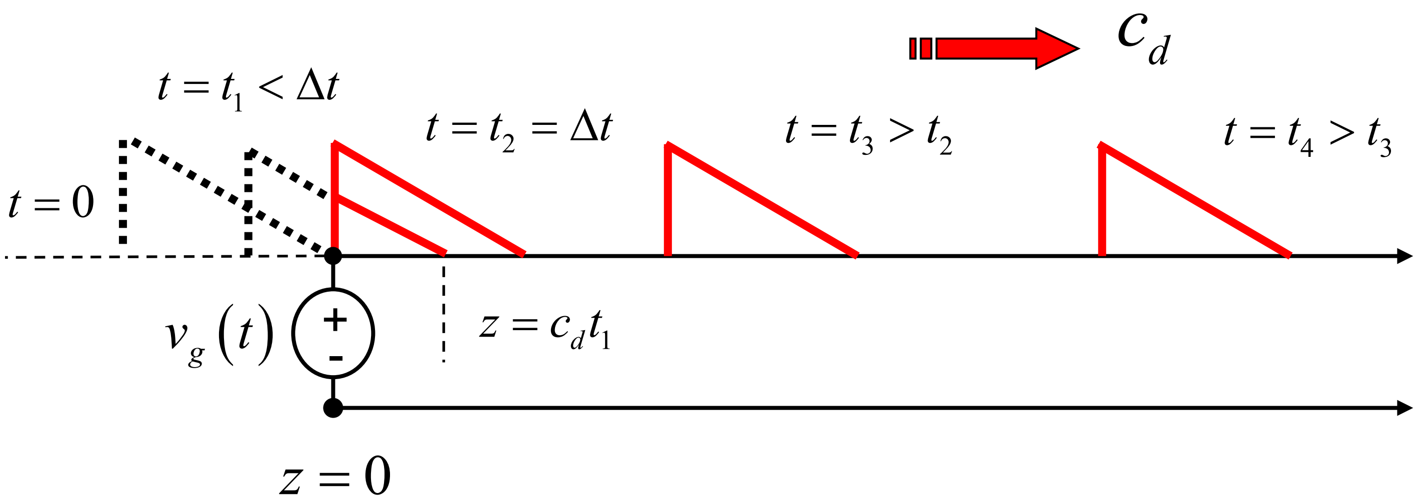
(The oscilloscope trace is the mirror image of the snapshot shape.)



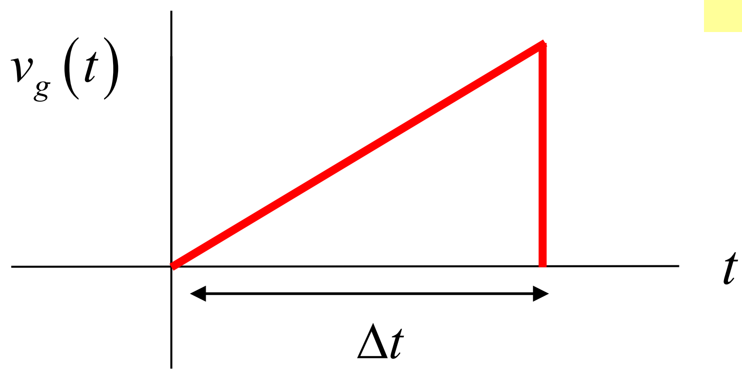
# Pulse on Transmission Line (cont.)

The pulse is shown emerging from the source end of the line.

A series of "snapshots" is shown.

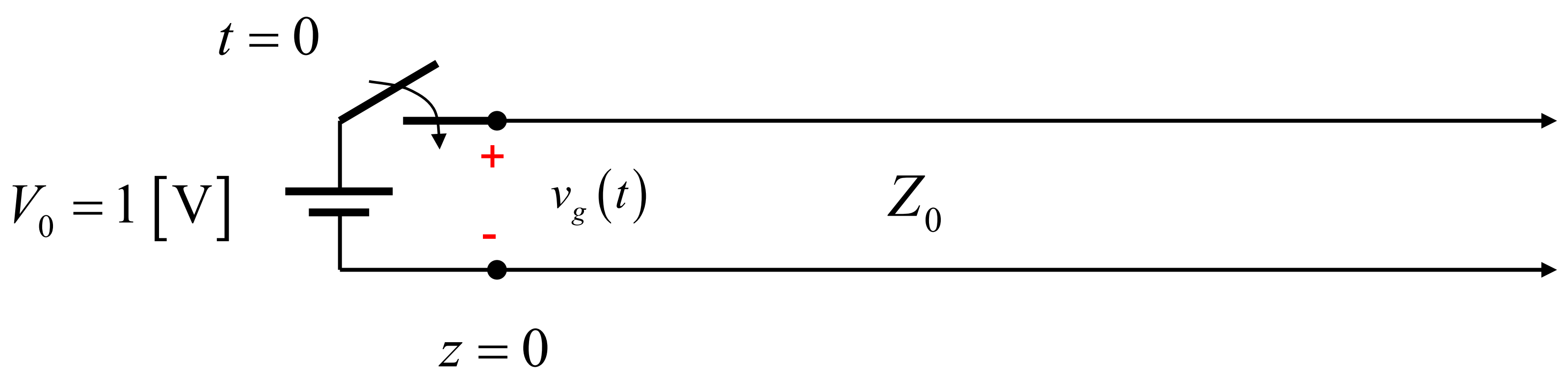


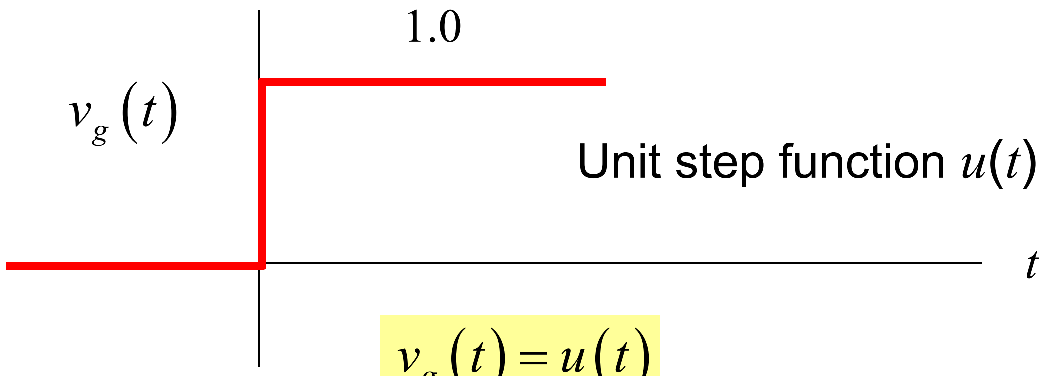

$$v^+(z,t) = v_g(t - z / c_d)$$



# Step Function Source

Another example (battery and switch)

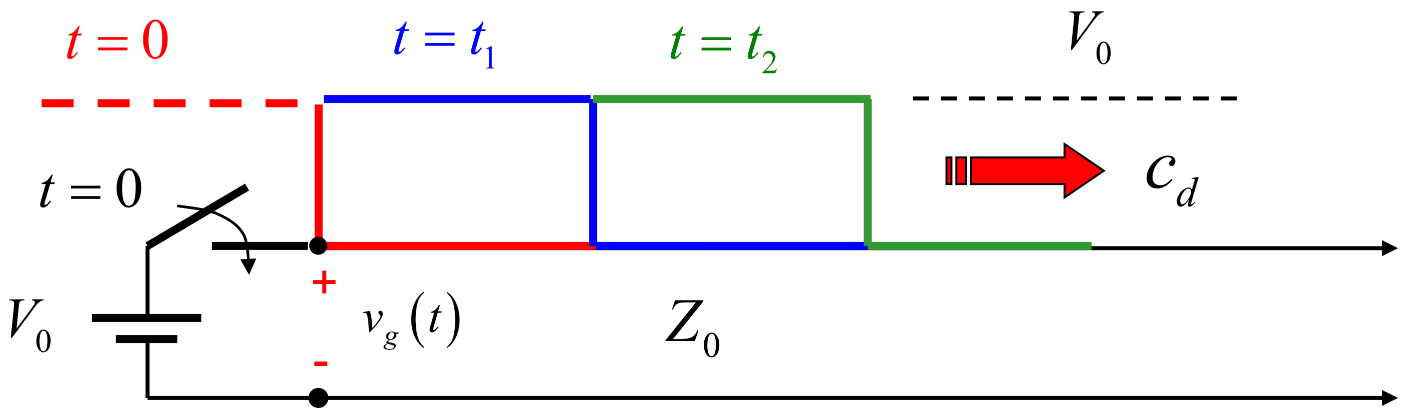


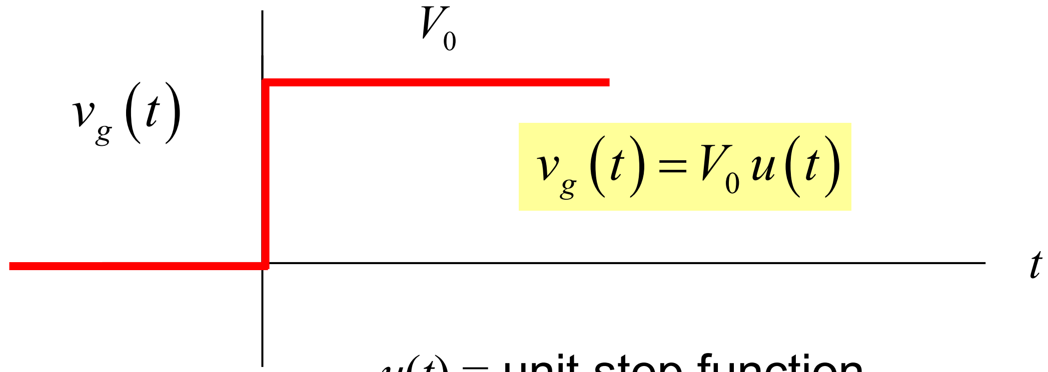




$$v_g(t) = u(t)$$

# Step Function Source (cont.)

The step function is shown propagating down the line.



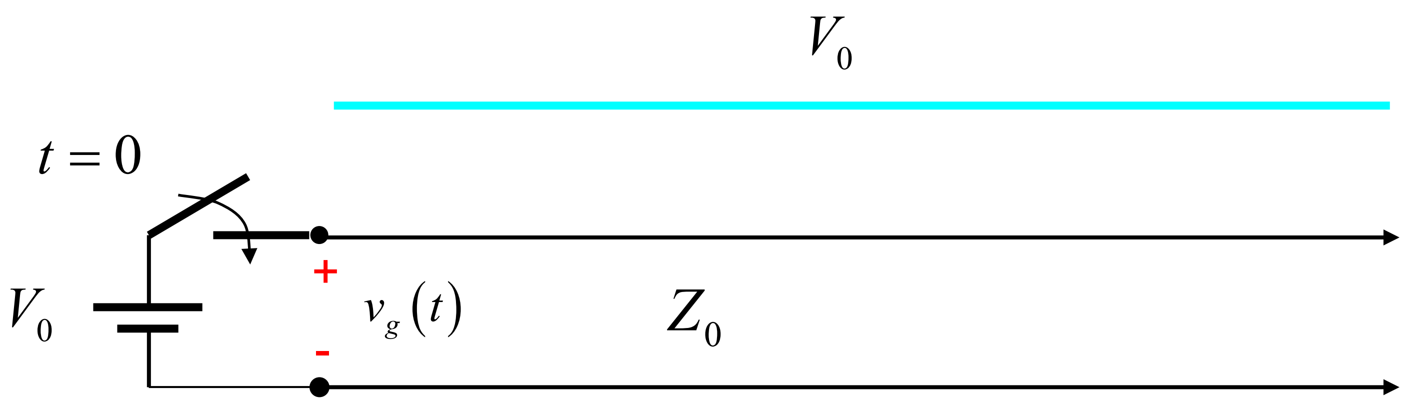




$u(t)$ = unit step function

# Step Function Source (cont.)

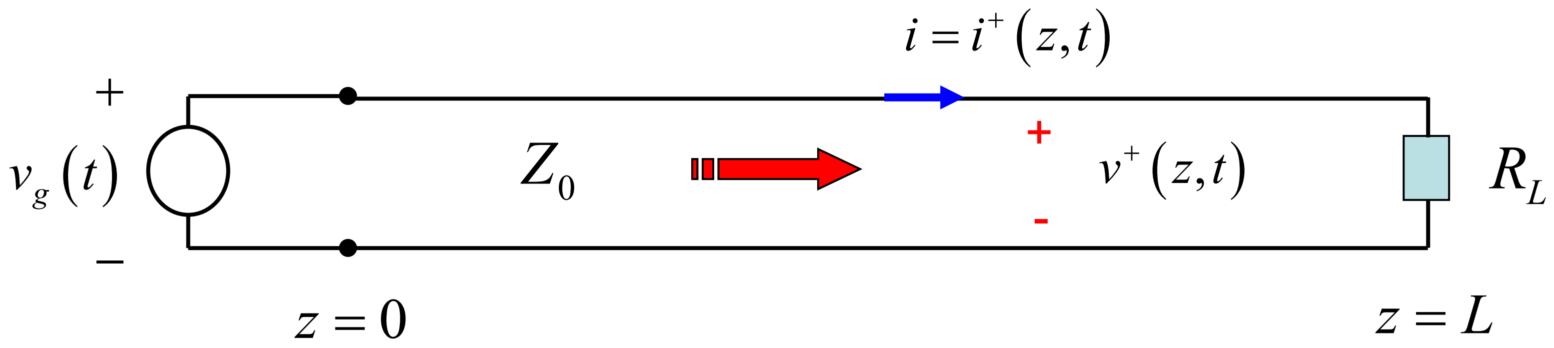
Steady-state solution ($t = \infty$)

The steady-state voltage is just the battery voltage.



# Matched Load

$$R_L = Z_0$$




On the line: $\dfrac{v^+(L,t)}{i^+(L,t)} = Z_0$

At the load: $\dfrac{v(L,t)}{i(L,t)} = R_L$

same

$$v^+(L,t) = v(L,t)$$

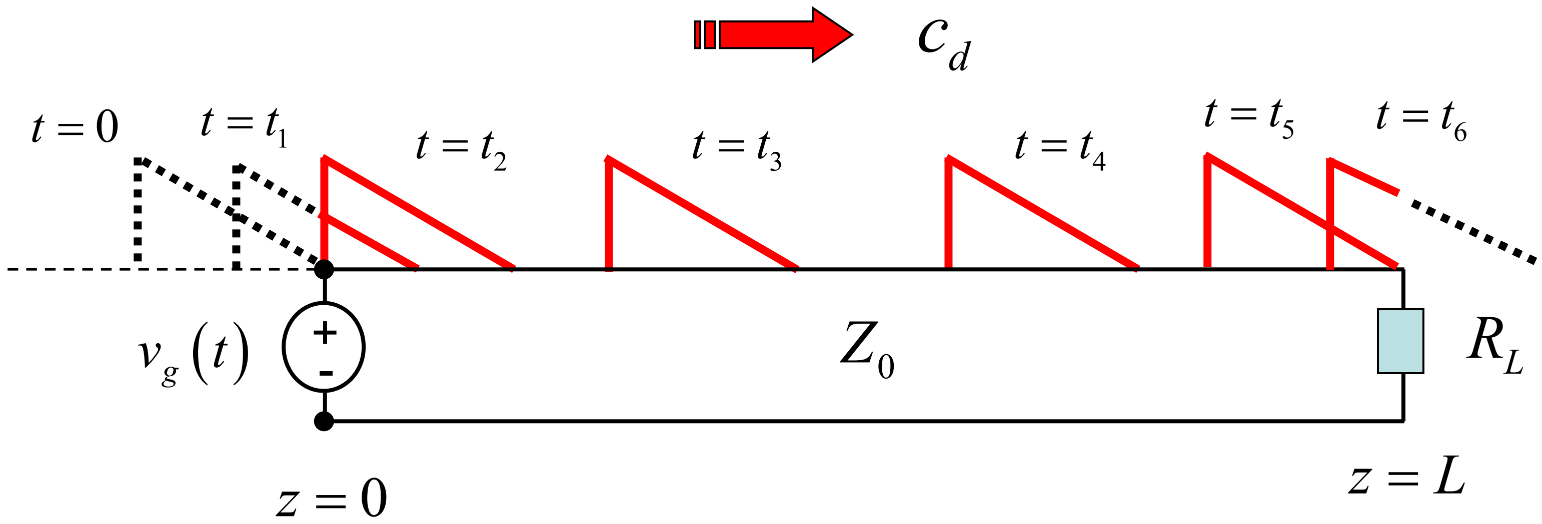
Recall:

$$Z_0 \equiv \frac{v^+(z,t)}{i^+(z,t)}$$

At $z = L$, the two right-hand side terms are the same since $R_L = Z_0$. Hence the total voltage at the load is the same as the incident voltage. There is thus no reflection. When the waveform hits the load, it "sees" a continuation of the line.

# Absorption by Load

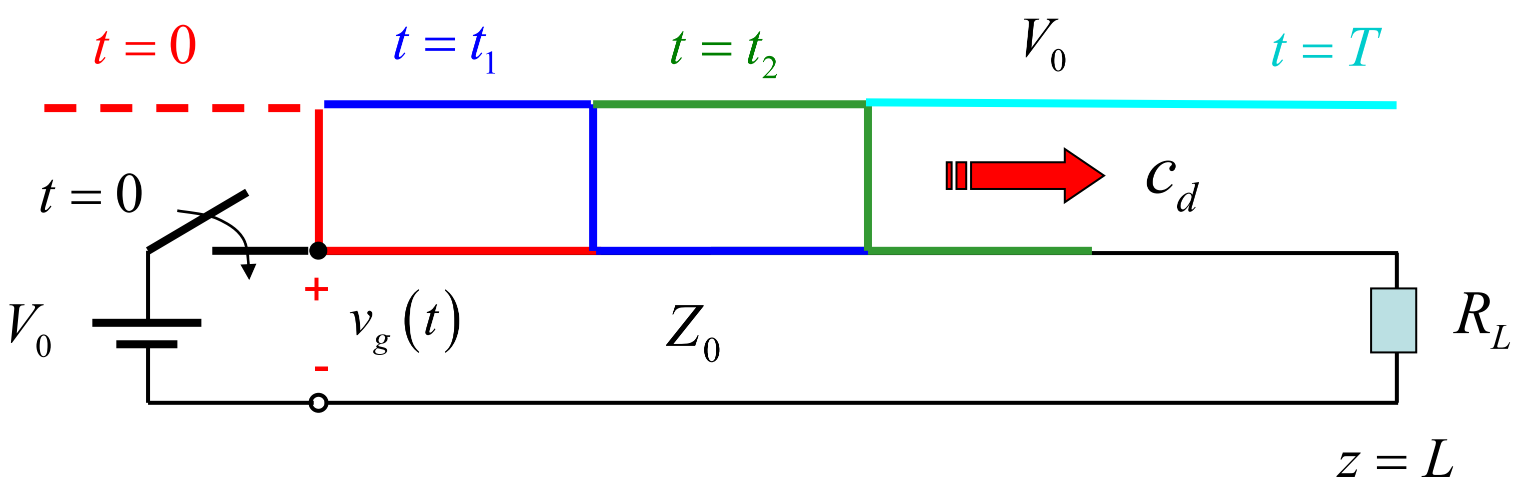
This shows the sawtooth waveform propagating on a matched line.




The pulse is shown emerging from the source end of the line, traveling down the line, and then being <u>absorbed</u> by the matched load.

# Absorption by Load (cont.)

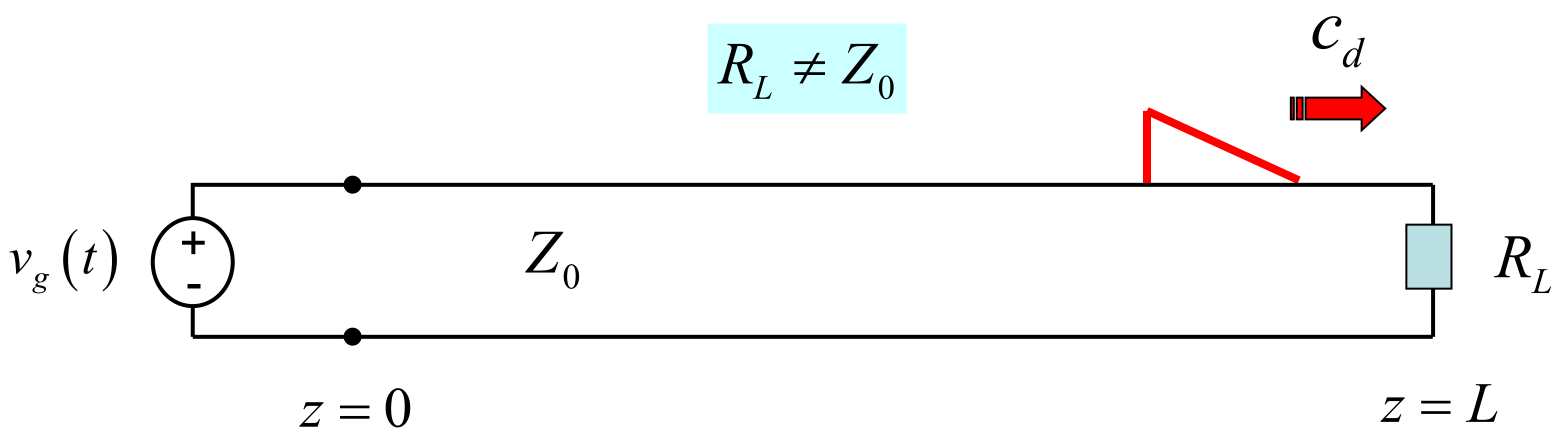
This shows the step function waveform propagating on a matched line.




Time to reach the load end: $$T=\frac{L}{c_d}$$

For $t>T$ we have reached steady state: $V(z,t)=V_0$ everywhere on the line.

# Load Reflection




On the line:

$$v(z,t) = f(t - z/c_d) + g(t + z/c_d)$$

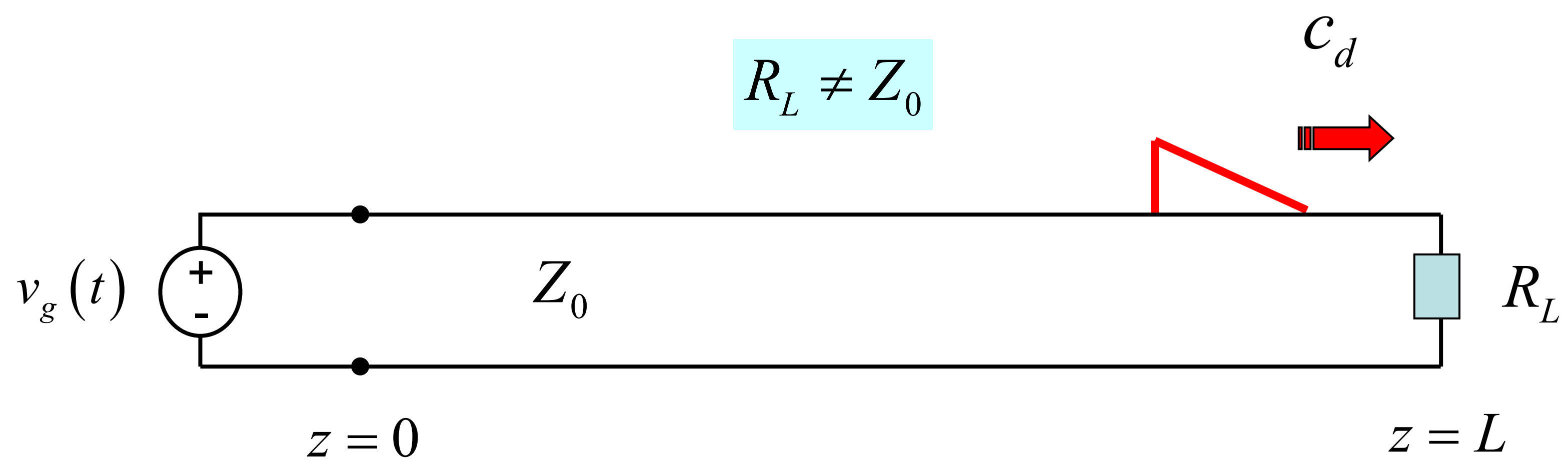
$$i(z,t) = \frac{1}{Z_0}\left[f(t - z/c_d) - g(t + z/c_d)\right]$$

where $v^+(z,t) = f(t - z/c_d) = v_g(t - z/c_d)$ (known function)

Goal : solve for $v^-(z,t) = g(t + z/c_d)$

At load: $$\frac{v(L,t)}{i(L,t)} = R_L$$

# Load Reflection (cont.)




At the load:

$$\frac{v(L,t)}{i(L,t)} = R_L$$

Hence, we have:

$$\frac{\overbrace{f\left(t - L/c_d\right)}^{v^+(L,t)} + \overbrace{g\left(t + L/c_d\right)}^{v^-(L,t)}}{\frac{1}{Z_0}\left[f\left(t - L/c_d\right) - g\left(t + L/c_d\right)\right]} = R_L$$

# Load Reflection (cont.)

$$f\left(t-L/c_d\right)+g\left(t+L/c_d\right)=R_L\left(\frac{1}{Z_0}\left[f\left(t-L/c_d\right)-g\left(t+L/c_d\right)\right]\right)$$

$$g\left(t+L/c_d\right)\left[1+\frac{R_L}{Z_0}\right]=f\left(t-L/c_d\right)\left[-1+\frac{R_L}{Z_0}\right]$$

$$g\left(t+L/c_d\right)=f\left(t-L/c_d\right)\left(\frac{-1+\frac{R_L}{Z_0}}{1+\frac{R_L}{Z_0}}\right)$$

$$g\left(t+L/c_d\right)=f\left(t-L/c_d\right)\left(\frac{R_L-Z_0}{R_L+Z_0}\right)$$

# Load Reflection (cont.)

$$g\left(t+L/c_d\right)=f\left(t-L/c_d\right)\left(\frac{R_L-Z_0}{R_L+Z_0}\right)$$

Define for simplicity:

$$v_L^+(t)\equiv f\left(t-L/c_d\right)$$ voltage of forward-traveling wave at the load

$$v_L^-(t)\equiv g\left(t+L/c_d\right)$$ voltage of backward-traveling wave at the load

Then

$$v_L^-(t)=v_L^+(t)\left(\frac{R_L-Z_0}{R_L+Z_0}\right)$$

Define

$$\Gamma_L=\left(\frac{R_L-Z_0}{R_L+Z_0}\right)$$ load reflection coefficient

We then have

$$v_L^-(t)=\Gamma_L\ v_L^+(t)$$

# Load Reflection (cont.)

## Summary for load reflection

$$v_L^-(t) = \Gamma_L \, v_L^+(t)$$

$$\Gamma_L = \left( \frac{R_L - Z_0}{R_L + Z_0} \right)$$

load reflection coefficient

**Note:**
There is no reflection for a matched system ($R_L = Z_0$).

Note:

$$-1 \le \Gamma_L \le +1$$

# Load Reflection (cont.)

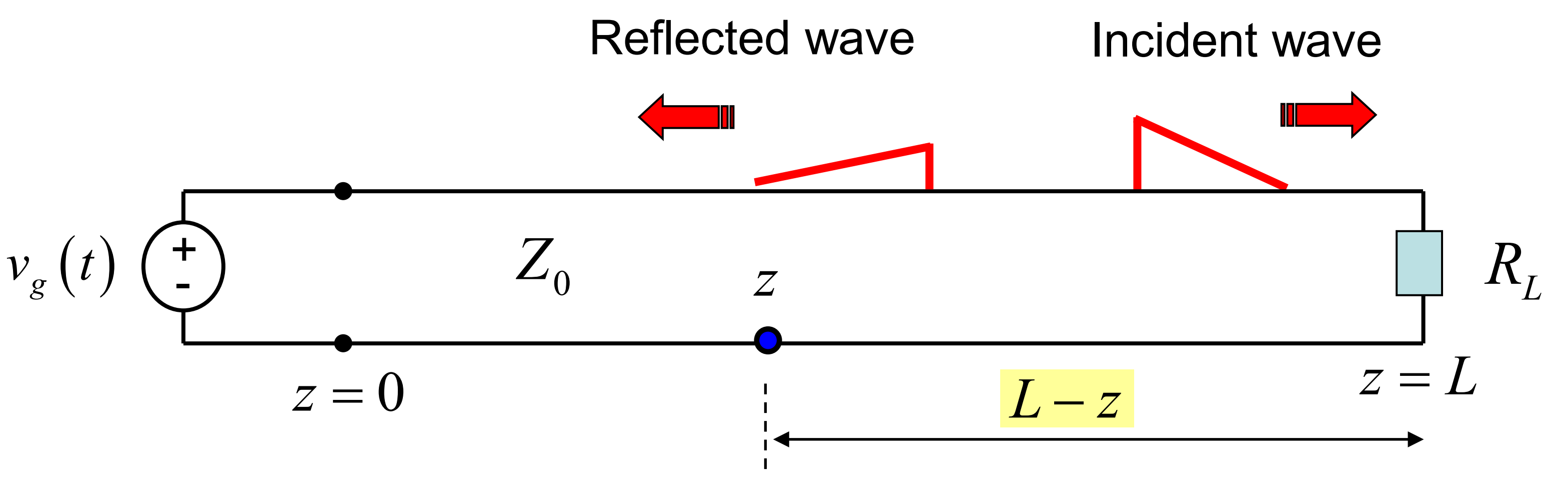
We now proceed to obtain $v^-(z,t)$ (for arbitrary $z$)

Reminder : $v^+(z,t) = v_g(t - z/c_d)$

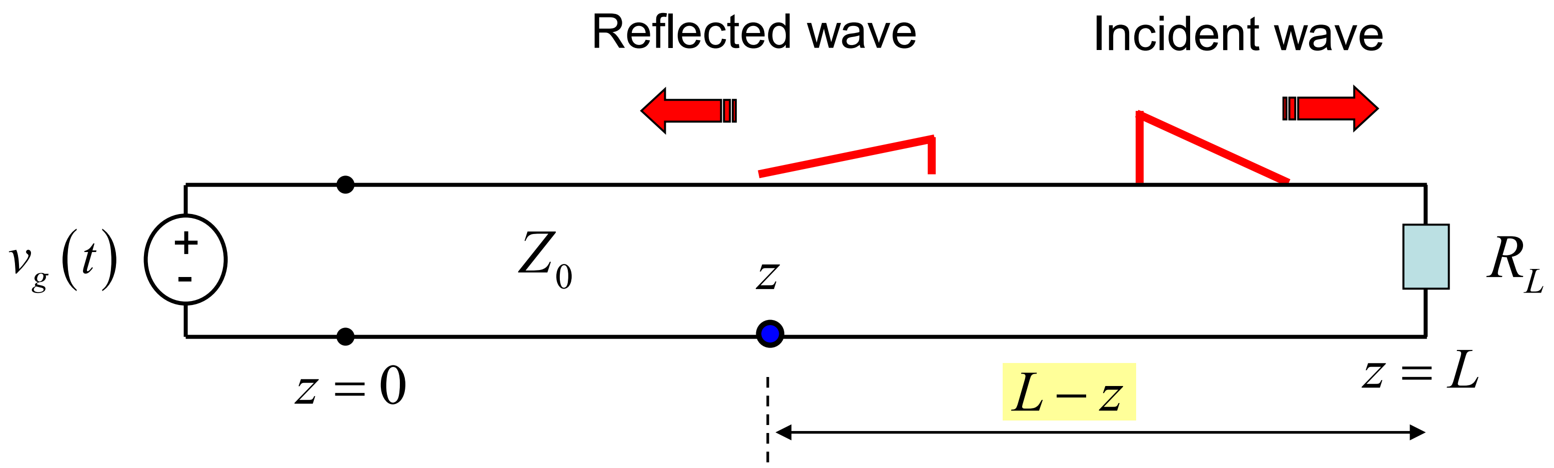
At the load:

$$v^-(L,t) = \Gamma_L v_g(t - L/c_d)$$

Moving back from the load, we see an additional delay of $(L-z)/c_d$



# Load Reflection (cont.)




Hence, we have for the reflected wave:

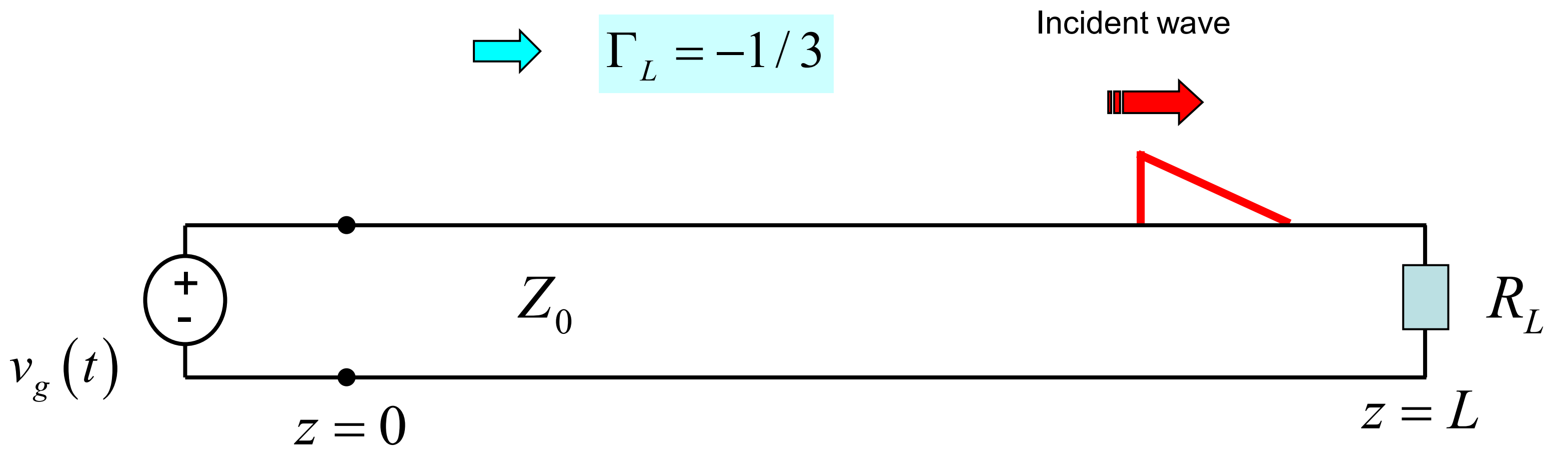
$$v^-(z,t) = \Gamma_L v_g\left(t - L/c_d - (L-z)/c_d\right)$$

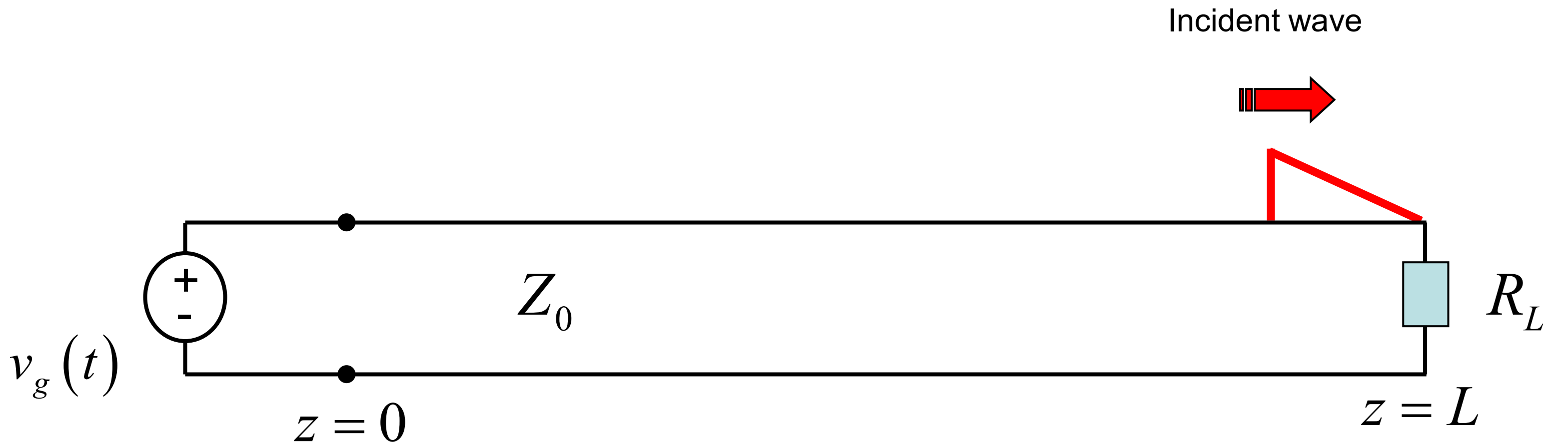
# Reflection Picture

Assume:

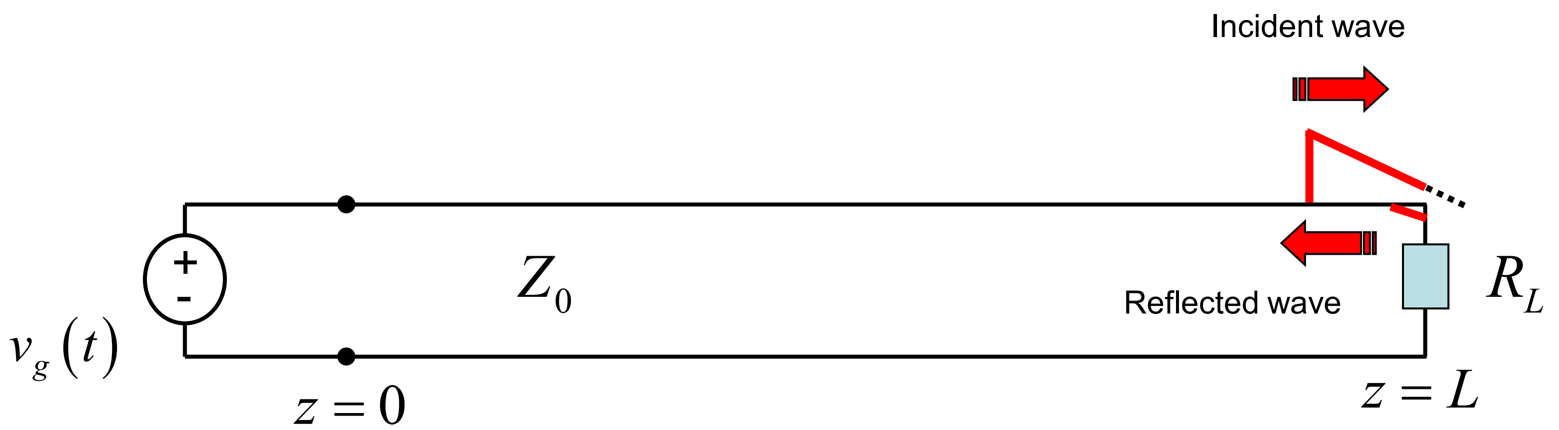
$$Z_0 = 50\ [\Omega]$$
$$R_L = 25\ [\Omega]$$

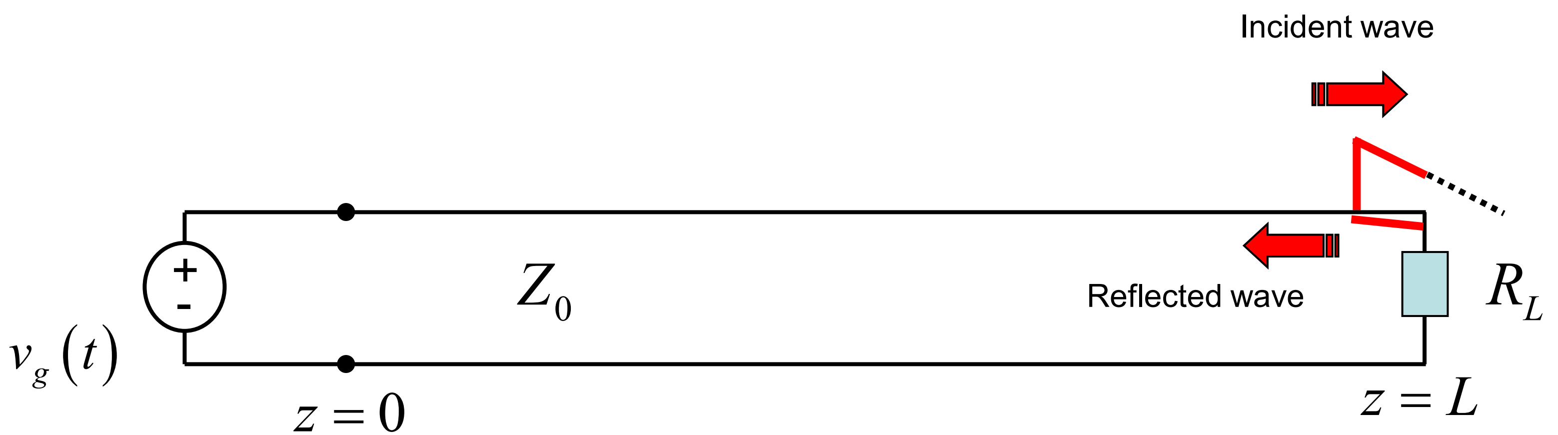
$$\Rightarrow \quad \Gamma_L = -1/3$$



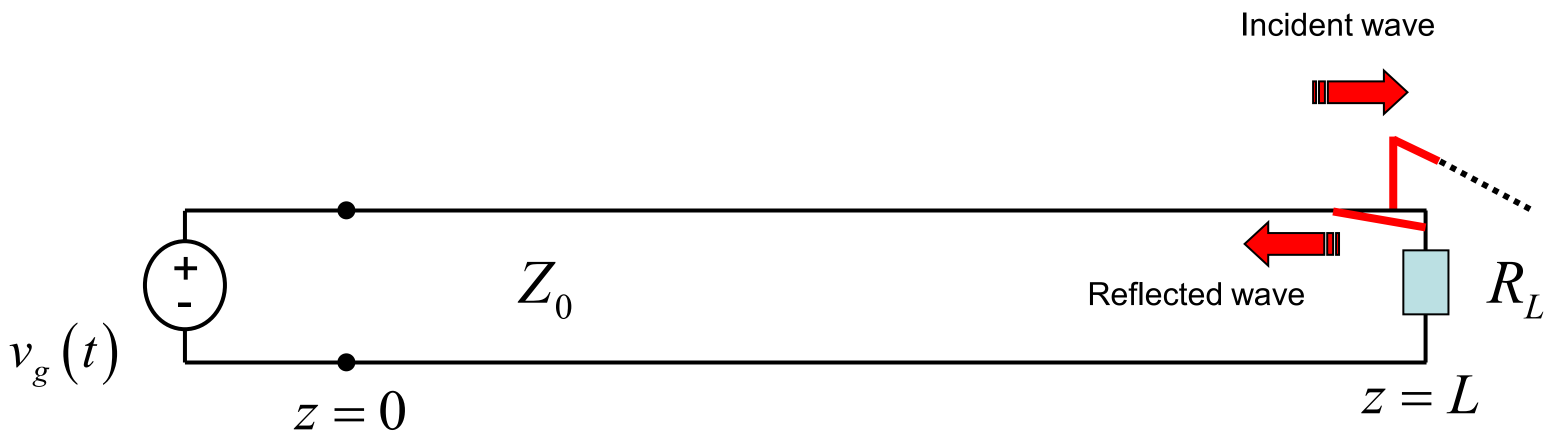



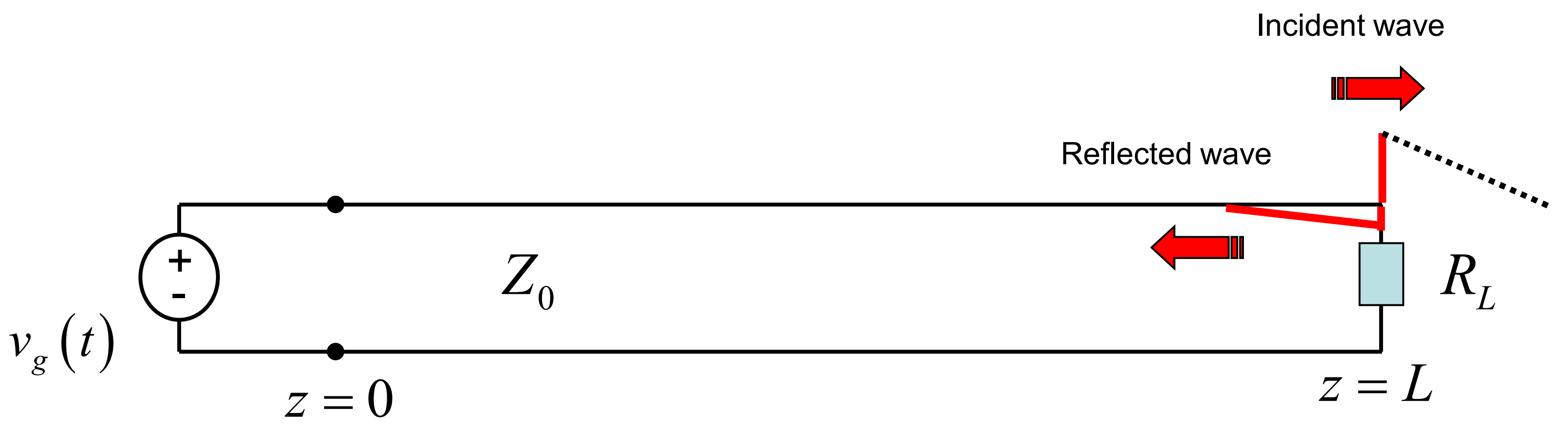
# Reflection Picture (cont.)



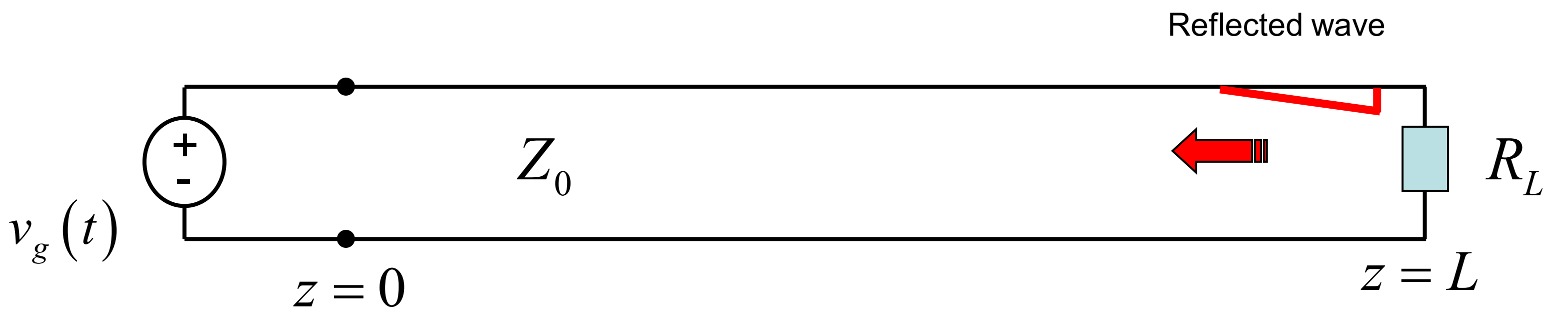



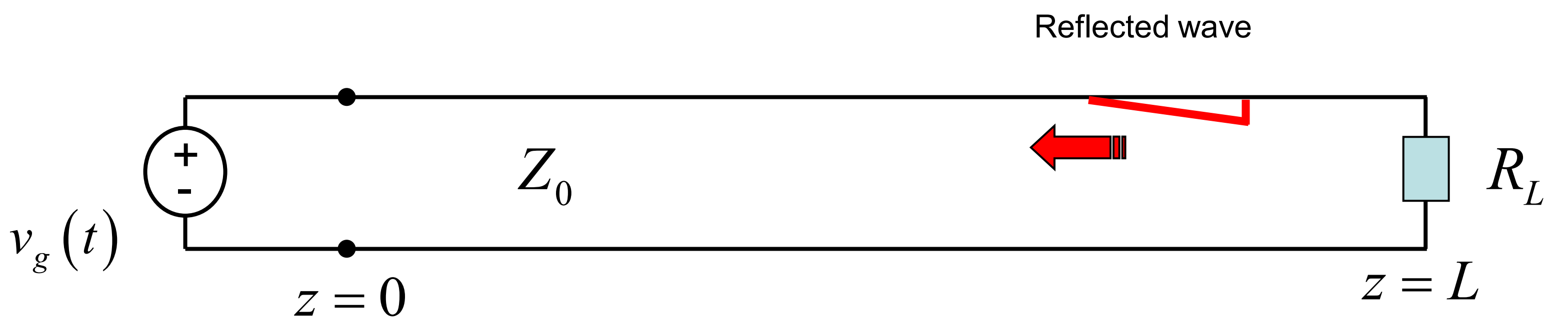
# Reflection Picture (cont.)



# Reflection Picture (cont.)

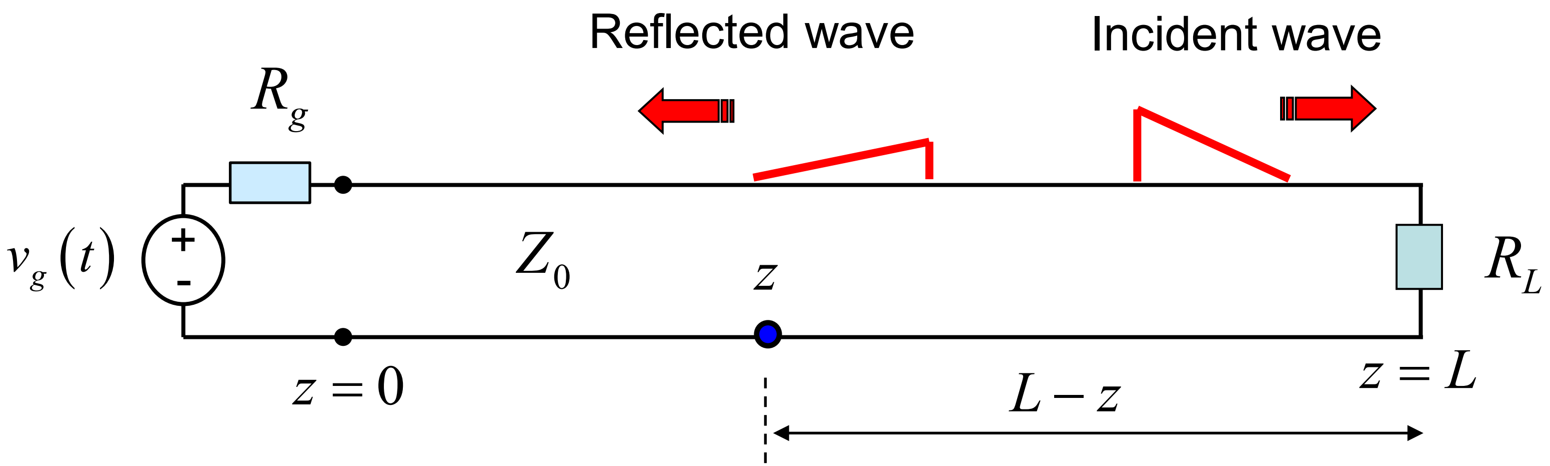


# Practical Voltage Generator

## Including Thévenin Resistance of Source

Voltage divider: $$v^+(0,t) = A v_g(t); \quad A = \frac{Z_0}{R_g + Z_0}$$



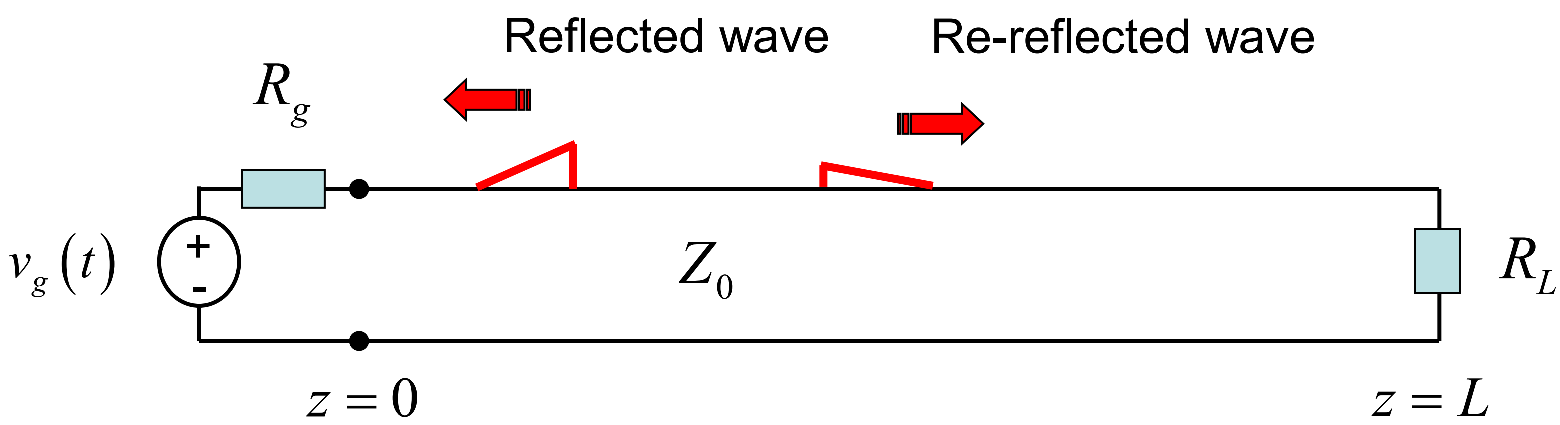

Incident wave: $$v^+(z,t) = A v_g(t - z/c_d)$$

Reflected wave: $$v^+(z,t) = \Gamma_L A v_g\left(t - L/c_d - (L - z)/c_d\right)$$

# Reflections at Source End

After the reflected wave hits the source end, there will be another reflection.

Here we allow the source to have a Thévenin resistance $R_g$.

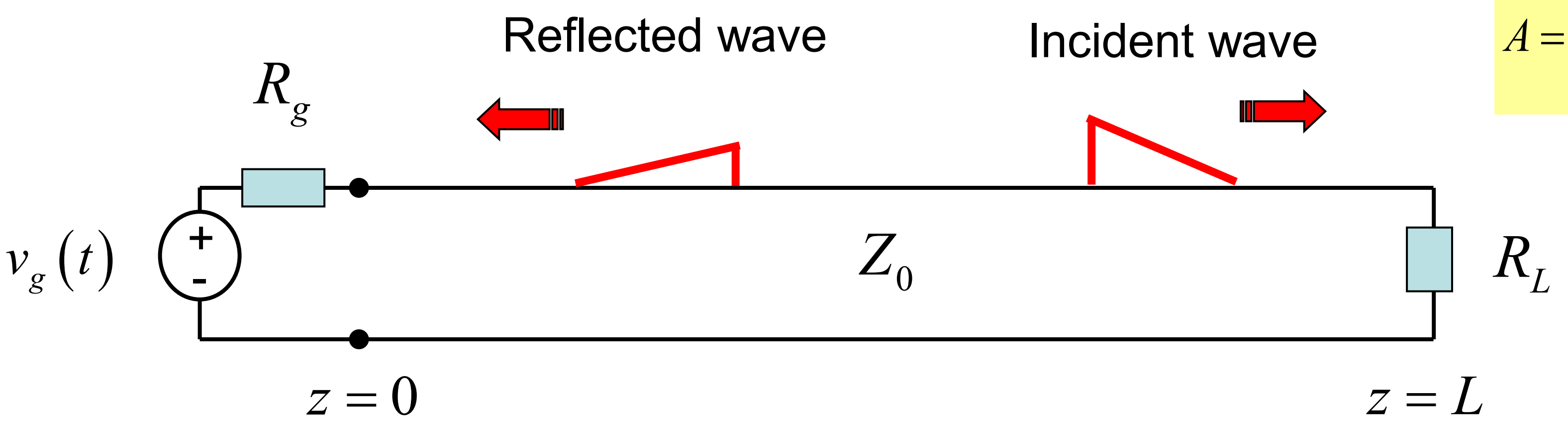



$$\Gamma_g = \left( \frac{R_g - Z_0}{R_g + Z_0} \right)$$

**Note:**
In calculating the reflection from the source, we ignore the source voltage.

# Complete Wave Picture

$$A = \frac{Z_0}{R_g + Z_0}$$




Here are the first four waves:

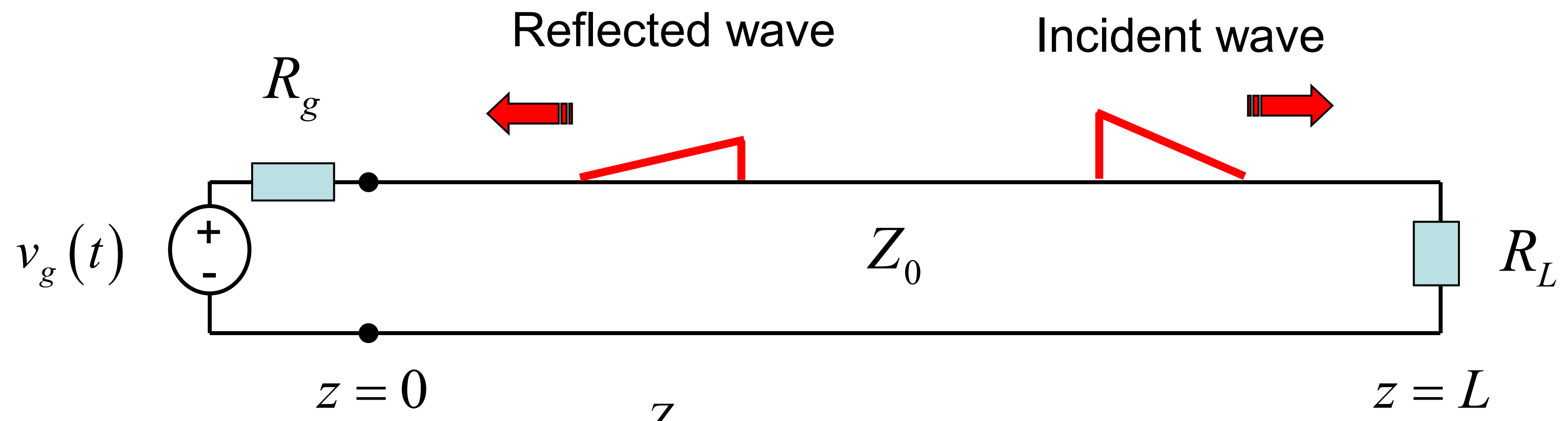
Incident: $$v^{+}(z,t) = A v_g\left(t - z / c_d\right)$$

Reflected: $$v^{-}(z,t) = \Gamma_L A v_g\left(t - L / c_d - (L - z) / c_d\right)$$

Re-reflected: $$v^{++}(z,t) = \Gamma_g \Gamma_L A v_g\left(t - 2L / c_d - z / c_d\right)$$

Re-re-reflected: $$v^{--}(z,t) = \Gamma_g \Gamma_L^2 A v_g\left(t - 3L / c_d - (L - z) / c_d\right)$$

# Exact Solution



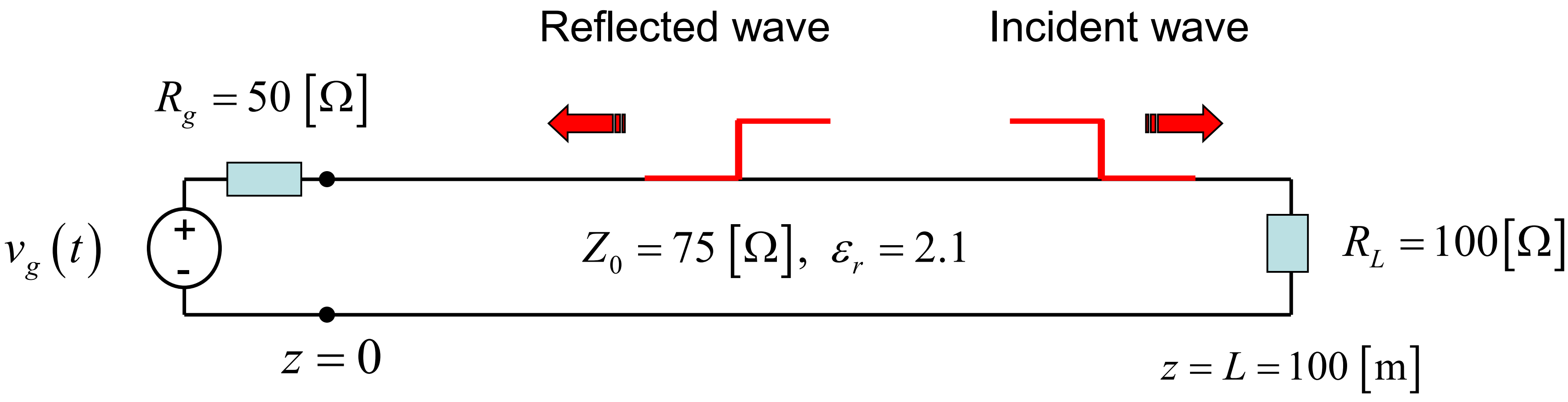

$$A=\frac{Z_0}{R_g+Z_0}$$

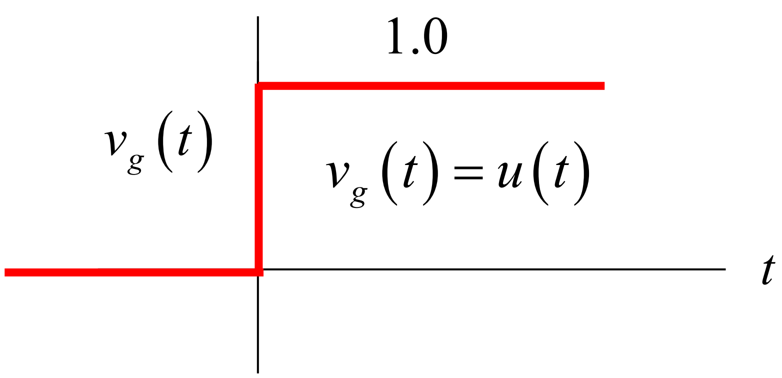
The exact solution (from summing all the reflections):

$$v(z,t)=A\sum_{\substack{n=0\\ \text{even}}}^{\infty}\left(\Gamma_g\Gamma_L\right)^{n/2} v_g\left(t-nL/c_d-z/c_d\right)$$

$$+A\Gamma_L\sum_{\substack{n=1\\ \text{odd}}}^{\infty}\left(\Gamma_g\Gamma_L\right)^{(n-1)/2} v_g\left(t-nL/c_d-(L-z)/c_d\right)$$

# Example



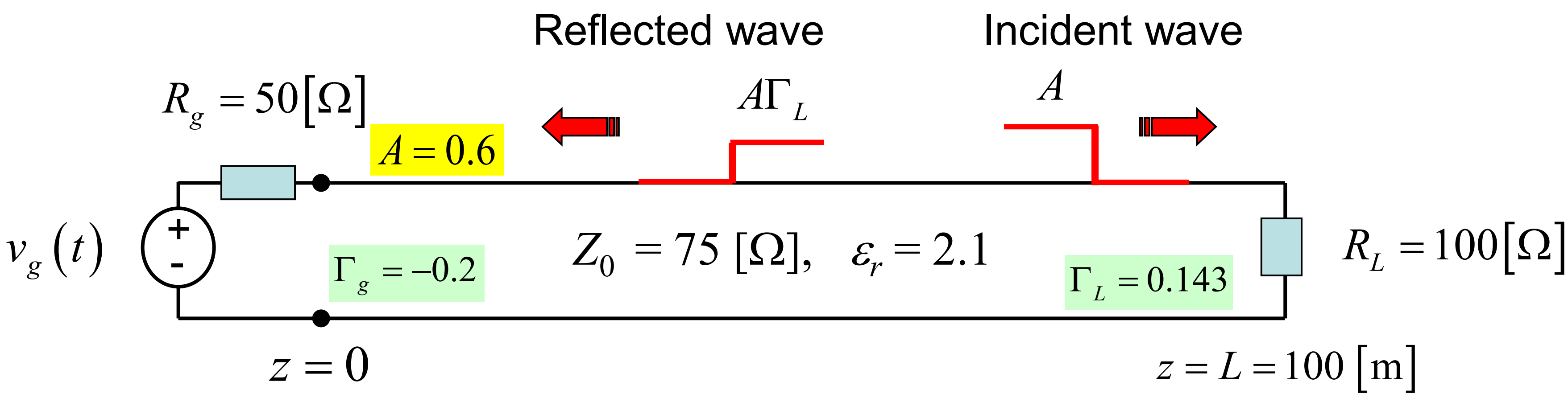

$$c_d = c / \sqrt{2.1} = 2.069 \times 10^8\ [\mathrm{m/s}]$$

$$A = \frac{75}{75+50} = 0.6000$$

$$\Gamma_L = \frac{100-75}{100+75} = 0.1429$$

$$\Gamma_g = \frac{50-75}{75+50} = -0.2000$$

# Example (cont.)



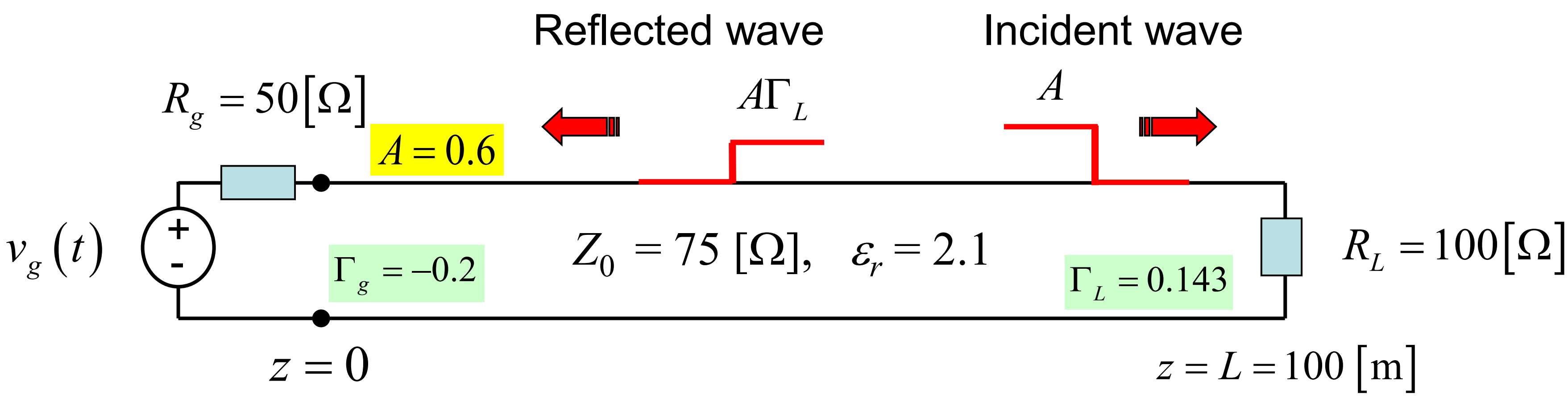

Incident: $v^{+}(z,t) = (0.6)\,u(t - z/c_d)$

Reflected: $v^{-}(z,t) = (0.1429)(0.6)\,u(t - L/c_d - (L-z)/c_d)$

Re-reflected: $v^{++}(z,t) = (-0.2)(0.1429)(0.6)\,u(t - 2L/c_d - z/c_d)$

Re-re-reflected: $v^{--}(z,t) = (-0.2)(0.1429)^2(0.6)\,u(t - 3L/c_d - (L-z)/c_d)$

# Example (cont.)




Total voltage:

$$
\begin{aligned}
v(z,t) = &(0.6)u(t - z/c_d) \\
&+(0.0858)u(t - L/c_d - (L-z)/c_d) \\
&+(-0.0172)u(t - 2L/c_d - z/c_d) \\
&+(-0.00245)u(t - 3L/c_d - (L-z)/c_d) \\
&+\ldots
\end{aligned}
$$

$L = 100\ [\text{m}]$

$c_d = c/\sqrt{2.1} = 2.069 \times 10^8\ [\text{m/s}]$

# Comments

- The higher-order reflected waves get smaller, due to the reflection coefficients.
- The bounce diagram (discussed in the next set of notes) gives us a convenient way of tracking all the waves and determining the waveform observed at any point on the line, when the generator voltage is a step function (or a rectangular pulse).