ECE 3317
Applied Electromagnetic Waves

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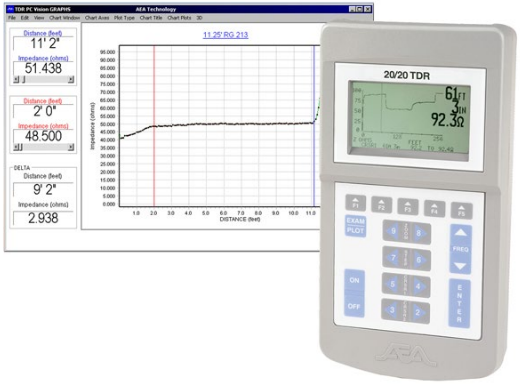
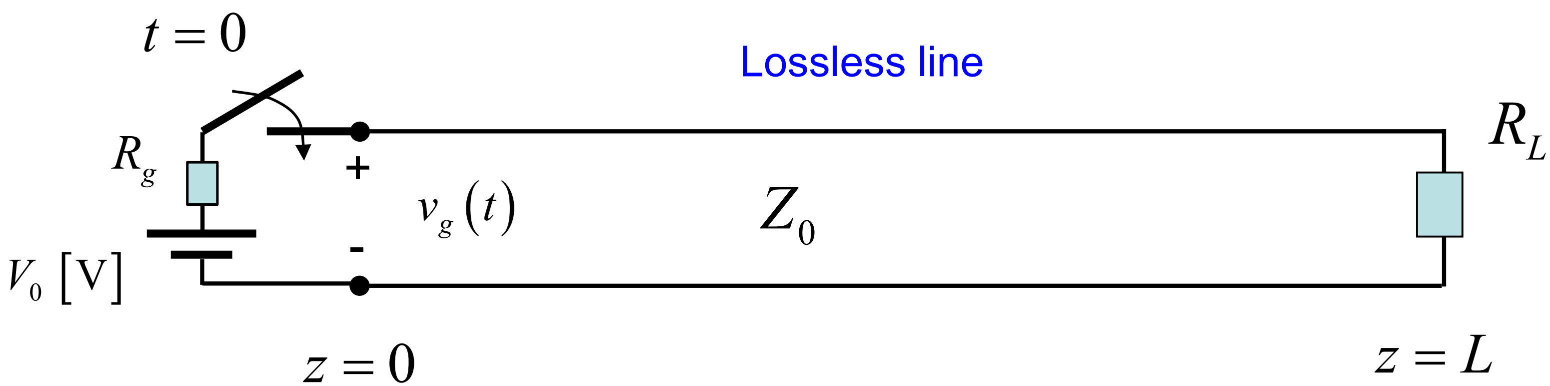
Fall 2023

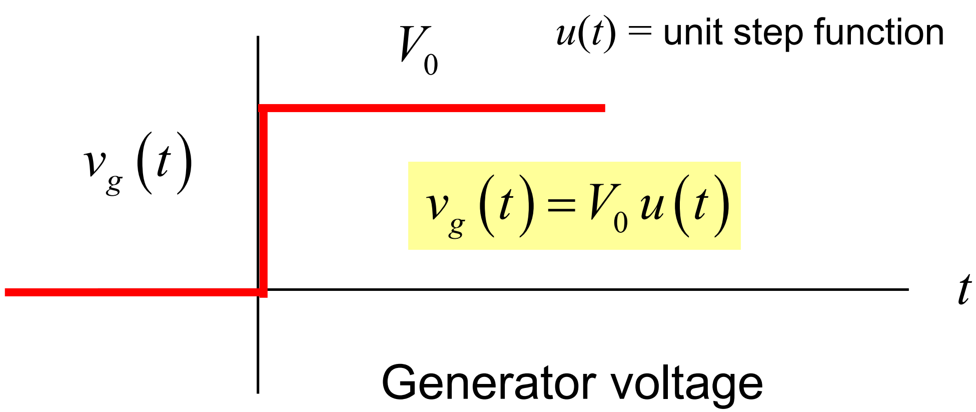
Notes 8
Transmission Lines
(Bounce Diagram)



Step Response

The concept of the bounce diagram is useful to find a <u>step response</u> on a terminated lossless line:

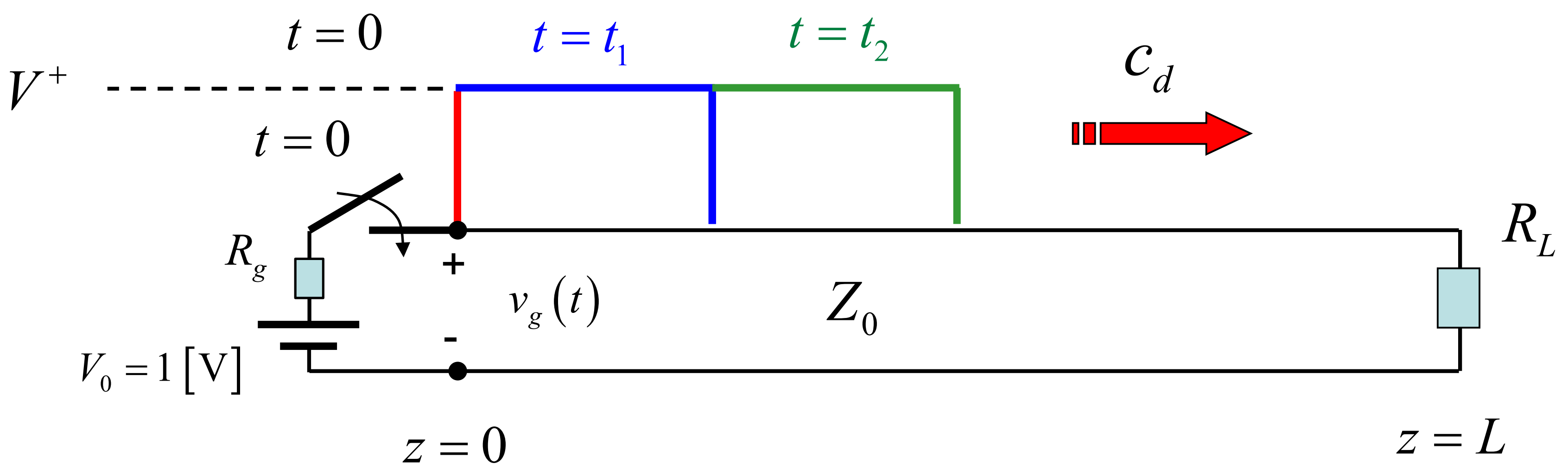




Note:
The bounce diagram is useful if the source is a step function or a rectangular pulse (discussed later in these notes). If the source is something else, it is better to use the general theory presented in Notes 7.

Step Response (cont.)

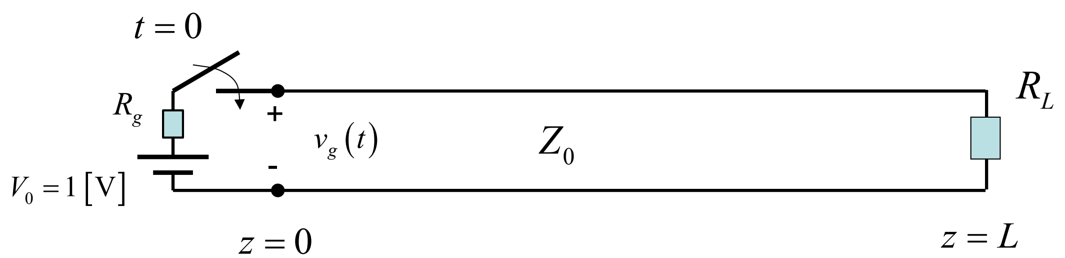
The voltage wave is shown approaching the load.



$$V^+ = \left(\frac{Z_0}{R_g + Z_0} \right) V_0$$

(from voltage divider)

Bounce Diagram



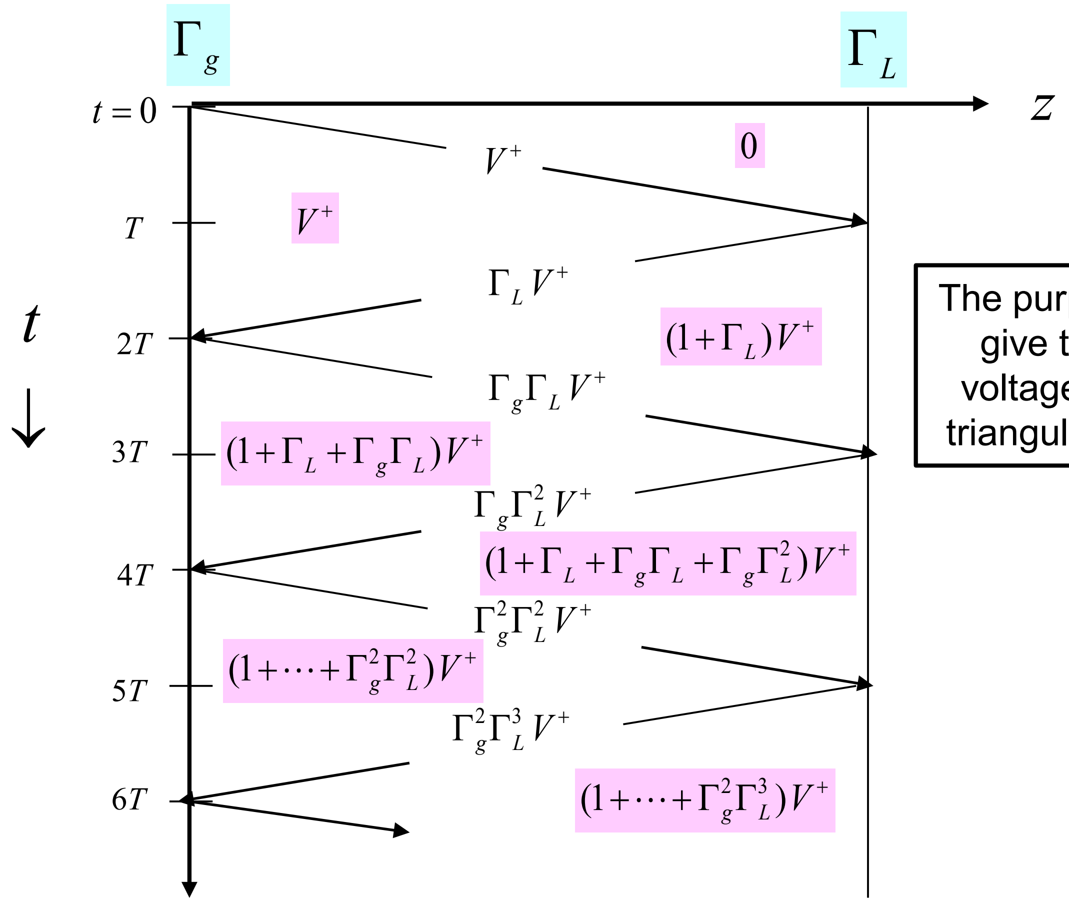
Basic constants

$$T = \frac{L}{c_d}$$

$$V^+ = \left(\frac{Z_0}{R_g + Z_0}\right) V_0$$

$$\Gamma_L = \left(\frac{R_L - Z_0}{R_L + Z_0}\right)$$

$$\Gamma_g = \left(\frac{R_g - Z_0}{R_g + Z_0}\right)$$



The purple values give the total voltage in each triangular region.

Steady-State Solution

Adding all infinite number of bounces ($t = \infty$), we have:

$$V(z,\infty) = \underbrace{V^+(1+\Gamma_g\Gamma_L+\Gamma_g^2\Gamma_L^2+\Gamma_g^3\Gamma_L^3+\cdots)}_{\text{Sum of all right-traveling waves}} + \underbrace{V^+\Gamma_L(1+\Gamma_g\Gamma_L+\Gamma_g^2\Gamma_L^2+\Gamma_g^3\Gamma_L^3+\cdots)}_{\text{Sum of all left-traveling waves}}$$

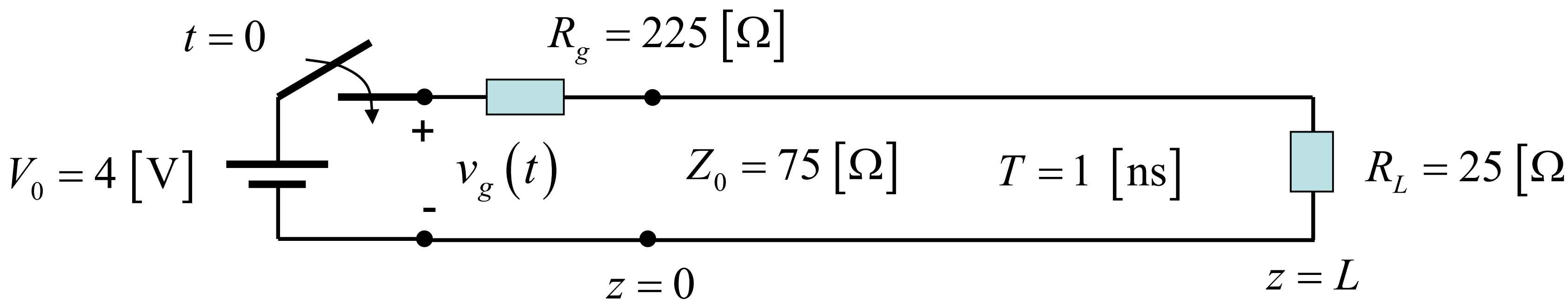
After some math, we have (please see the Appendix):

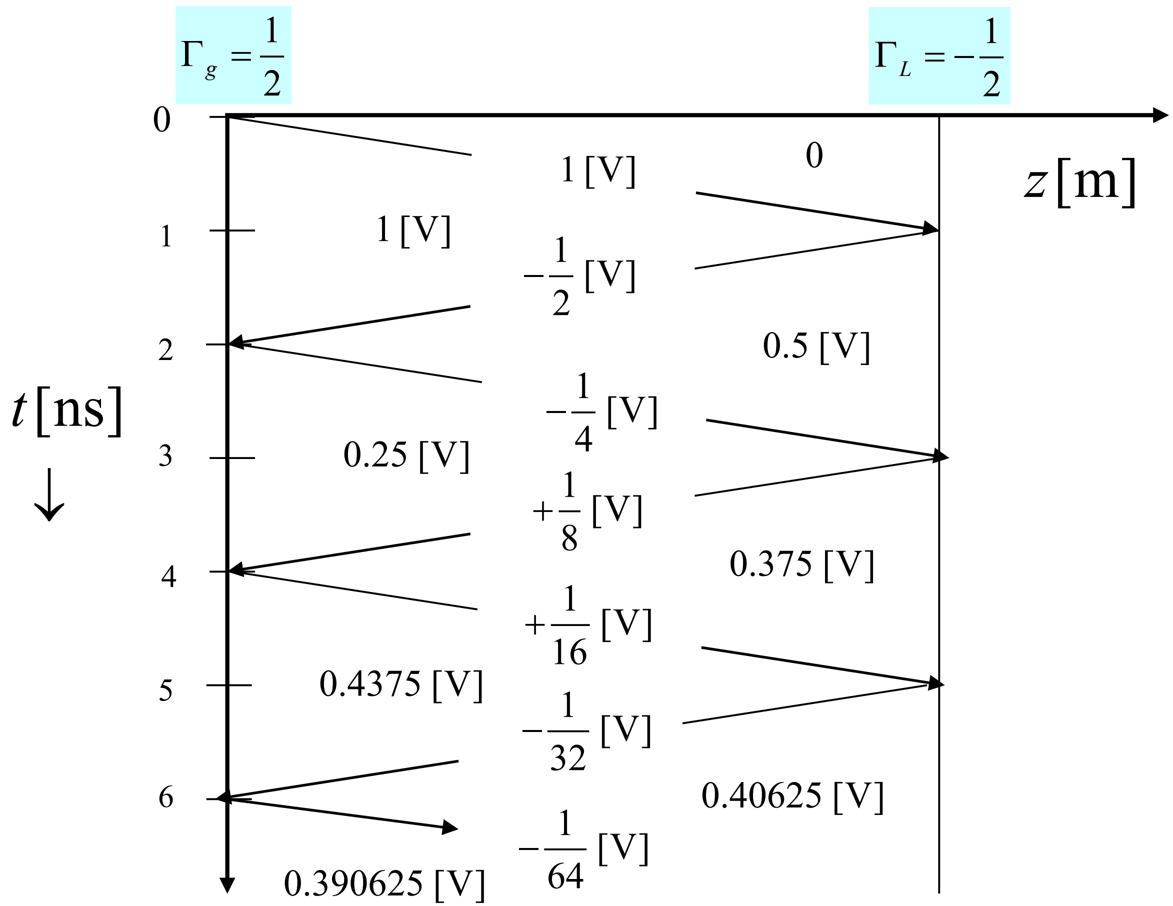
$$V(z,\infty) = \left(\frac{R_L}{R_L+R_g}\right)V_0$$

Note:
The steady-state solution does not depend on the transmission line length or the characteristic impedance!

This is the DC circuit-theory voltage divider equation!

Example





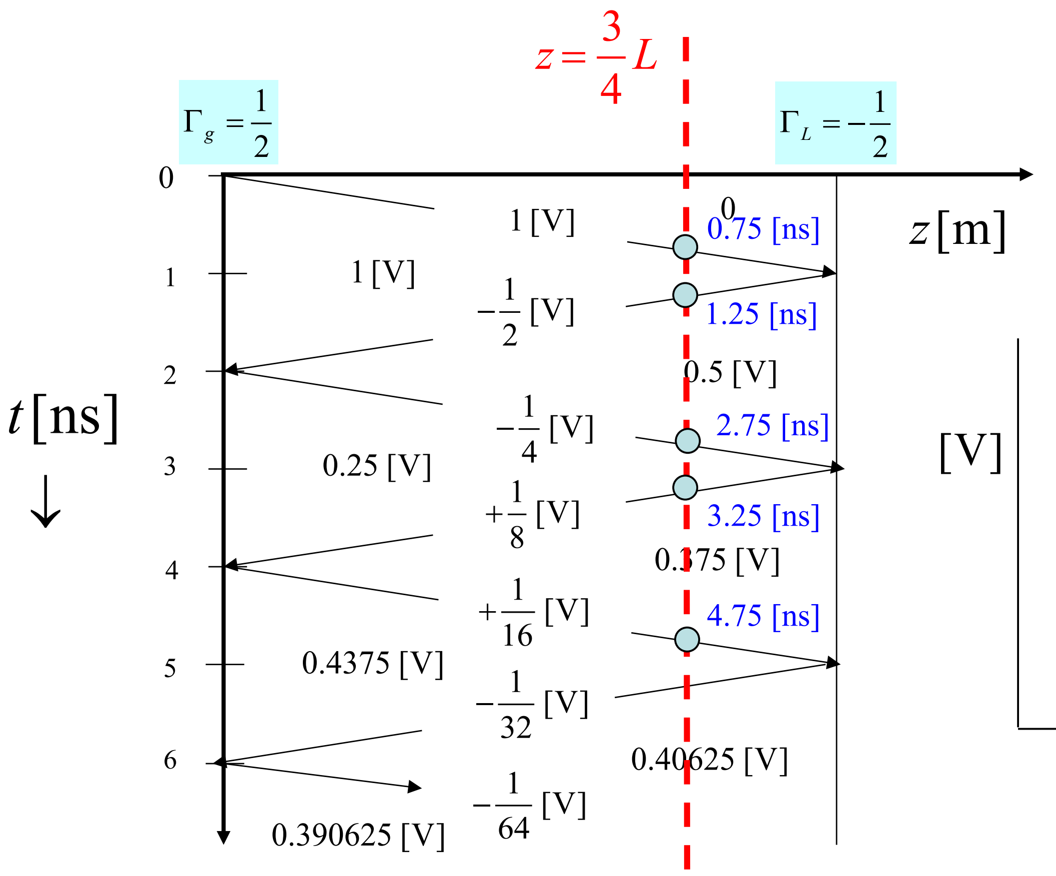
$$V^+ = \left(\frac{Z_0}{R_g + Z_0}\right) V_0 = 1\ [\mathrm{V}]$$

$$\Gamma_L = \left(\frac{R_L - Z_0}{R_L + Z_0}\right) = -\frac{1}{2}$$

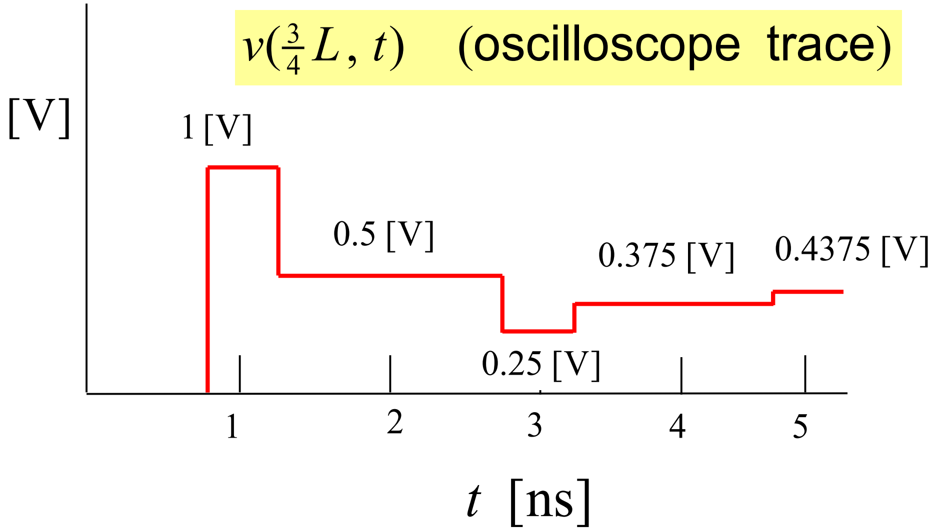
$$\Gamma_g = \left(\frac{R_g - Z_0}{R_g + Z_0}\right) = \frac{1}{2}$$

Example (cont.)

The bounce diagram can be used to get an "oscilloscope trace" of the voltage at any point on the line.



$z = \frac{3}{4}L$

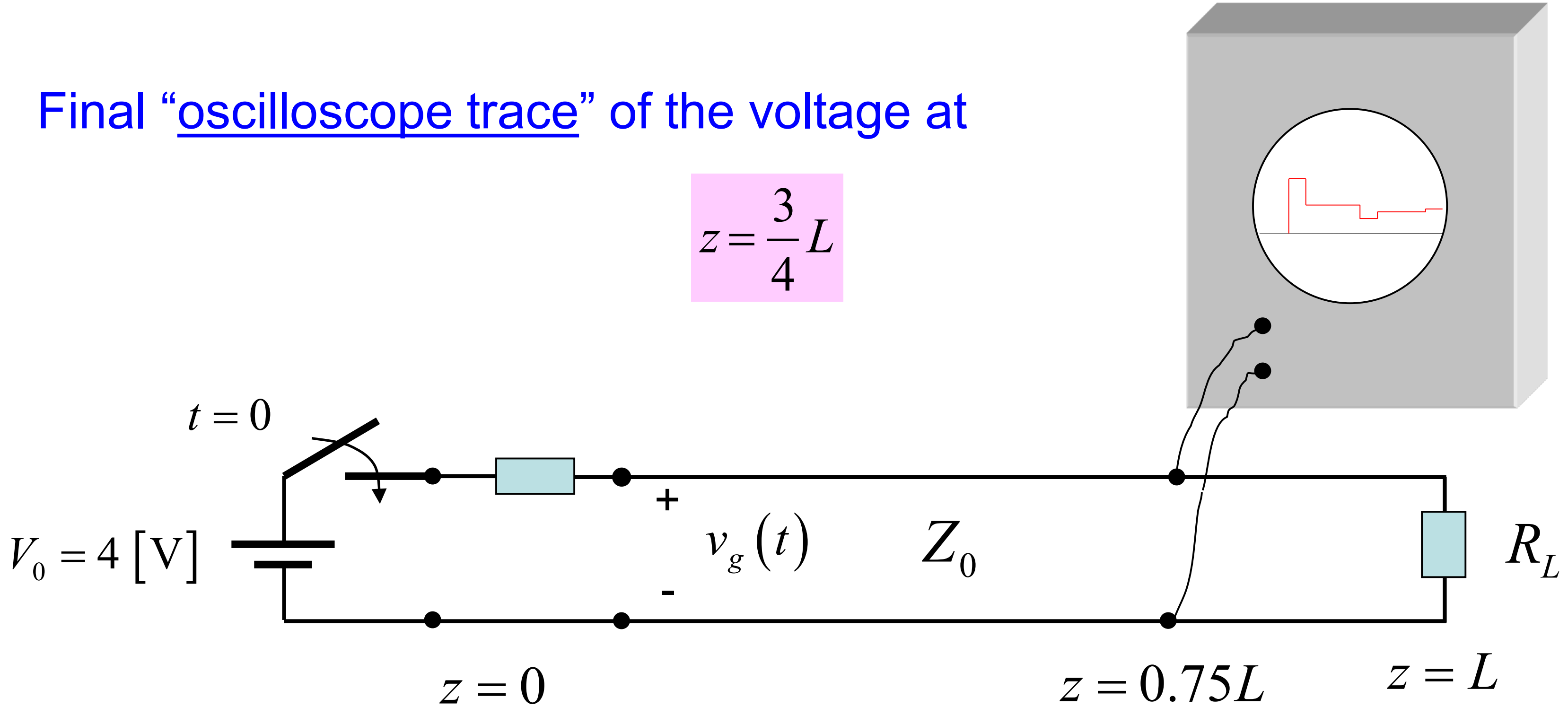


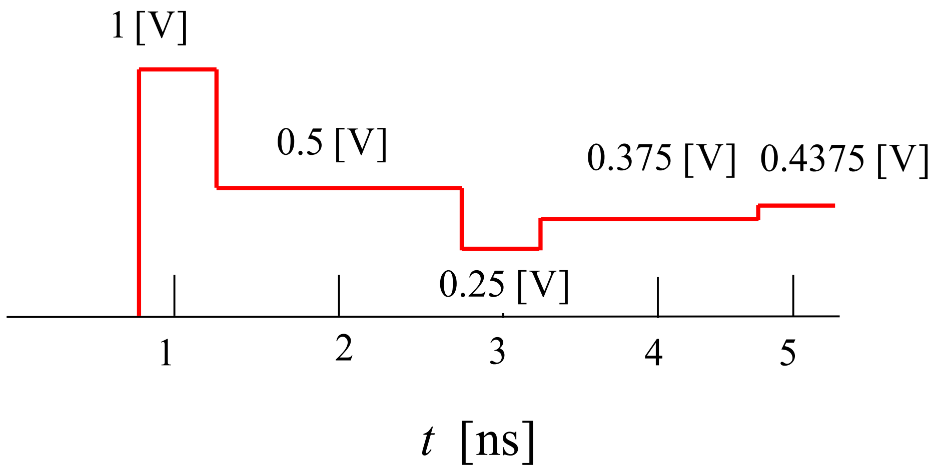
Steady state voltage: $v(z,\infty) = \left(\frac{R_L}{R_L + R_g}\right) V_0 = 0.400$ [V]

Example (cont.)

Final "oscilloscope trace" of the voltage at

$$z = \frac{3}{4}L$$

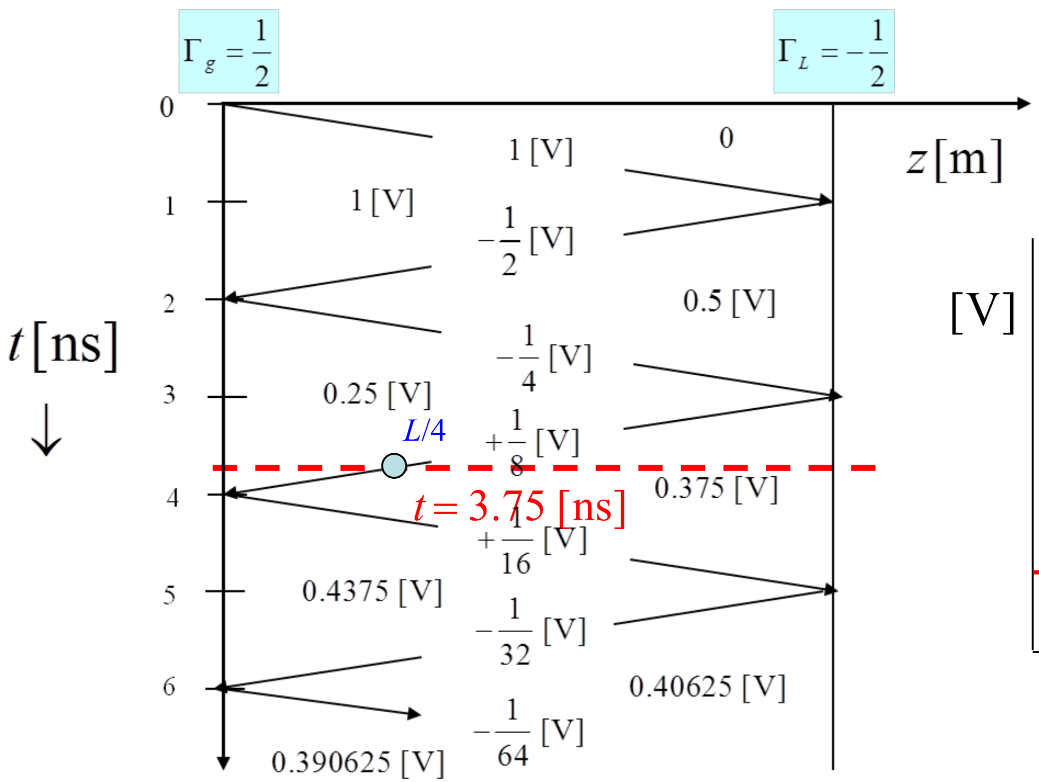


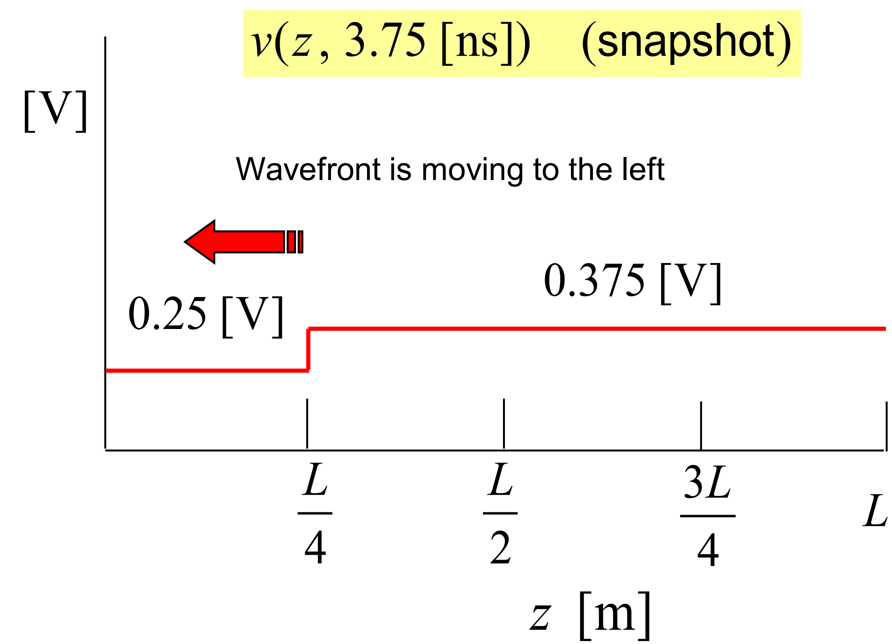


Example (cont.)

The bounce diagram can also be used to get a "snapshot" of the line voltage at any point in time.

$t = 3.75\,[\text{ns}]$

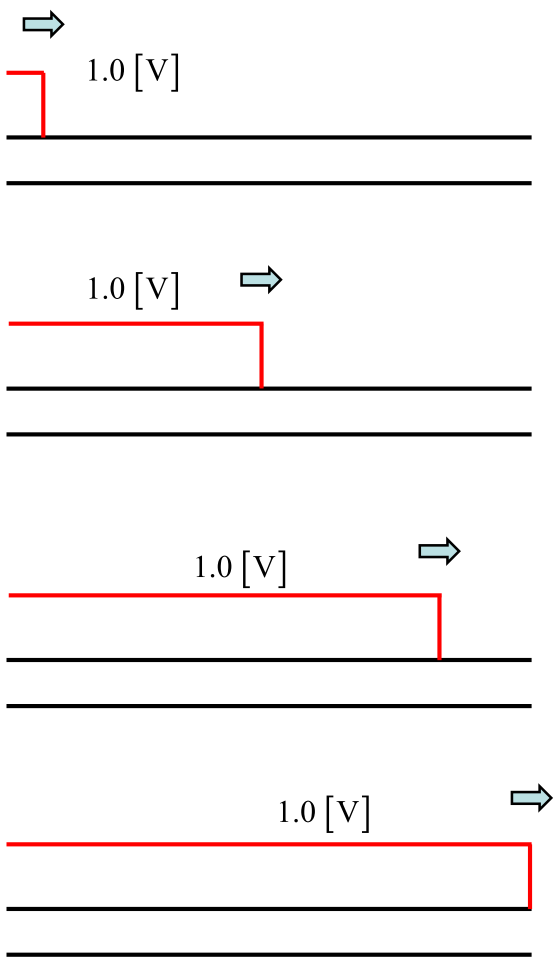




Example (cont.)

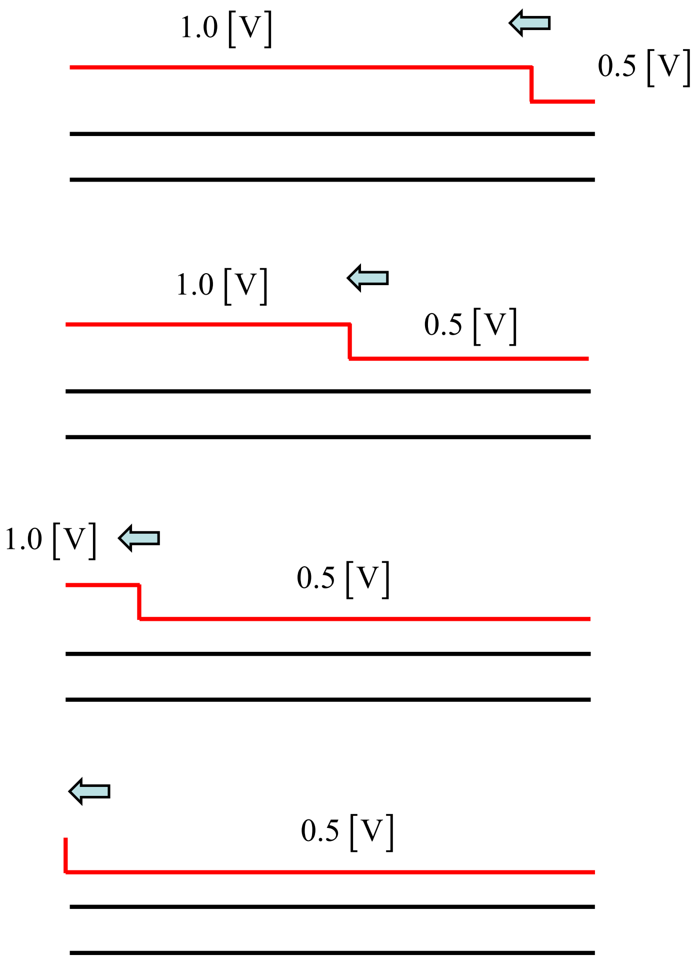
This set of snapshots shows the waves bouncing back and forth.

Initial wave



$$\Gamma_L = -\frac{1}{2}$$

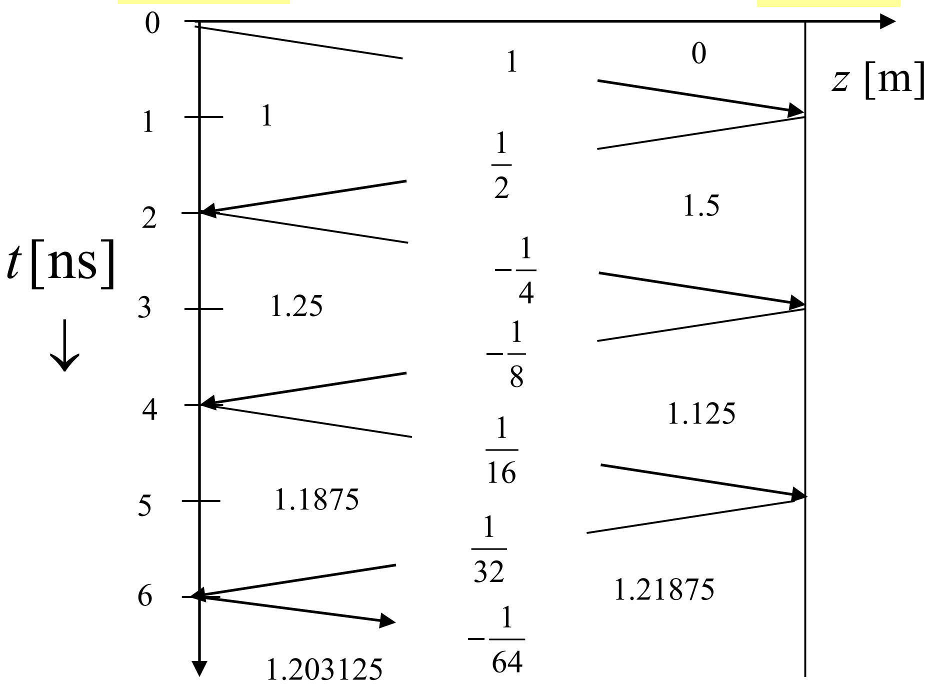
First reflected wave



Bounce Diagram for Current

We just change the signs of the reflection coefficients, as shown.

$$\Gamma^I = -\Gamma$$



$$\Gamma_L^I = \frac{i^-(L,t)}{i^+(L,t)}$$

$$= \frac{v^-(L,t)(-1/Z_0)}{v^+(L,t)(1/Z_0)}$$

$$= -\frac{v^-(L,t)}{v^+(L,t)}$$

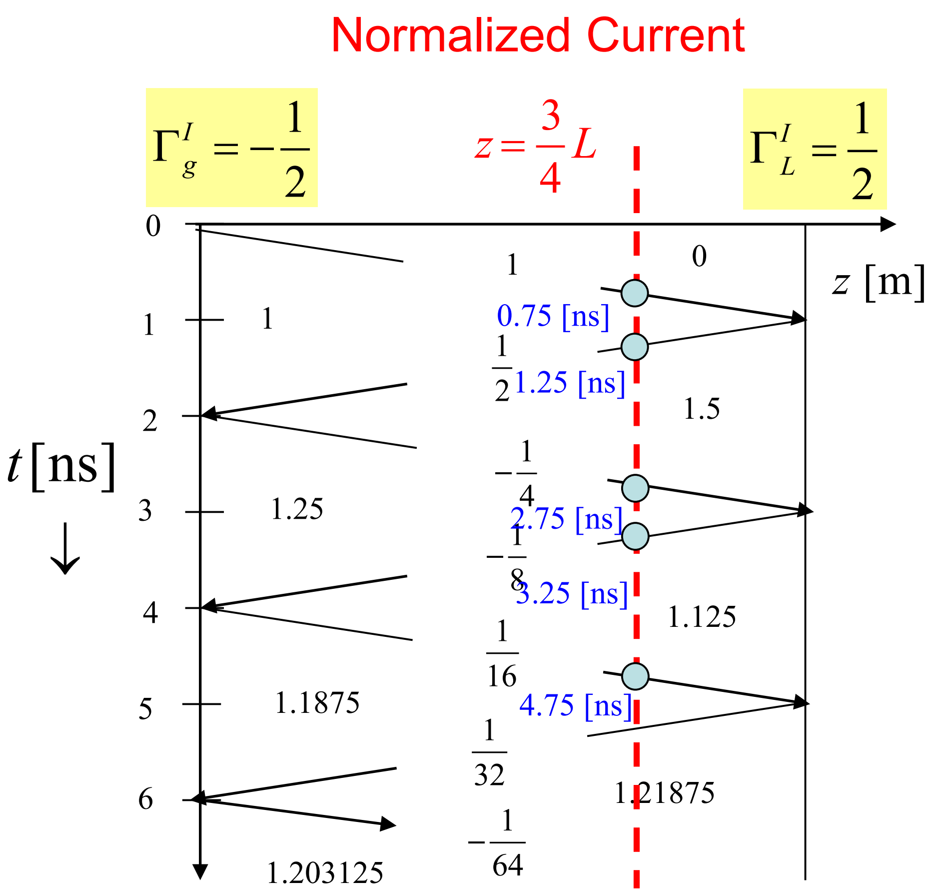
$$= -\Gamma_L$$

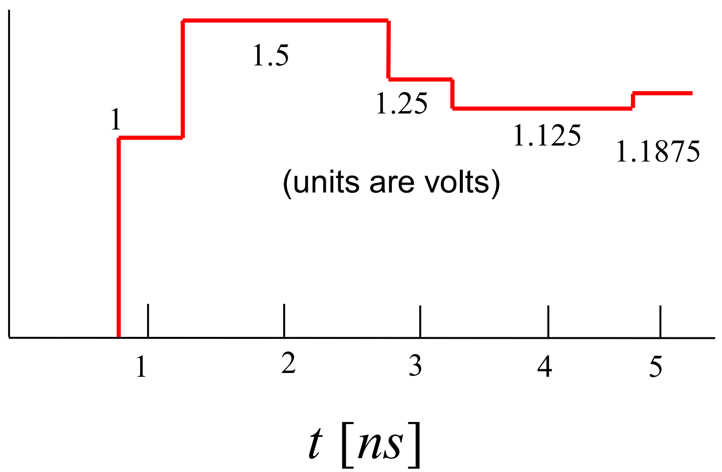
Note: The normalized current is defined as $Z_0\, i\,(z,t)$.

Example for Current

Oscilloscope Trace for Current

$$z = \frac{3}{4}L$$





Steady state current: $i(z,\infty) = \left(\dfrac{V_0}{R_L + R_g}\right) = 0.016$ [A] $\quad Z_0 i(z,\infty) = (0.016)(75) = 1.20$ [V]

Example for Current (cont.)

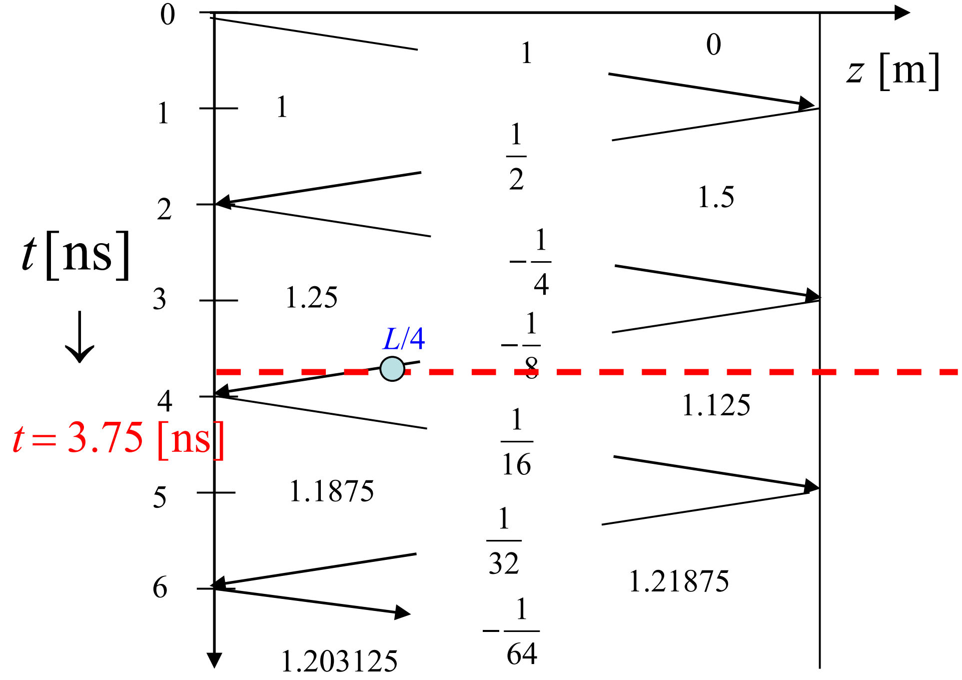
Snapshot for Current

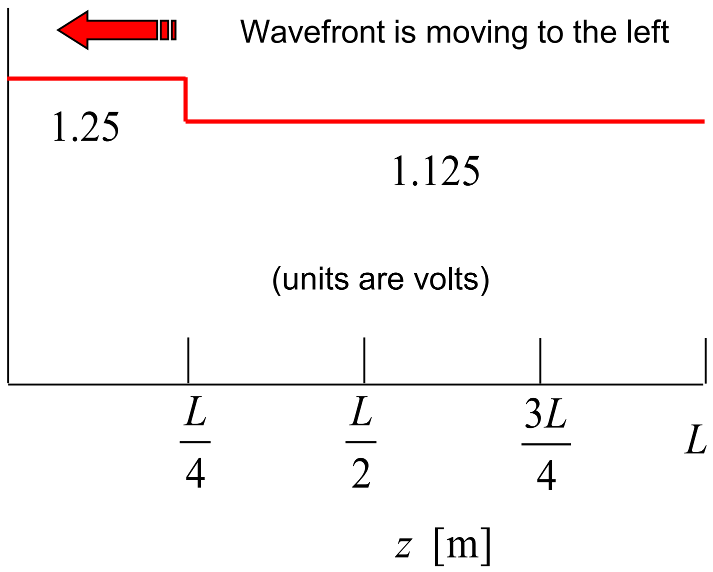
$t = 3.75\ [\text{ns}]$

$Z_0 i(z, 3.75\ [\text{ns}])$

(snapshot of current)

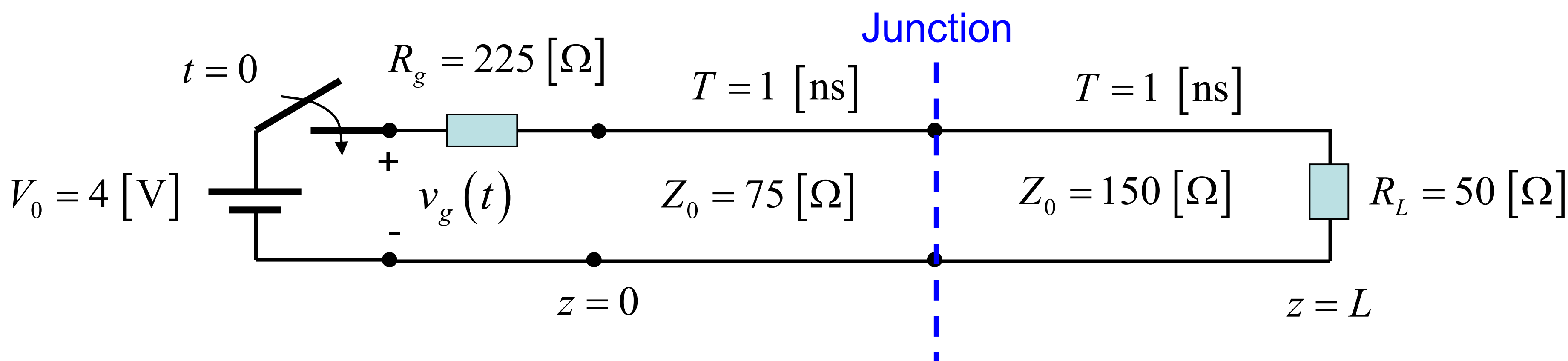
$\Gamma_g^I = -\frac{1}{2}$ Normalized Current $\Gamma_L^I = \frac{1}{2}$





Two Transmission Lines

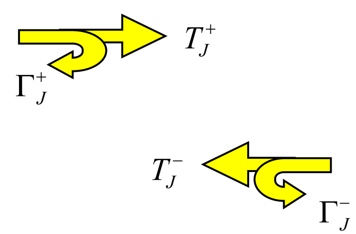
Here we have a reflection <u>and</u> transmission coefficient at the junction between two lines.



Note:
When the wave first hits the junction, it sees the characteristic impedance of the line on the other side as a load.

$$\Gamma_J^+ = \frac{150-75}{225} = \frac{1}{3}$$

$$T_J^+ = 1 + \Gamma_J^+ = \frac{4}{3}$$



$$\Gamma_J^- = \frac{75-150}{225} = -\frac{1}{3}$$

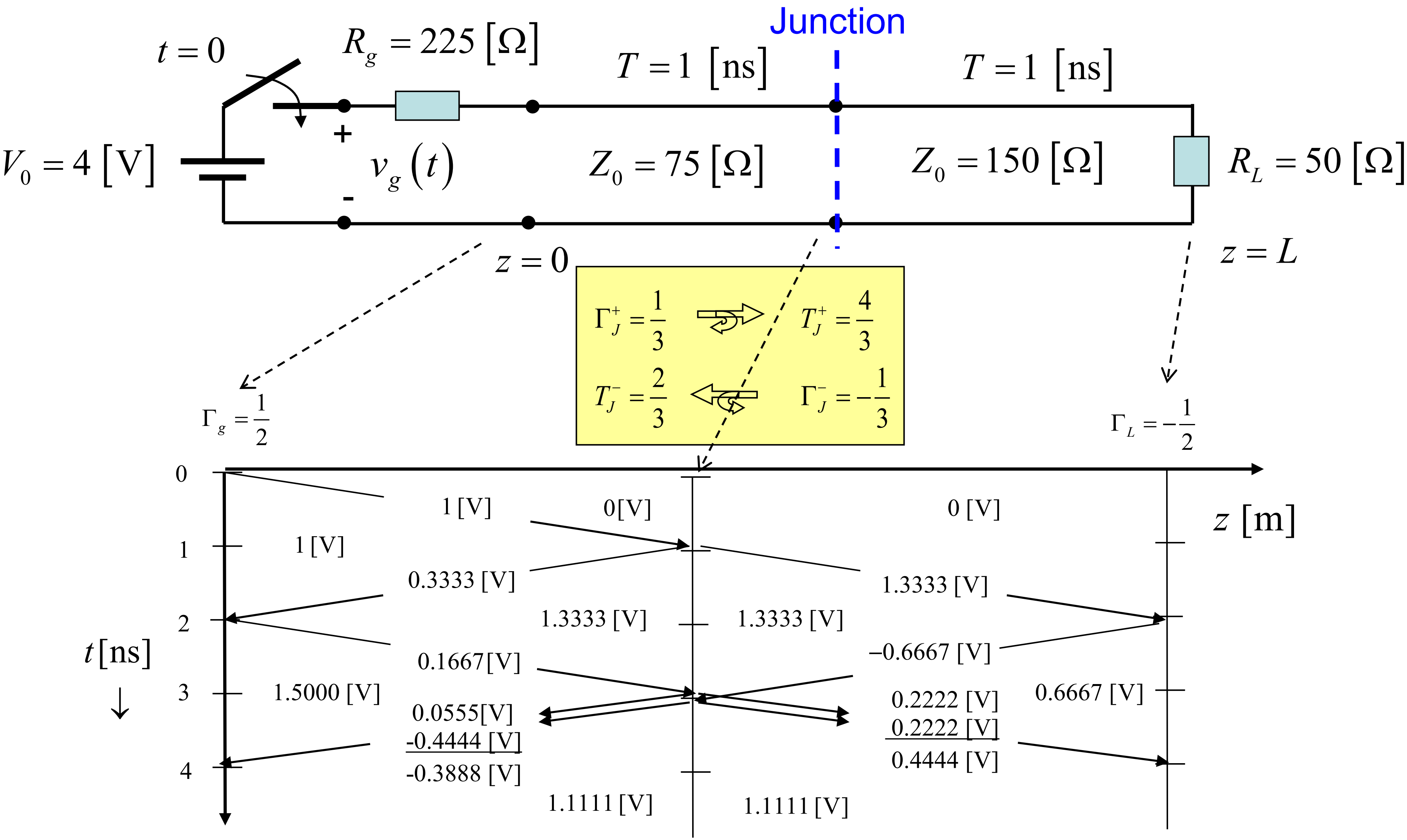
$$T_J^- = 1 + \Gamma_J^- = \frac{2}{3}$$

KVL: $T_J = 1 + \Gamma_J$

(This follows from the fact that voltage must be continuous across the junction.)

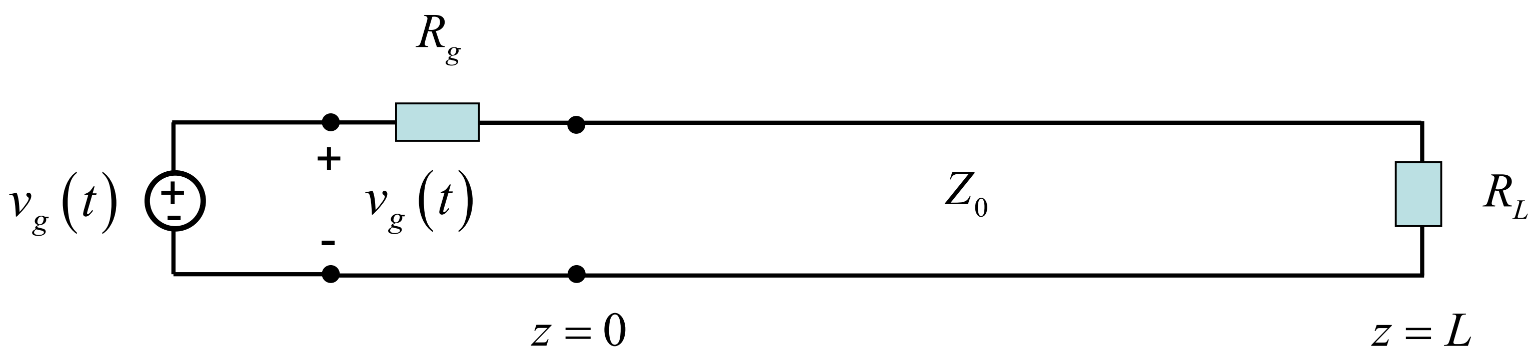
Two Transmission Lines (cont.)

Bounce Diagram for Two Lines

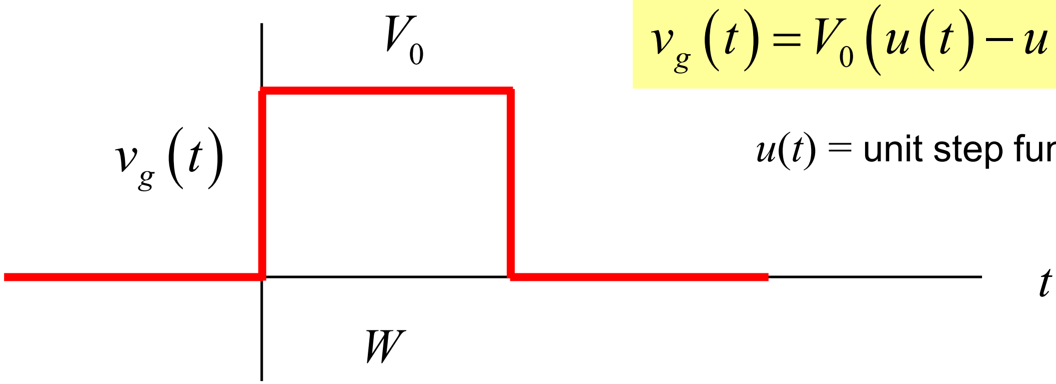


Pulse Response

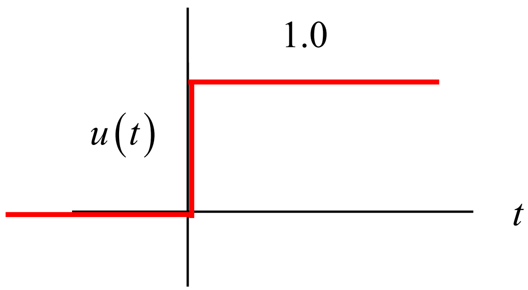
Superposition can be used to get the response due to a rectangular pulse.



$$v_g(t) = V_0\left(u(t) - u(t - W)\right)$$



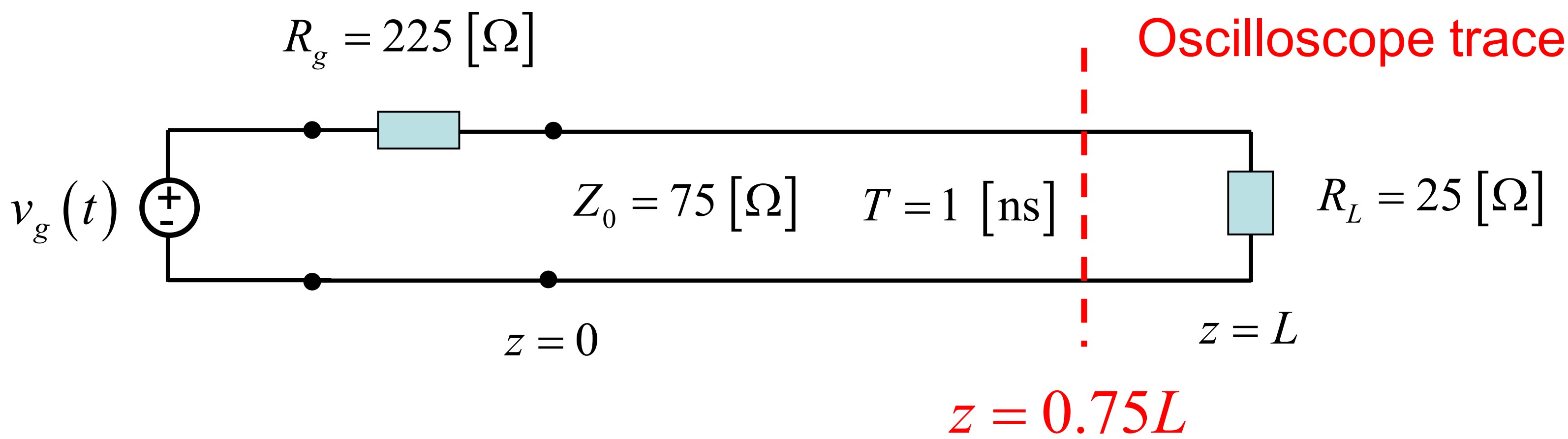
$u(t)$ = unit step function



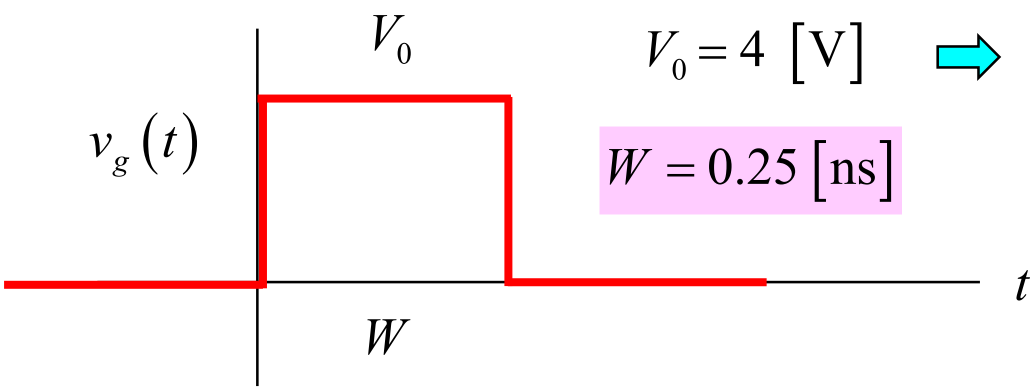
We thus subtract two bounce diagrams, with the second one being a shifted version of the first one.

Example: Pulse

Find an "oscilloscope trace" of the voltage at $z = 0.75\,L$



Given pulse:



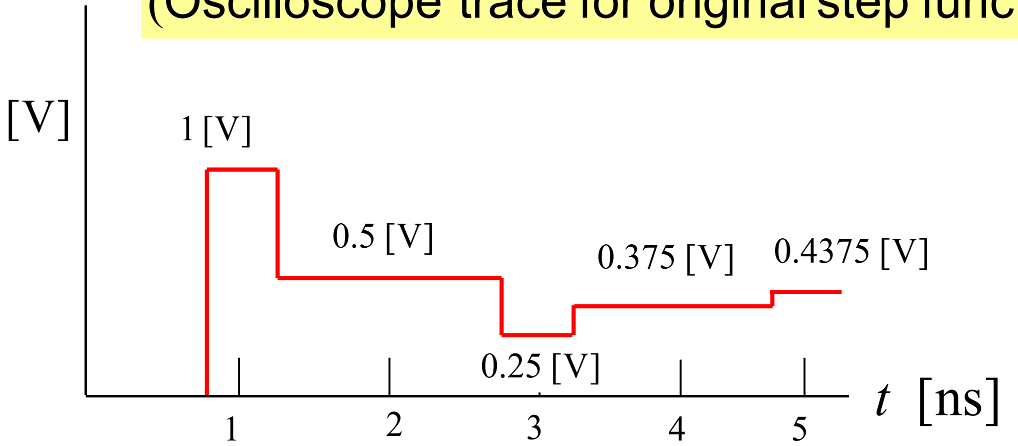
$V_0 = 4\ [\text{V}]$ ⇨ $V^+ = 1\ [\text{V}]$

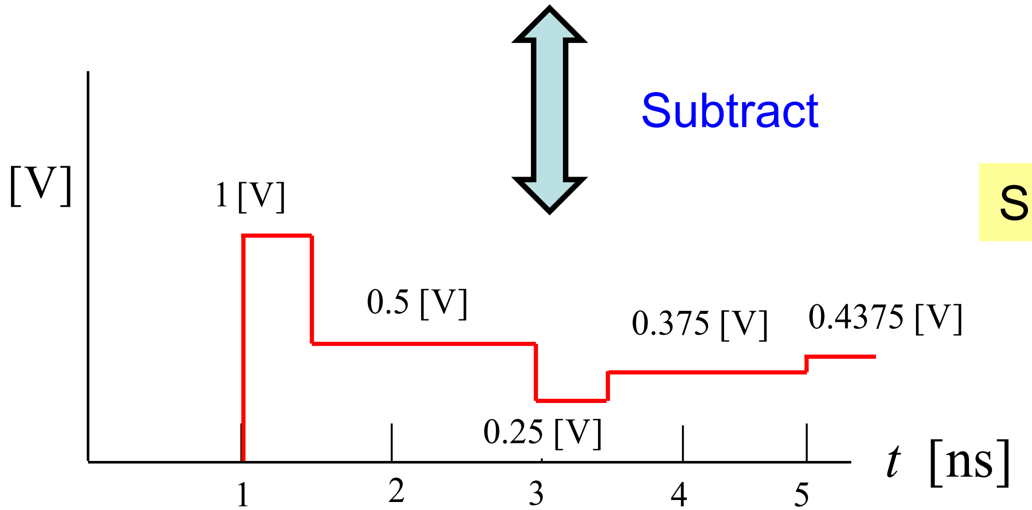
$W = 0.25\ [\text{ns}]$

Recall : $V^+ = \left(\dfrac{Z_0}{R_g + Z_0}\right) V_0 = 1\ [\text{V}]$

Example (cont.)

(Oscilloscope trace for original step function)

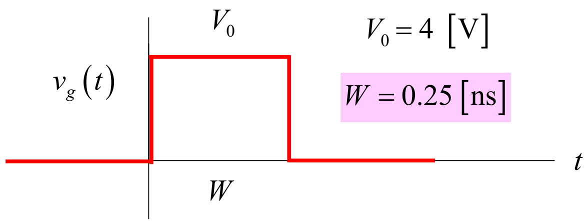


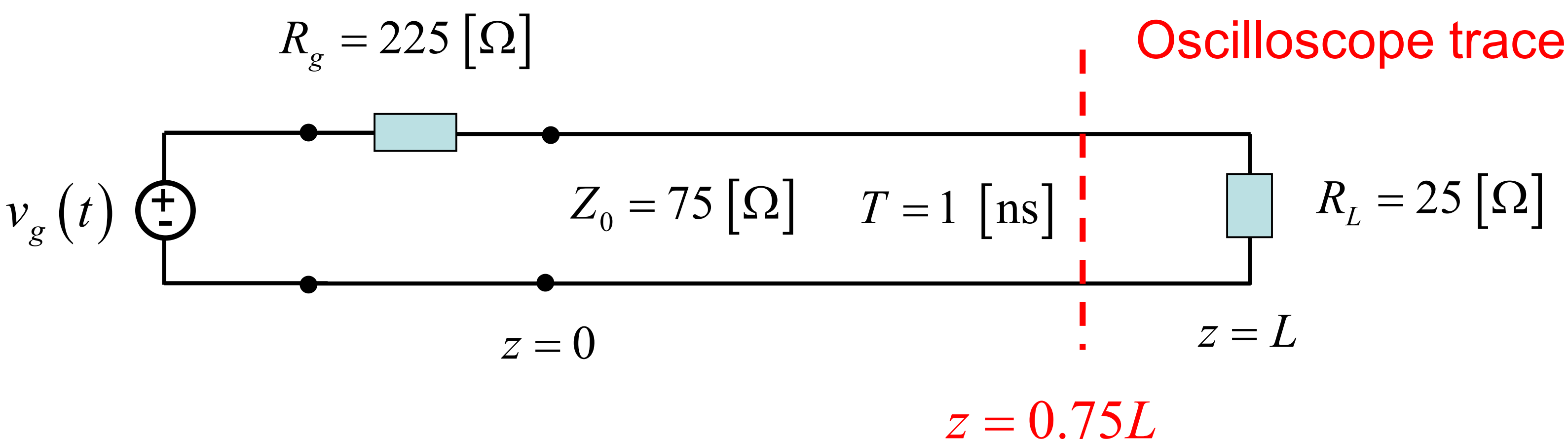


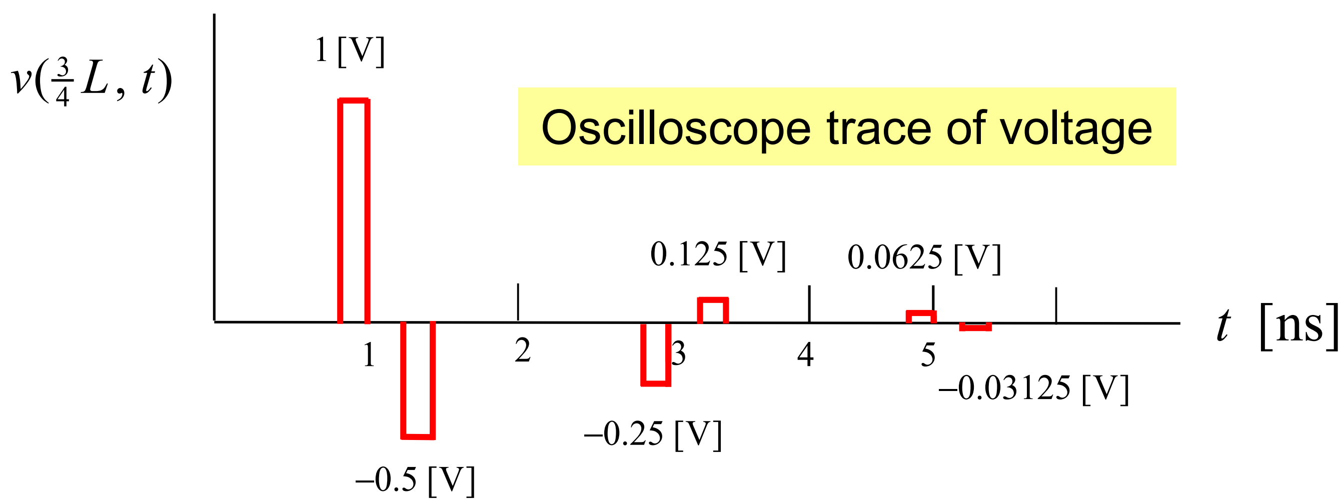
Shifted trace

(shifted by 0.25 [ns])

Example: Pulse (cont.)



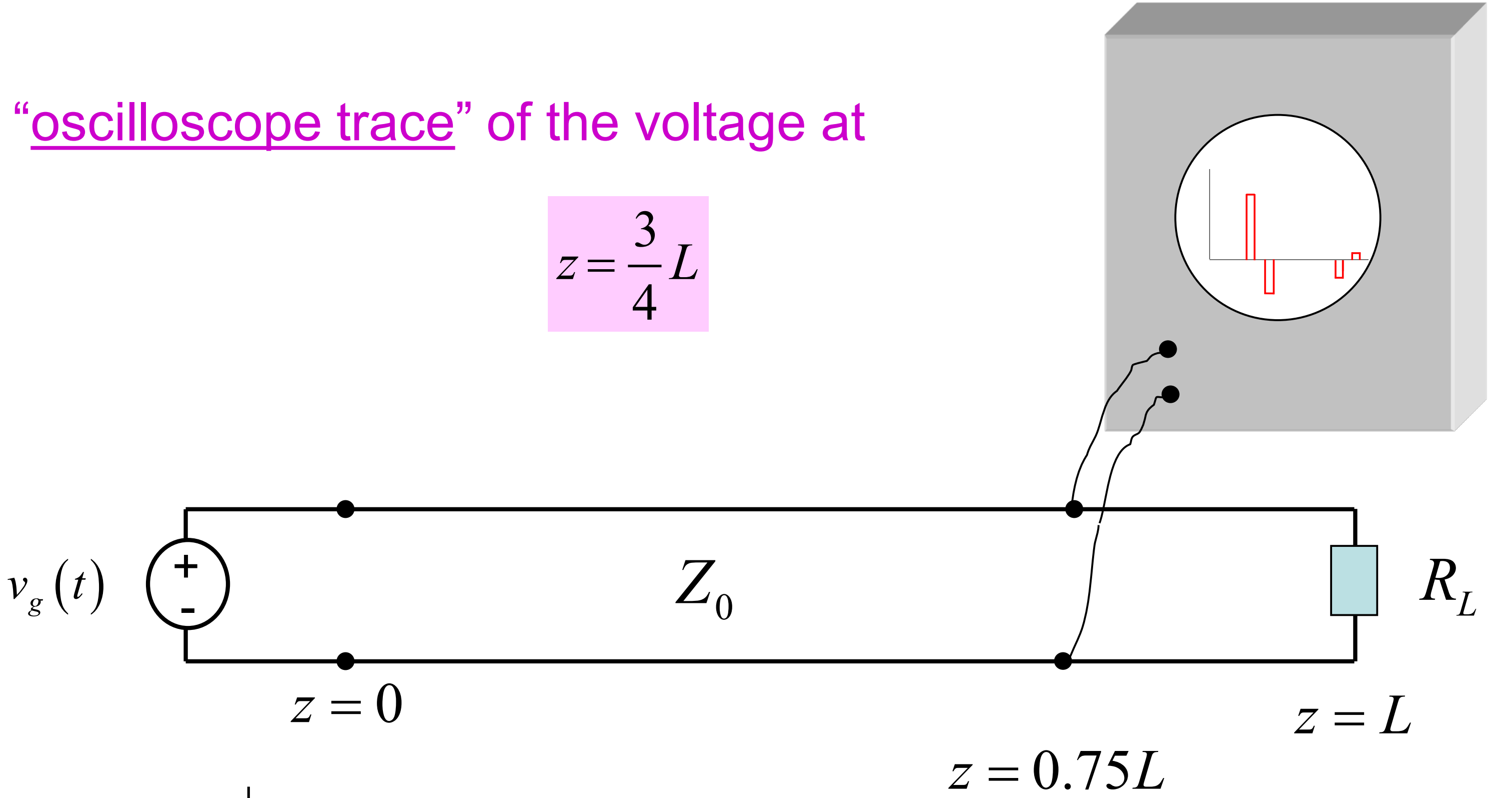


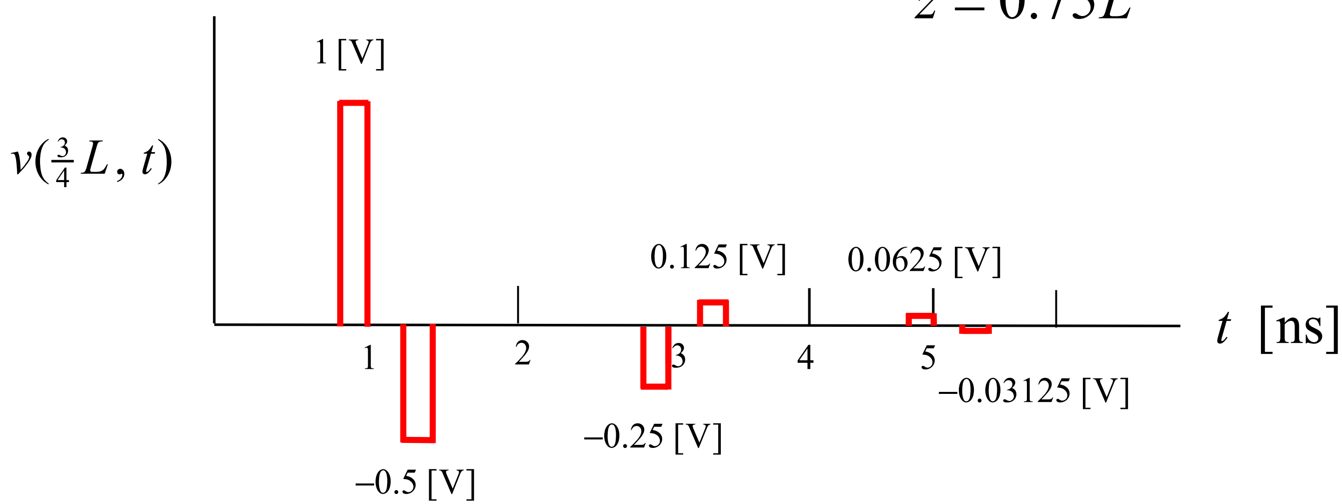


Example (cont.)

Final "oscilloscope trace" of the voltage at

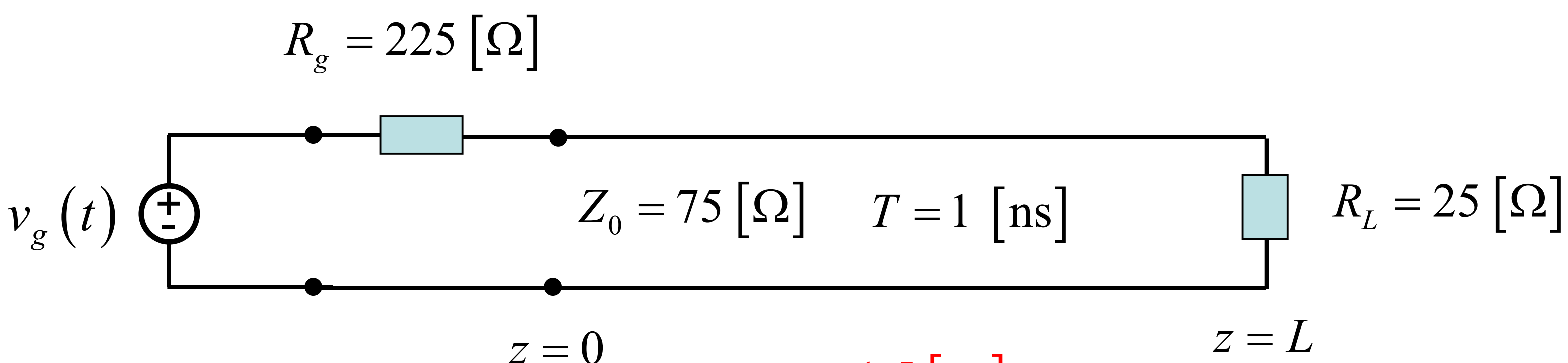
$$z = \frac{3}{4}L$$



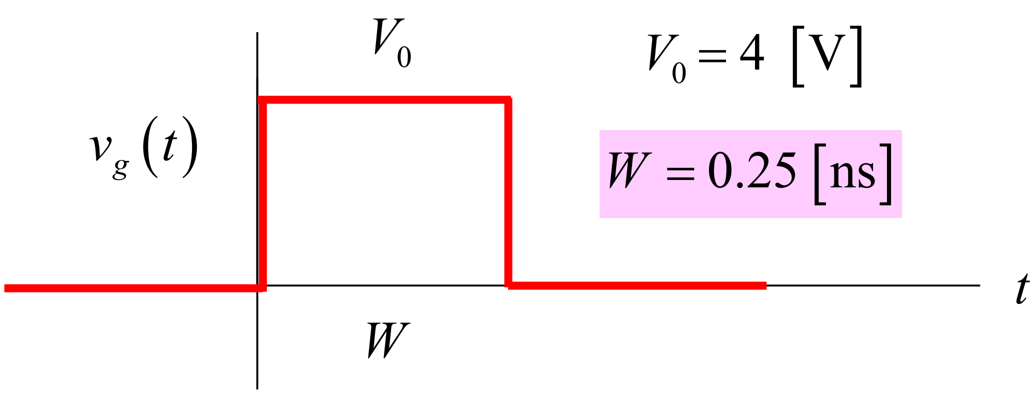


Example: Pulse (cont.)

Find a "<u>snapshot</u>" of the voltage at $t = 1.5$ [ns].



$t = 1.5\,[\text{ns}]$



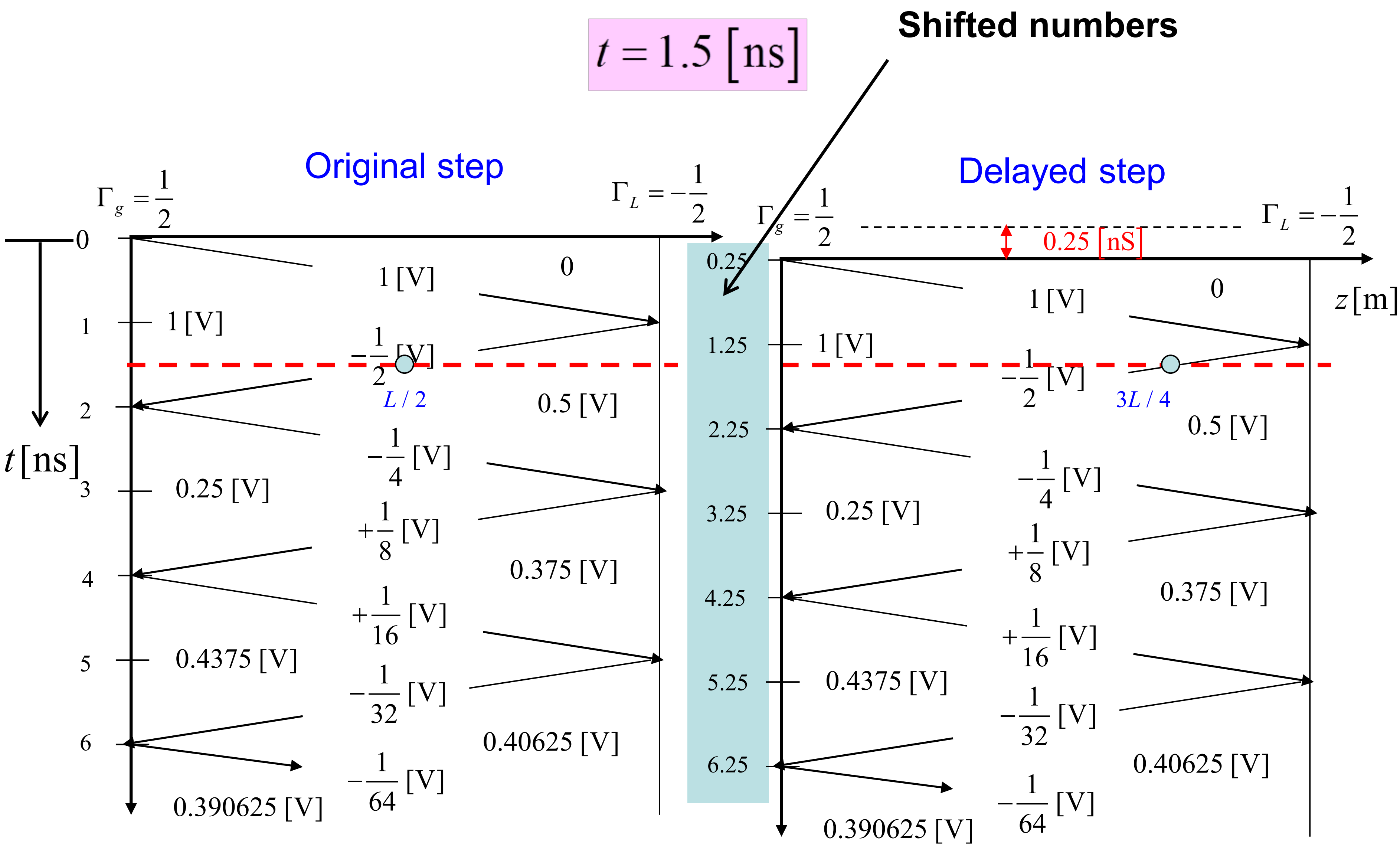
$V_0 = 4\,[\text{V}]$

$W = 0.25\,[\text{ns}]$

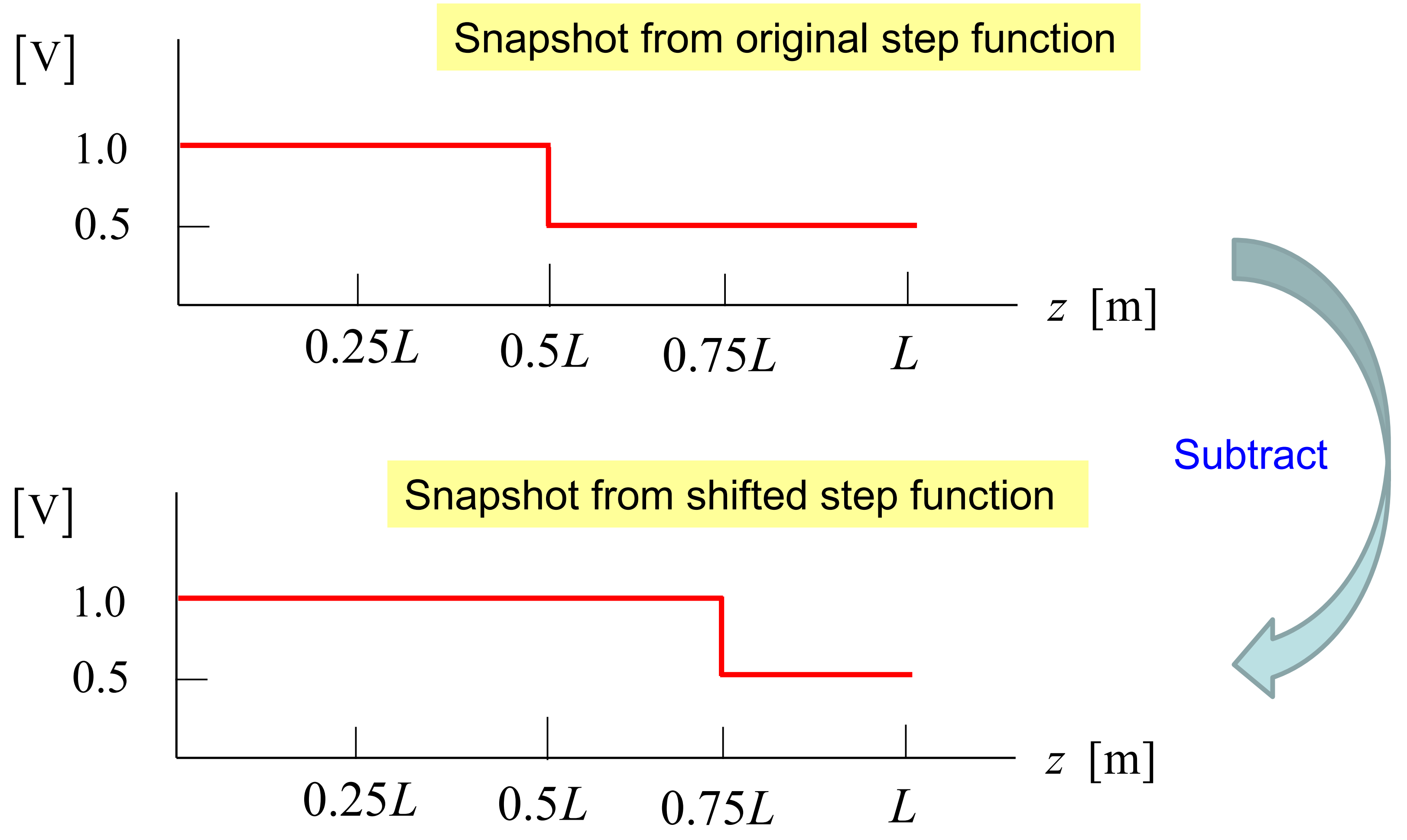
$V^+ = 1\,[\text{V}]$

Example: Pulse (cont.)

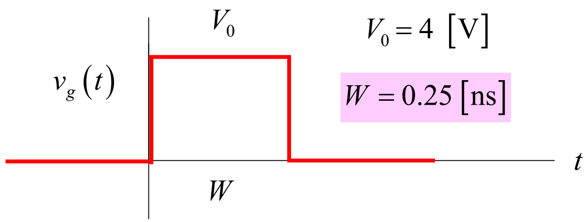
$t = 1.5\ [\text{ns}]$



Example: Pulse (cont.)



Example: Pulse (cont.)

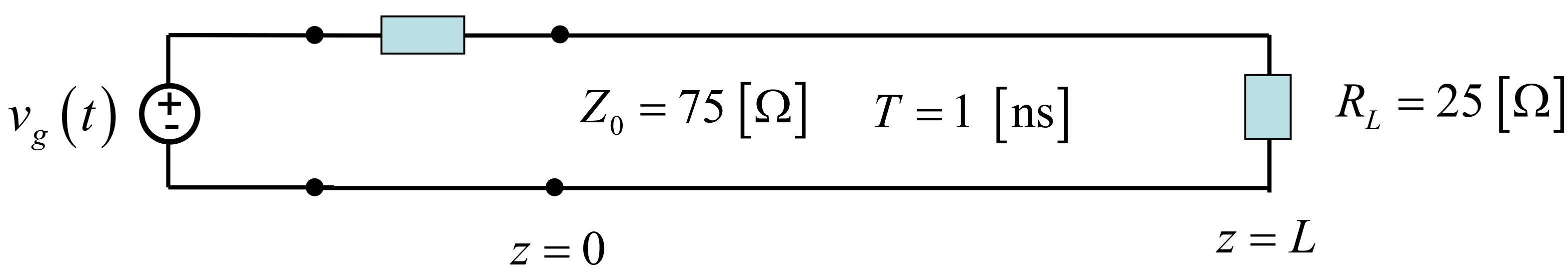


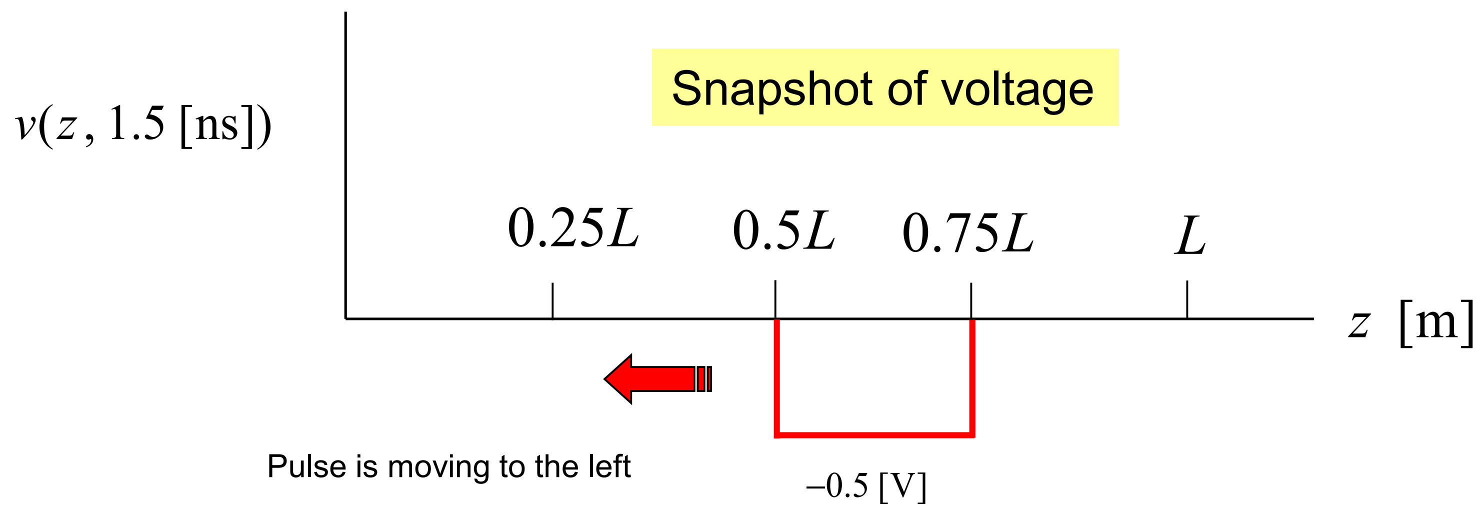
$V_0 = 4$ [V]

$W = 0.25$ [ns]

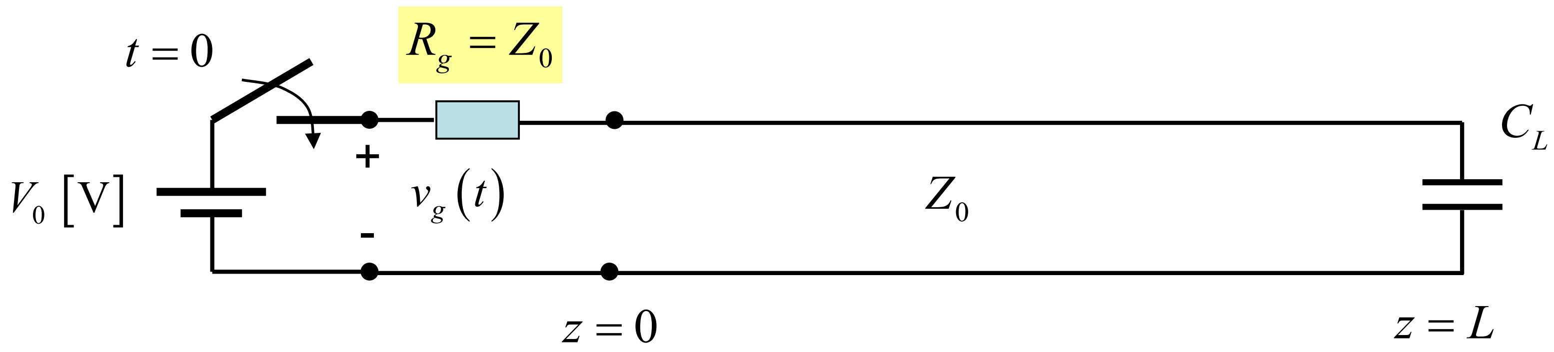
Snapshot

$t = 1.5$ [ns]





Capacitive Load



Note: The generator is assumed to be matched to the transmission line for convenience (we wish to focus on the effects of the capacitive load).

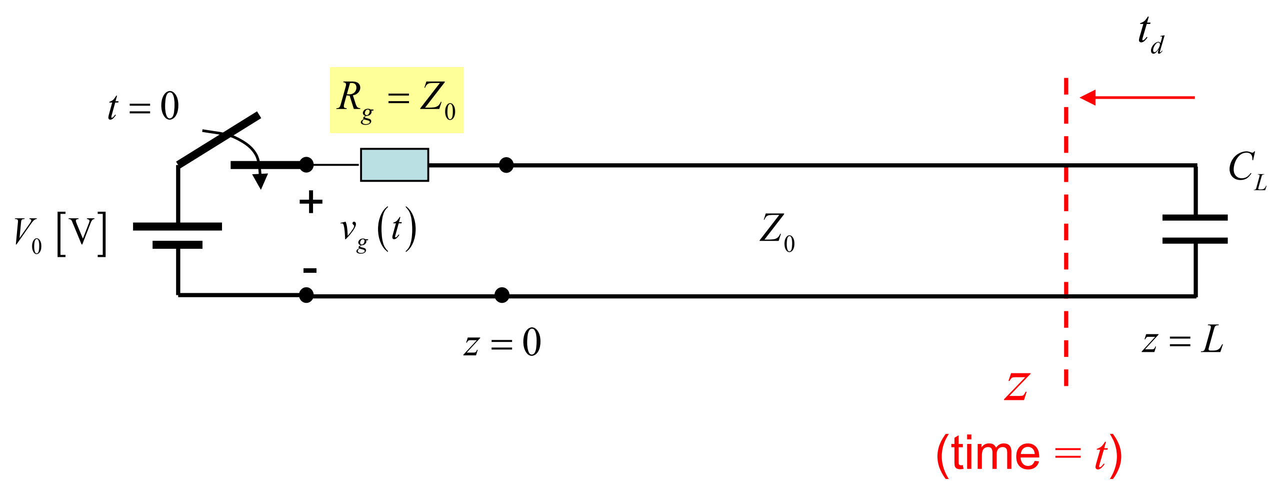
Hence $\Gamma_g = 0$

The reflection coefficient is now a function of time.

Capacitive Load (cont.)

$$t_d = (L - z)/c_d$$

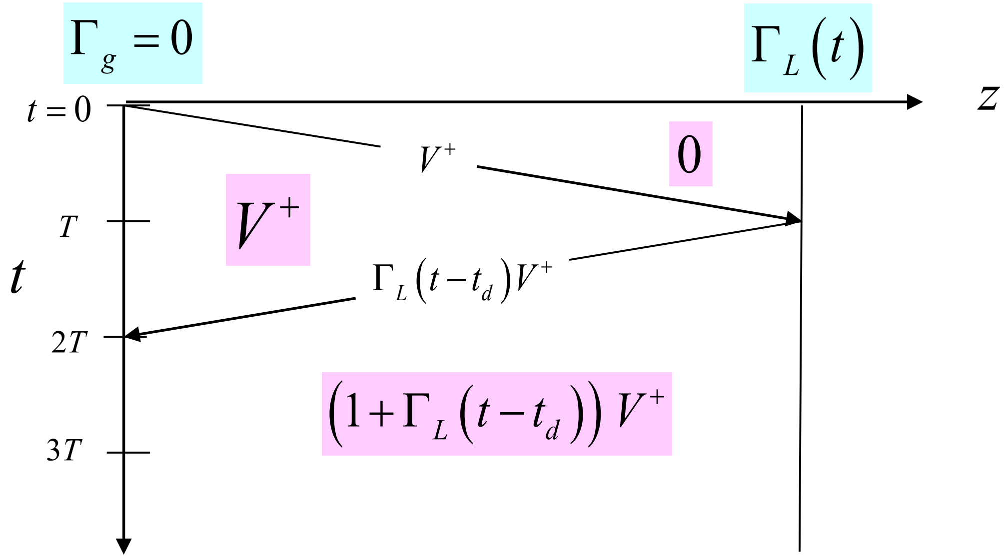
$$(\text{delay} = t_d)$$



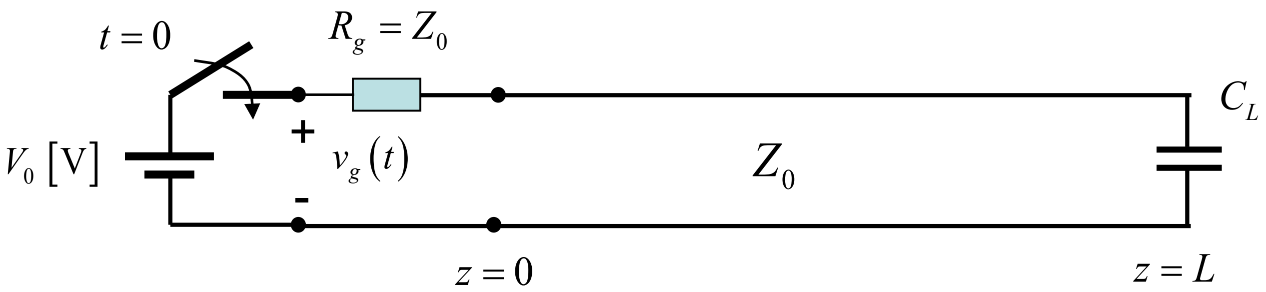
$$V^+ = \left(\frac{Z_0}{R_g + Z_0}\right) V_0$$

$$= V_0 / 2$$

$$V^+ = V_0 / 2$$



Capacitive Load (cont.)



At $t=T$: The capacitor acts as a short circuit: $\Gamma_L(T) = -1$

At $t=\infty$: The capacitor acts as an open circuit: $\Gamma_L(\infty) = 1$

Between $t=T$ and $t=\infty$, there is an exponential time-constant behavior.

General time-constant formula:

$$F(t) = F(\infty) + \left[F(T) - F(\infty)\right] e^{-(t-T)/\tau}$$

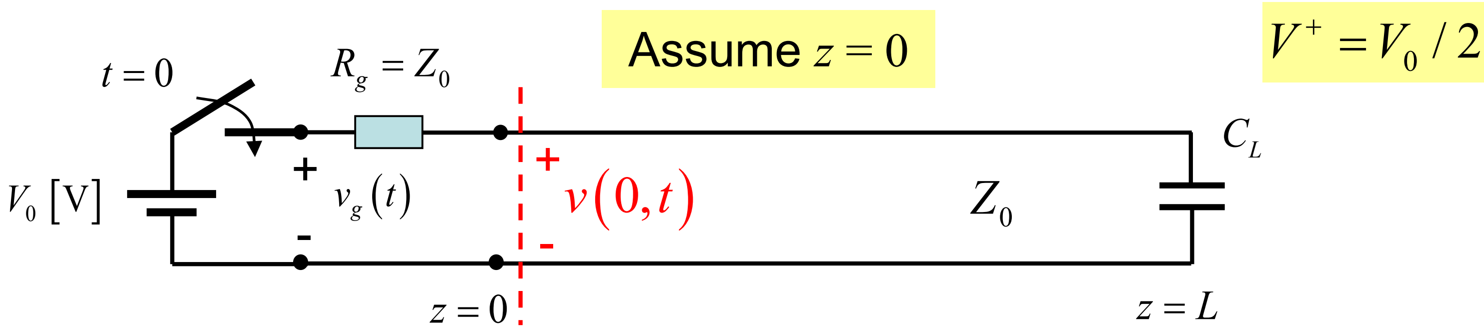
$$t \geq T$$

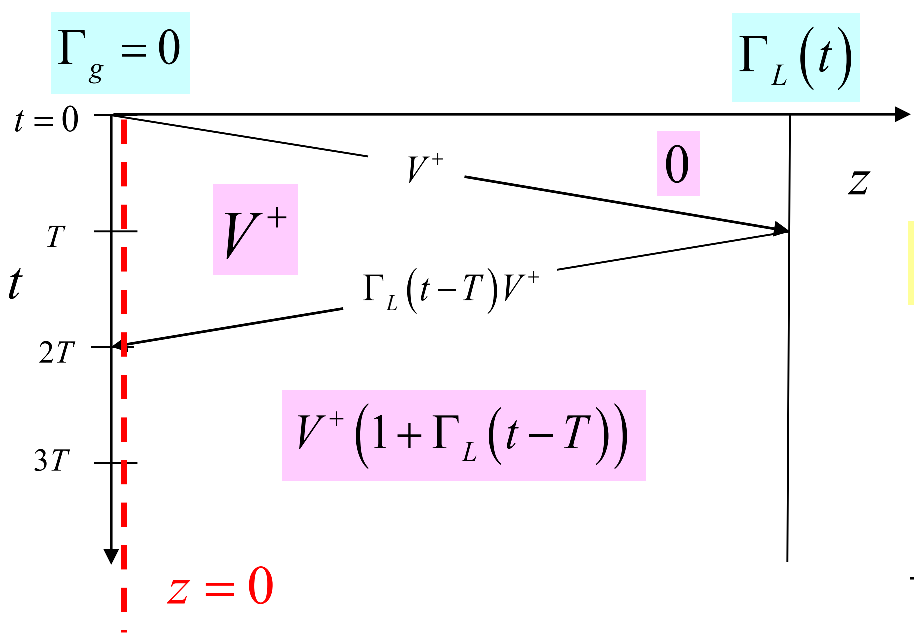
Hence, we have:

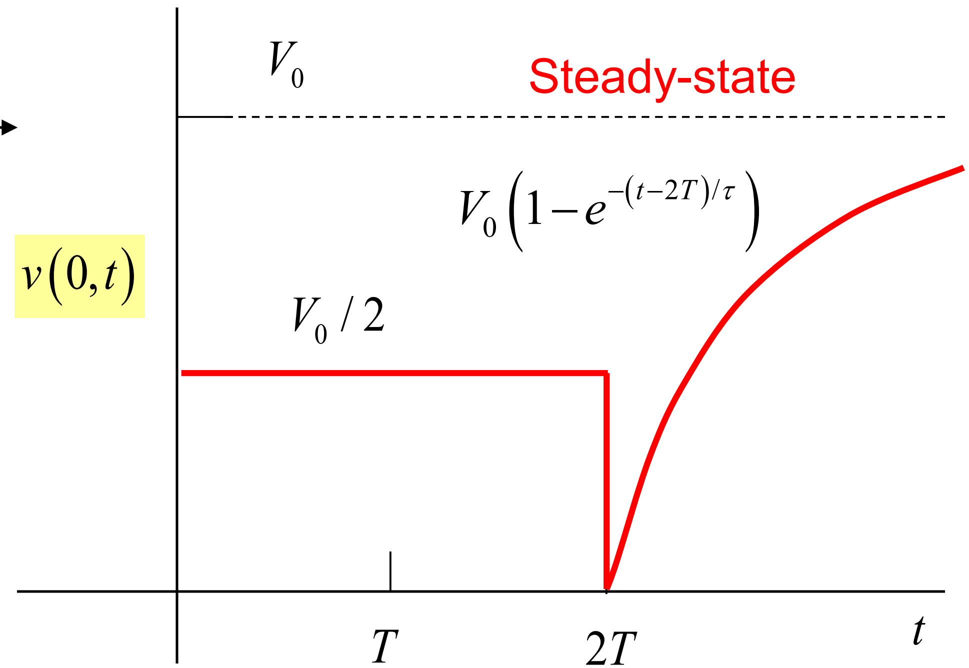
$$\Gamma_L(t) = 1 - 2\,e^{-(t-T)/\tau}, \quad t \geq T$$

$$\tau = Z_0 C_L$$

Capacitive Load (cont.)

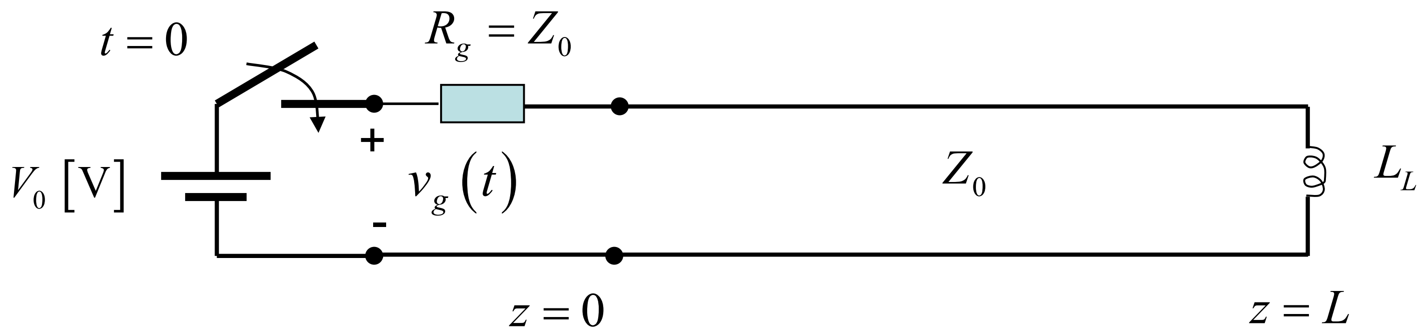






$$\Gamma_L(t) = 1-2e^{-(t-T)/\tau}, \quad 1+\Gamma_L(t-T) = 2\left(1-e^{-(t-2T)/\tau}\right), \quad V^+\left(1+\Gamma_L(t-T)\right) = V_0\left(1-e^{-(t-2T)/\tau}\right)$$

Inductive Load



At $t=T$: inductor acts as an open circuit: $\Gamma_L(T)=1$

At $t=\infty$: inductor acts as a short circuit: $\Gamma_L(\infty)=-1$

Between $t=T$ and $t=\infty$, there is an exponential time-constant behavior.

$$\Gamma_L(t) = -1 + 2e^{-(t-T)/\tau}, \quad t \geq T$$

$$\tau = L_L / Z_0$$

Recall:

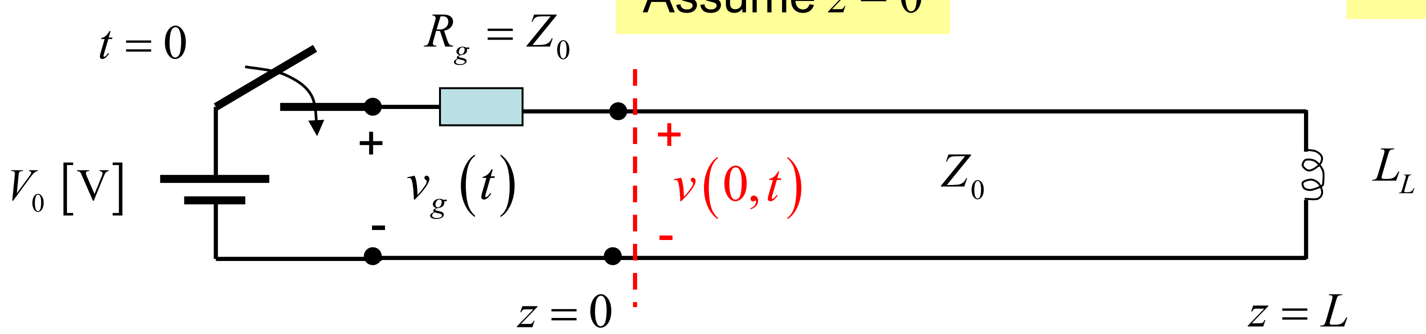
$$F(t) = F(\infty) + \left[F(T) - F(\infty)\right]e^{-(t-T)/\tau}$$

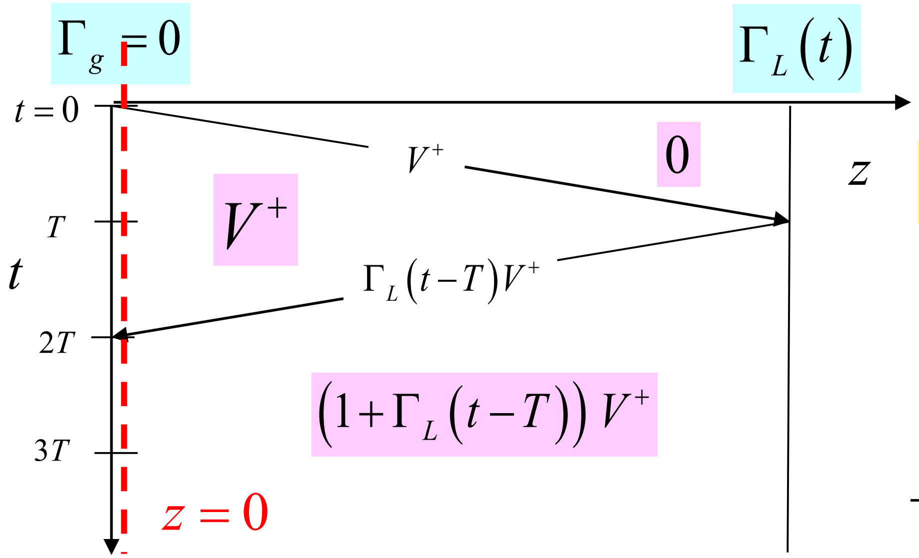
$$t \geq T$$

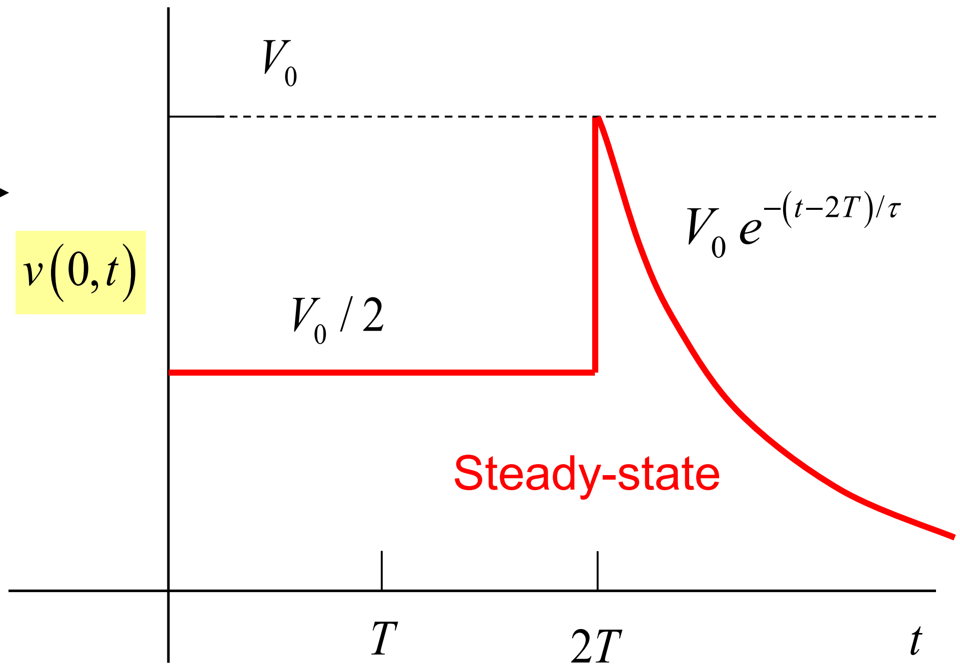
Inductive Load (cont.)

Assume $z = 0$

$V^+ = V_0 / 2$



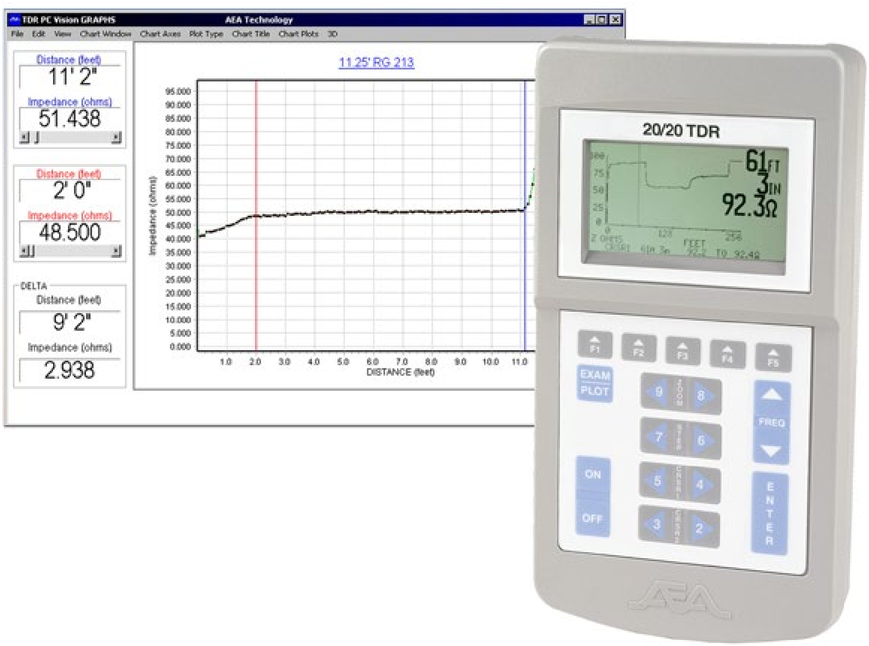




$$\Gamma_L(t) = -1 + 2e^{-(t-T)/\tau}, \quad 1+\Gamma_L(t-T) = 2e^{-(t-2T)/\tau}, \quad V^+\left(1+\Gamma_L(t-T)\right) = V_0 e^{-(t-2T)/\tau}$$

Time-Domain Reflectometer (TDR)

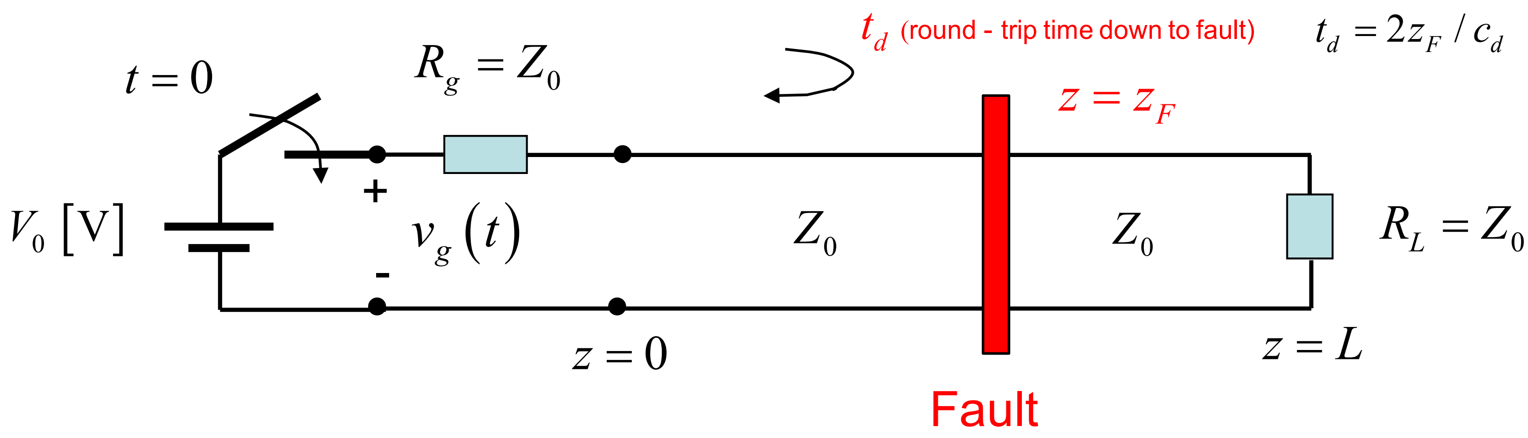
This is a device that is used to look at reflections on a line, to look for potential problems such as breaks on the line.



"The **20/20 Step Time Domain Reflectometer (TDR)** was designed to provide the clearest picture of coaxial or twisted pair cable lengths and to pin-point cable faults."

AEA Technology, Inc.

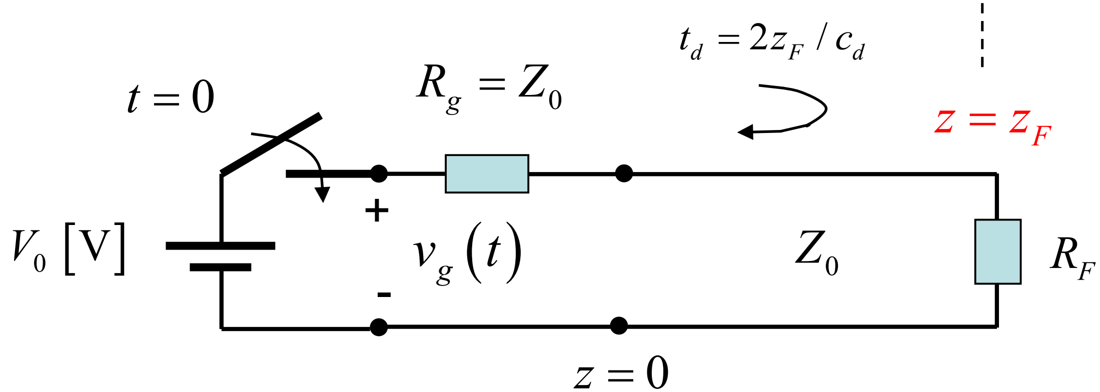
Time-Domain Reflectometer (cont.)



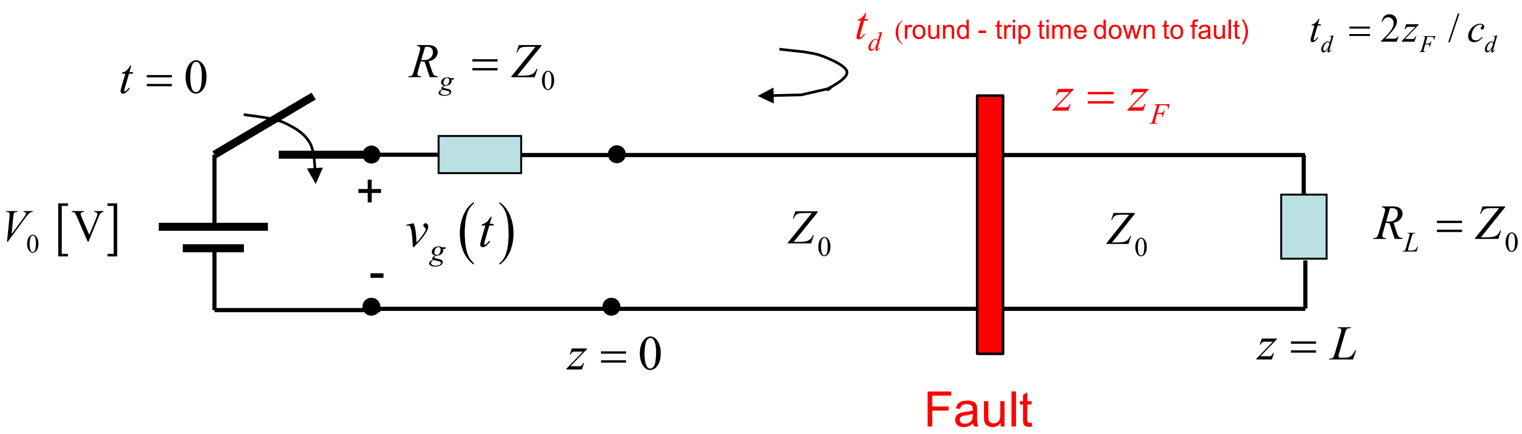
The fault is modeled as a load resistor R_F at $z = z_F$:

The fault might be acting as a parallel resistor or a series resistor.

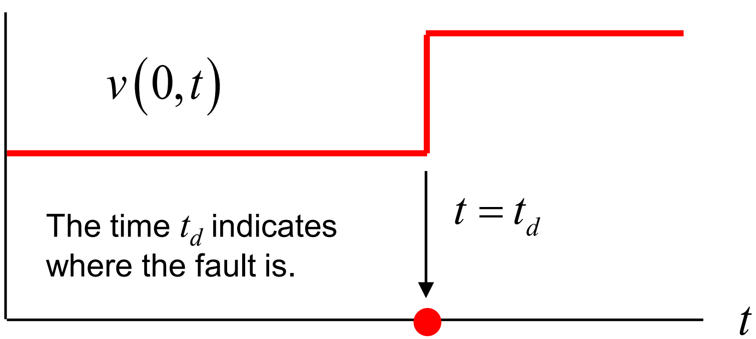
$R_F < Z_0$ or $R_F > Z_0$



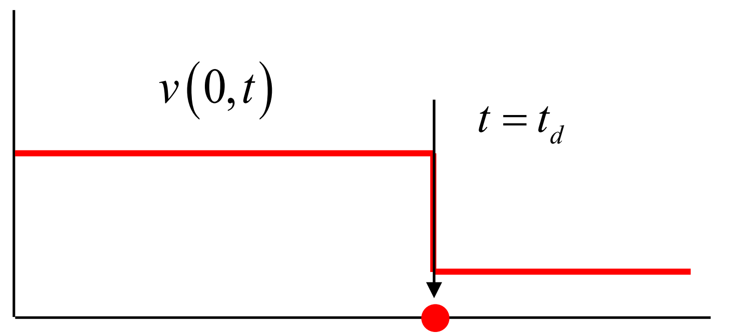
Time-Domain Reflectometer (cont.)



The fault is modeled as a load resistor R_F at $z = z_F$.



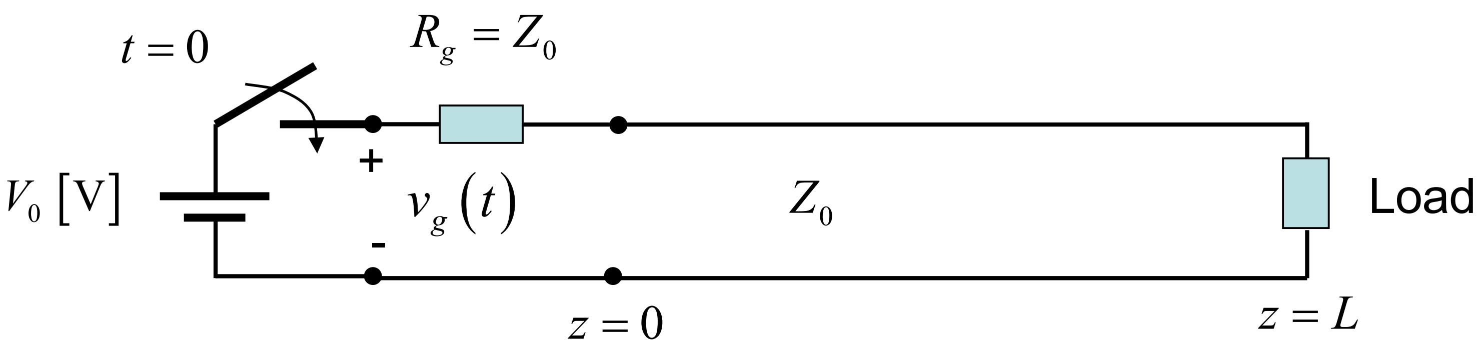
Resistive load, $R_F > Z_0$

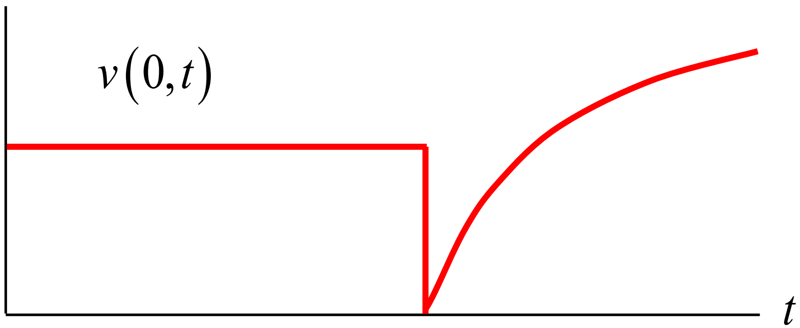


Resistive load, $R_F < Z_0$

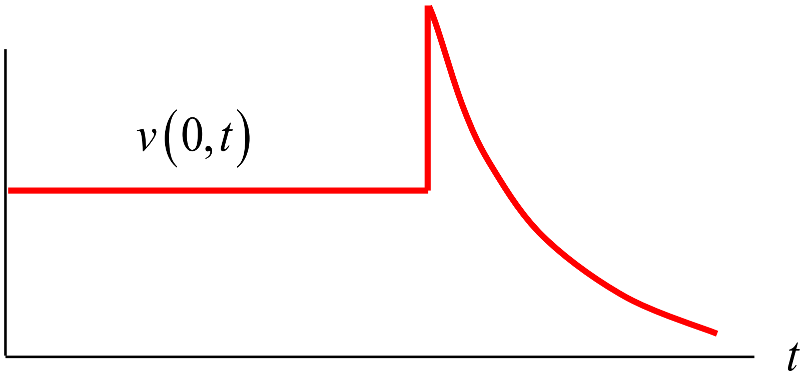
Time-Domain Reflectometer (cont.)

The TDR can also tell us what kind of a load we have at the end of a line.





Capacitive load



Inductive load

Appendix: Steady-State Solution

Adding all infinite number of bounces ($t = \infty$) for the step response, we have:

$$V(z,\infty) = \underbrace{V^+(1+\Gamma_g\Gamma_L+\Gamma_g^2\Gamma_L^2+\Gamma_g^3\Gamma_L^3+\cdots)}_{\text{Sum of all right-traveling waves}} + \underbrace{V^+\Gamma_L(1+\Gamma_g\Gamma_L+\Gamma_g^2\Gamma_L^2+\Gamma_g^3\Gamma_L^3+\cdots)}_{\text{Sum of all left-traveling waves}}$$

$$= \frac{V^+}{1-\Gamma_g\Gamma_L} + \frac{V^+\Gamma_L}{1-\Gamma_g\Gamma_L} = \frac{(1+\Gamma_L)}{1-\Gamma_g\Gamma_L}V^+$$

$$= \frac{\left(1+\dfrac{R_L-Z_0}{R_L+Z_0}\right)}{1-\left(\dfrac{R_g-Z_0}{R_g+Z_0}\right)\left(\dfrac{R_L-Z_0}{R_L+Z_0}\right)}\left(\frac{Z_0}{R_g+Z_0}V_0\right)$$

$$= \frac{\left(1+\dfrac{R_L-Z_0}{R_L+Z_0}\right)(R_g+Z_0)(R_L+Z_0)}{(R_g+Z_0)(R_L+Z_0)-(R_g-Z_0)(R_L-Z_0)}\left(\frac{Z_0}{R_g+Z_0}V_0\right)$$

Note: We have used

$$\sum_{n=0}^{\infty} z^n = \frac{1}{1-z}$$
$$\left(|z|<1\right)$$

Appendix: Steady-State Solution (cont.)

Simplifying, we have:

$$V(z,\infty) = \frac{\left(1+\dfrac{R_L - Z_0}{R_L + Z_0}\right)(R_g + Z_0)(R_L + Z_0)}{(R_g + Z_0)(R_L + Z_0) - (R_g - Z_0)(R_L - Z_0)} \left(\frac{Z_0}{R_g + Z_0} V_0\right)$$

$$= \frac{\left(\dfrac{2R_L}{R_L + Z_0}\right)(R_g + Z_0)(R_L + Z_0)}{(R_g + Z_0)(R_L + Z_0) - (R_g - Z_0)(R_L - Z_0)} \left(\frac{Z_0}{R_g + Z_0} V_0\right)$$

$$= \frac{2R_L(R_g + Z_0)}{(R_g + Z_0)(R_L + Z_0) - (R_g - Z_0)(R_L - Z_0)} \left(\frac{Z_0}{R_g + Z_0} V_0\right)$$

$$= \frac{2R_L Z_0 V_0}{(R_g + Z_0)(R_L + Z_0) - (R_g - Z_0)(R_L - Z_0)}$$

$$= \frac{2R_L Z_0 V_0}{R_g R_L + Z_0^2 + R_L Z_0 + R_g Z_0 - R_g R_L - Z_0^2 + R_L Z_0 + R_g Z_0}$$

$$= \frac{2R_L Z_0 V_0}{+R_L Z_0 + R_g Z_0 + R_L Z_0 + R_g Z_0}$$

Appendix: Steady-State Solution (cont.)

Continuing with the simplification:

$$
\begin{aligned}
V(z,\infty) &= \frac{2R_L Z_0 V_0}{R_L Z_0 + R_g Z_0 + R_L Z_0 + R_g Z_0} \\
&= \frac{2R_L Z_0 V_0}{2\left(R_L Z_0 + R_g Z_0\right)} \\
&= \frac{R_L Z_0 V_0}{\left(R_L Z_0 + R_g Z_0\right)} \\
&= \frac{R_L V_0}{\left(R_L + R_g\right)}
\end{aligned}
$$

Hence we finally have:

$$
V(z,\infty) = \left(\frac{R_L}{R_L + R_g}\right) V_0
$$