ECE 3317
Applied Electromagnetic Waves

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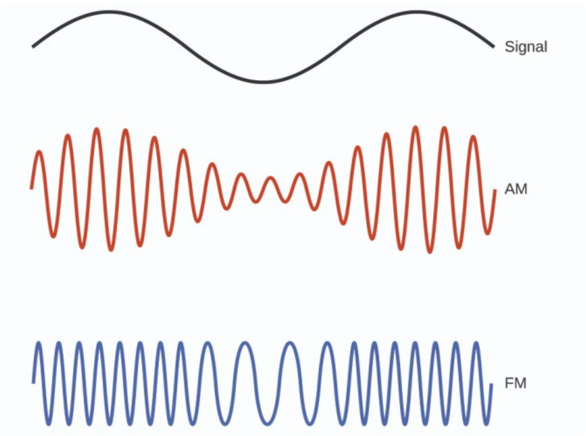
Fall 2023



Notes 9
Transmission Lines
(Frequency Domain)

Frequency Domain

Why is the frequency domain important?

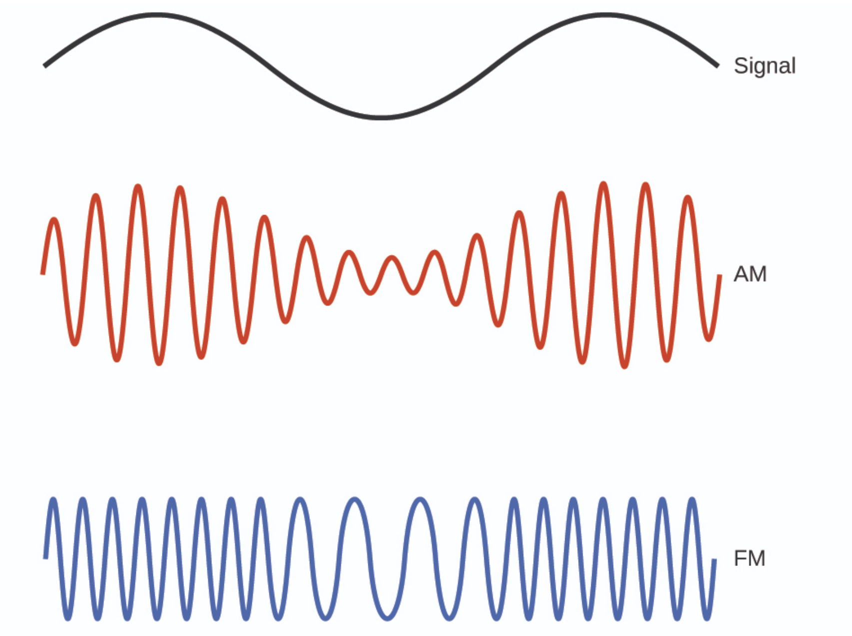
- Most communication systems use a sinusoidal signal (which may be modulated).

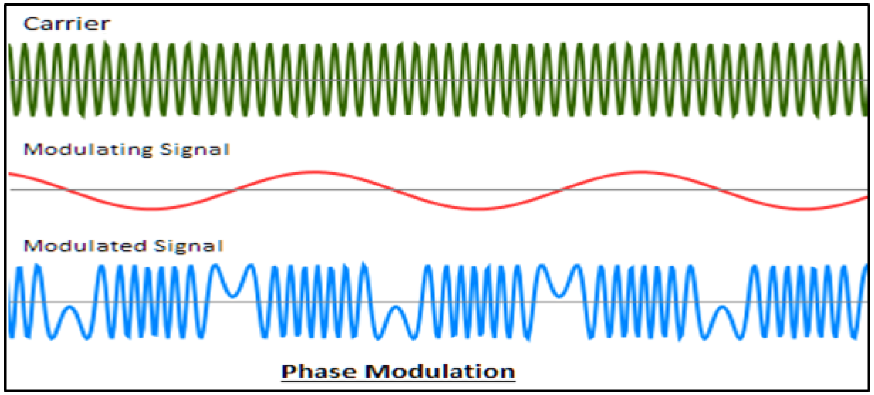
(Some systems, like Ethernet, communicate in "baseband", meaning that there is no carrier.)

↑

Examples:

100BASET, 10GBASET





Frequency Domain (cont.)

Why is the frequency domain important?

- A solution in the frequency-domain allows to solve for an arbitrary time-varying signal on a lossy line (by using the Fourier transform method).

Fourier-transform pair

$$\tilde{v}(\omega) = \int_{-\infty}^{\infty} v(t)\, e^{-j\omega t}\, dt$$

$$v(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{v}(\omega)\, e^{j\omega t}\, d\omega$$

For a physically-realizable (real-valued) signal, we can also write

$$v(t) = \frac{1}{\pi} \int_{0}^{\infty} \mathrm{Re}\left\{ \tilde{v}(\omega)\, e^{j\omega t} \right\} d\omega$$

$\left(\text{since } \tilde{v}(-\omega) = \tilde{v}^*(\omega)\right)$

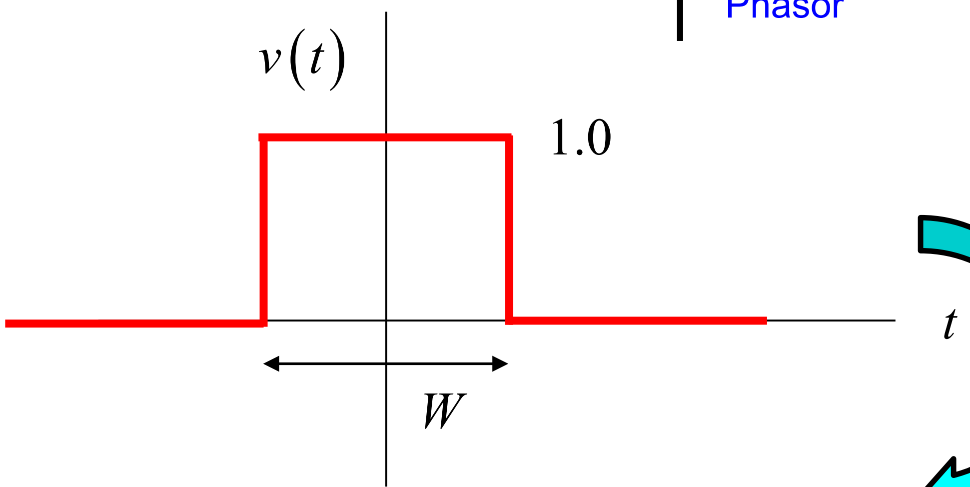
Jean-Baptiste-Joseph Fourier

Frequency Domain (cont.)

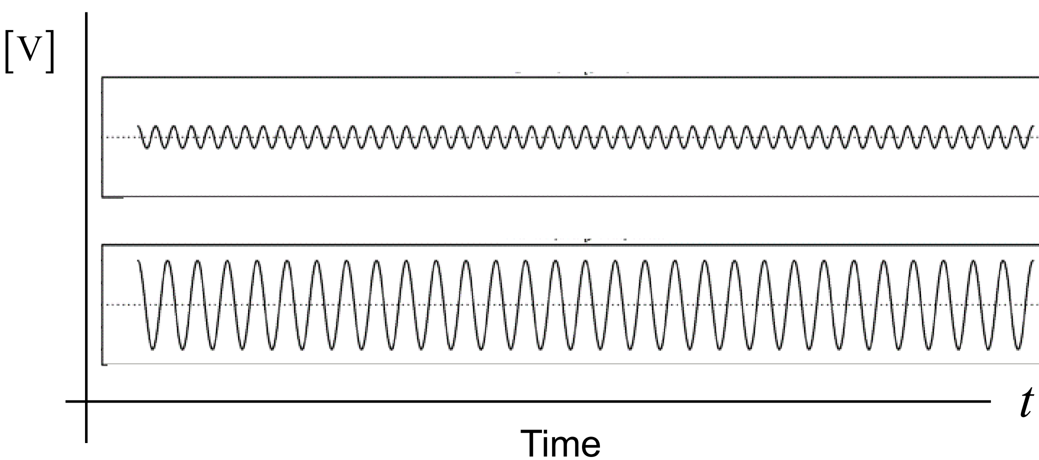
$$v(t) = \int_0^{\infty} \mathrm{Re}\left\{ \left(\frac{1}{\pi} \tilde{v}(\omega)\, d\omega \right) e^{j\omega t} \right\}$$

A collection of phasor-domain signals!

Phasor

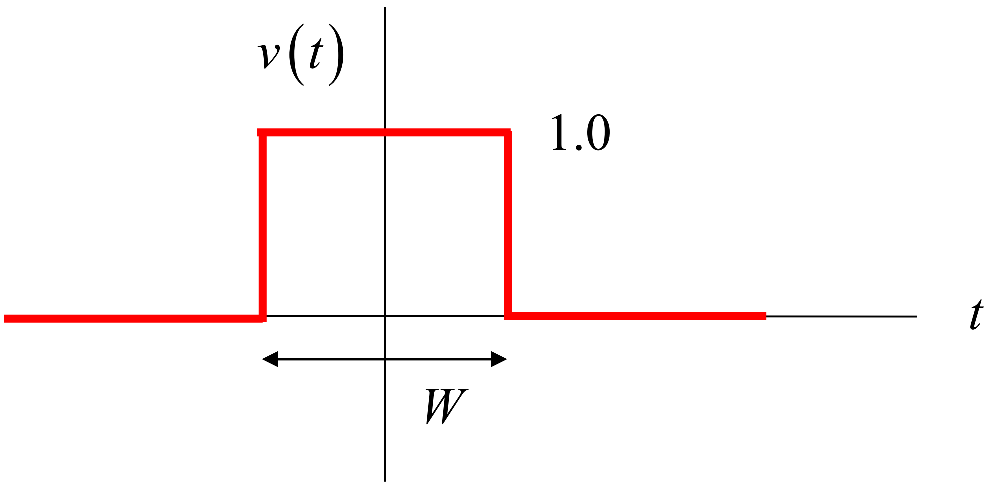


A pulse is resolved into a collection (spectrum) of infinite sinusoidal waves with different frequencies, amplitudes, and phases.

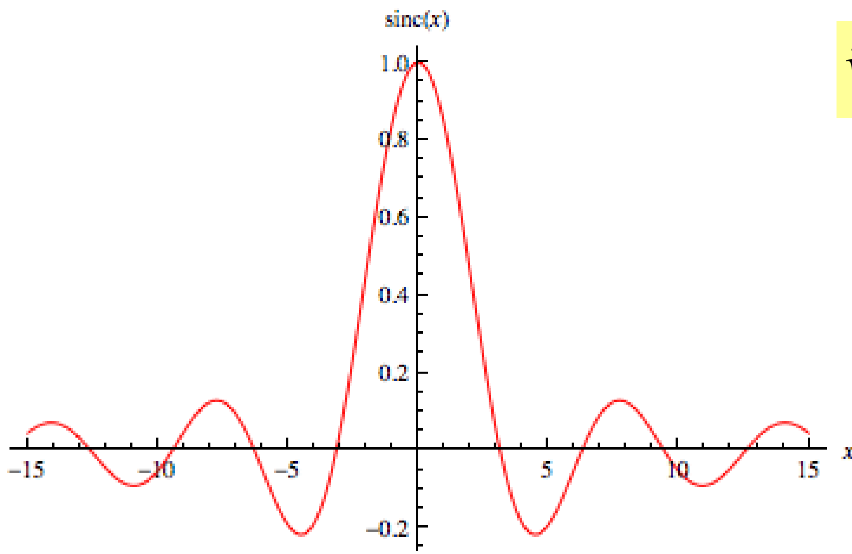


Frequency Domain (cont.)

Example: Rectangular pulse



$$\tilde{v}(\omega) = \int_{-\infty}^{\infty} v(t)\, e^{-j\omega t} dt$$

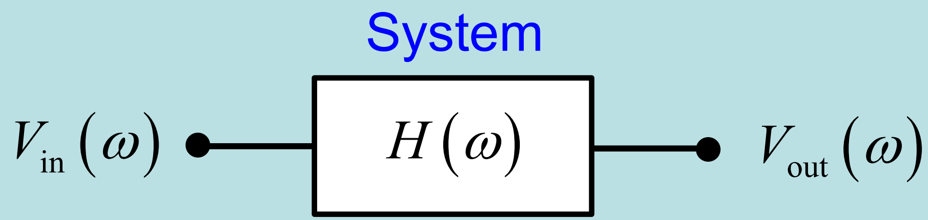


$$\tilde{v}(\omega) = W \operatorname{sinc}\left(W\omega/2\right)$$

$$\operatorname{sinc}(x) \equiv \frac{\sin(x)}{x}$$

Frequency Domain (cont.)

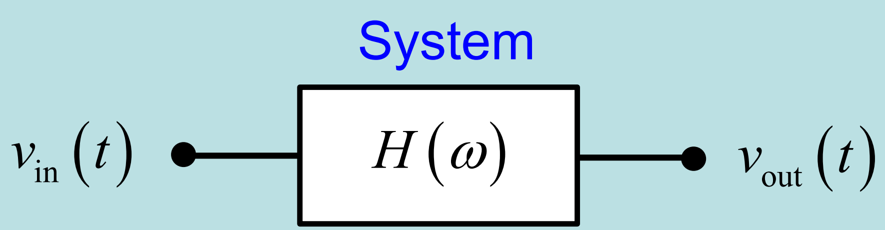
Phasor domain:



In the frequency domain, the system has a transfer function $H(\omega)$:

$$V_{\text{out}}(\omega) = H(\omega) V_{\text{in}}(\omega)$$

Time domain:



The time-domain response of the system to an input signal is:

$$v_{\text{out}}(t) = \frac{1}{\pi} \int_0^{\infty} \text{Re}\left\{ H(\omega) \tilde{v}_{\text{in}}(\omega) e^{j\omega t} \right\} d\omega$$

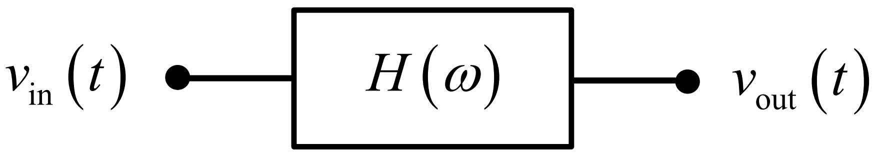
Frequency Domain (cont.)

Summary

Phasor domain:

$$H(\omega) \equiv \frac{V_{\text{out}}(\omega)}{V_{\text{in}}(\omega)}$$

System



$$v_{\text{out}}(t) = \frac{1}{\pi}\int_0^{\infty} \text{Re}\left\{H(\omega)\,\tilde{v}_{\text{in}}(\omega)\,e^{j\omega t}\right\} d\omega$$

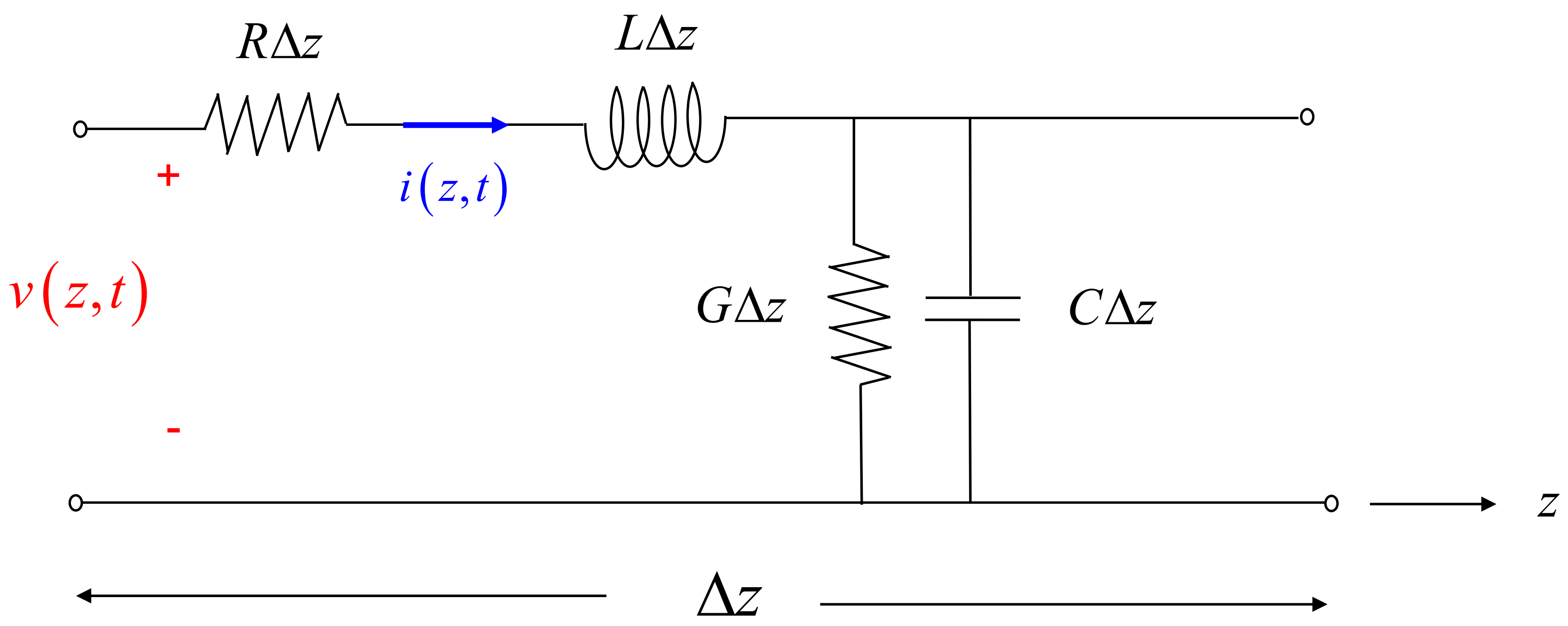
If we can solve the system in the phasor domain (i.e., get the transfer function $H(\omega)$), we can get the output for any time-varying input signal.

This is one reason why the phasor domain is so important!

This applies for transmission lines also!

Telegrapher's Equations

$$\frac{\partial^2 v}{\partial z^2} - (RG)v - (RC + LG)\frac{\partial v}{\partial t} - LC\left(\frac{\partial^2 v}{\partial t^2}\right) = 0$$



Frequency Domain

To convert to the phasor domain, we use: $\dfrac{\partial}{\partial t} \rightarrow j\omega$

$$\frac{\partial^2 v}{\partial z^2} - (RG)v - (RC + LG)\frac{\partial v}{\partial t} - LC\left(\frac{\partial^2 v}{\partial t^2}\right) = 0$$

$$\frac{\partial^2 V}{\partial z^2} - (RG)V - j\omega(RC + LG)V - (j\omega)^2 LC\,V = 0$$

or

$$\frac{d^2 V}{dz^2} = (RG)V + j\omega(RC + LG)V - (\omega^2 LC)V$$

Frequency Domain (cont.)

$$\frac{d^2V}{dz^2} = \left[\left(RG\right) + j\omega(RC + LG) - \left(\omega^2 LC\right)\right]V$$

Note that

$$RG + j\omega(RC + LG) - \omega^2 LC = (R + j\omega L)(G + j\omega C)$$

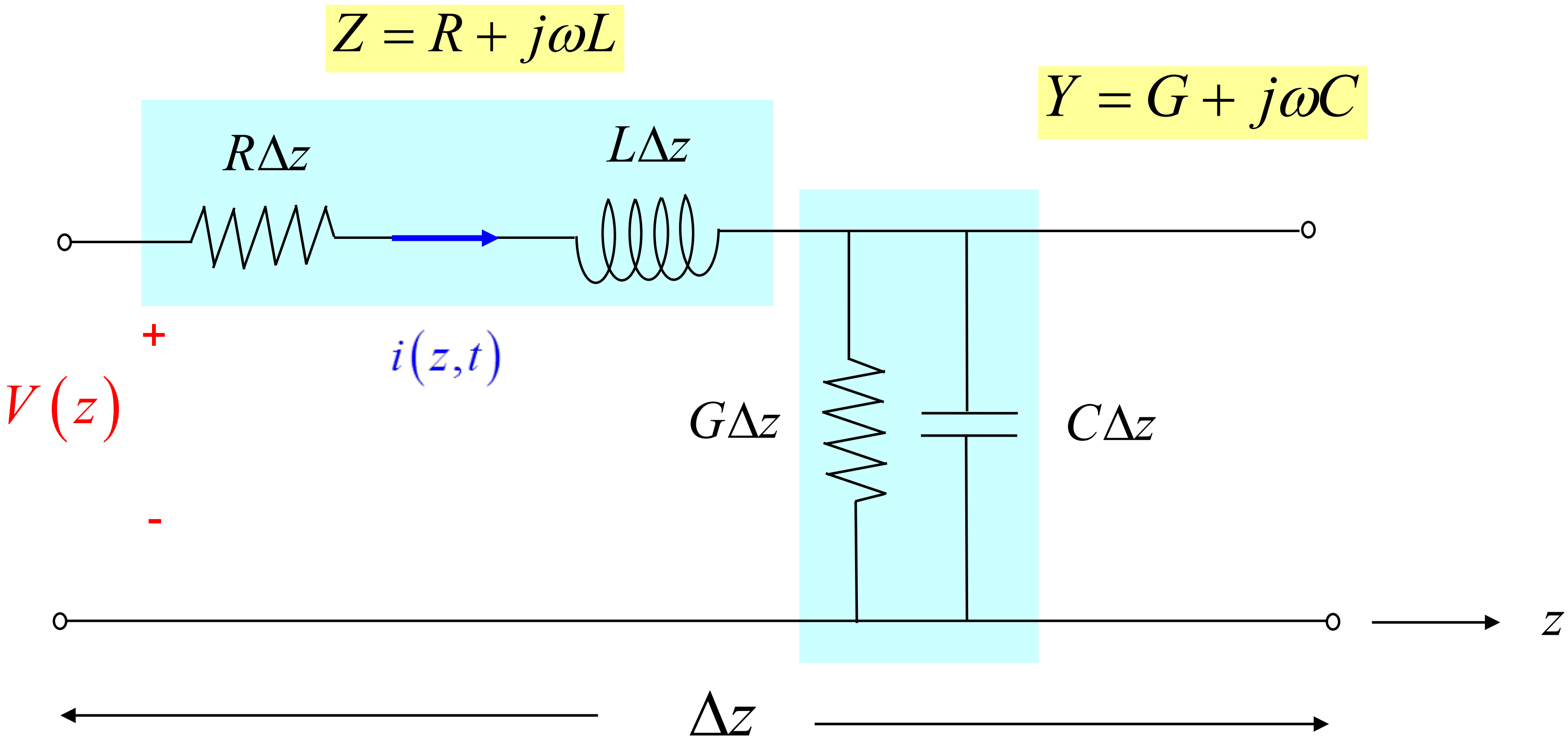
$Z = R + j\omega L =$ series impedance / length

$Y = G + j\omega C =$ parallel admittance / length

We can therefore write: $$\frac{d^2V}{dz^2} = (ZY)V$$

Telegrapher's Equations

$$\frac{d^2V}{dz^2} = (ZY)V$$



Frequency Domain (cont.)

$$\frac{d^2V}{dz^2} = (ZY)V$$

Define

$$\gamma^2 \equiv ZY = (R + j\omega L)(G + j\omega C)$$

Then

$$\frac{d^2V}{dz^2} = (\gamma^2)V$$

Solution:

$$V(z) = Ae^{-\gamma z} + B\,e^{+\gamma z}$$

Note: We have an exact solution, even for a lossy line, in the phasor domain!

Propagation Constant

Convention: We choose the (complex) square root to be the principal branch:

$$\gamma \equiv \sqrt{(R + j\omega L)(G + j\omega C)}$$ (lossy case)

γ is called the propagation constant, with units of [1/m]

Review of principal branch of square root:

$$c = |c| e^{j\phi}$$

$$\sqrt{c} = \sqrt{|c|}\; e^{j(\phi/2)}$$

$$-\pi < \phi \le \pi$$

Note:

$$-\pi / 2 < \phi / 2 \le \pi / 2$$

$$\Downarrow$$

$$\operatorname{Re}\sqrt{c} \ge 0$$

Propagation Constant (cont.)

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

Denote: $\gamma = \alpha + j\beta$

γ = propagation constant [1/m]
α = attenuation constant [nepers/m]
β = phase constant [radians/m]

Choosing the principle branch means that

$$\operatorname{Re}\gamma \geq 0 \quad \Rightarrow \quad \alpha \geq 0$$

Propagation Constant (cont.)

For a lossless line, we consider this as the limit of a lossy line, in the limit as the loss tends to zero:

$$\gamma = \sqrt{(\cancel{R} + j\omega L)(\cancel{G} + j\omega C)} = \left(\omega\sqrt{LC}\right)\sqrt{-1}$$

Hence $\gamma = j\omega\sqrt{LC}$ (lossless case)

Hence, we have that

$$\alpha = 0$$

$$\beta = \omega\sqrt{LC}$$

Note: $\alpha = 0$ for a lossless line.

Propagation Constant (cont.)

Physical interpretation of waves:

Note:
The waves must decay in the direction of propagation.

$$V^{+}(z) = A\,e^{-\gamma z} \qquad \text{(forward traveling wave)}$$

$$V^{-}(z) = B\,e^{+\gamma z} \qquad \text{(backward traveling wave)}$$

$$\gamma = \alpha + j\beta$$

Forward traveling wave: $V^{+}(z) = A\,e^{-\alpha z}\,e^{-j\beta z}$

Backward traveling wave: $V^{-}(z) = B\,e^{+\alpha z}\,e^{+j\beta z}$

Propagation Wavenumber

Alternative notations:

$$\gamma = \alpha + j\beta \quad \text{(propagation constant)}$$

$$k_z = \beta - j\alpha \quad \text{(propagation wavenumber)}$$

Note: $\gamma = jk_z$

$$V^{+}(z) = Ae^{-\gamma z} = Ae^{-jk_z z} = Ae^{-\alpha z}e^{-j\beta z}$$

Forward Wave

Forward traveling wave: $$V^+(z) = A\,e^{-\alpha z}\,e^{-j\beta z}$$

Denote $A = |A|\,e^{j\phi}$

Then $$V^+(z) = |A|\,e^{j\phi}\,e^{-\alpha z}\,e^{-j\beta z}$$

In the time domain we have:

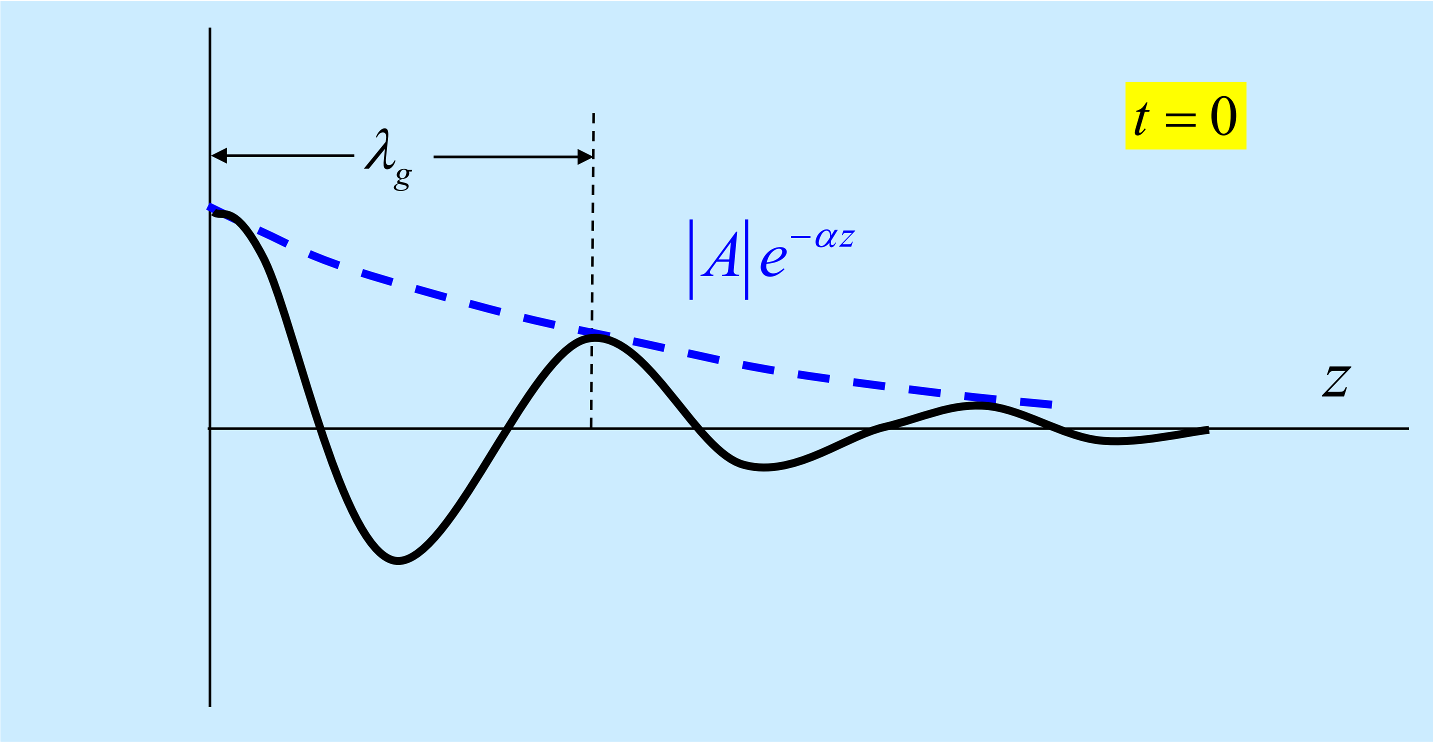
$$v^+(z,t) = \mathrm{Re}\left\{V^+(z)\,e^{j\omega t}\right\}$$

Hence we have $$v^+(z,t) = |A|\,e^{-\alpha z}\cos(\omega t - \beta z + \phi)$$

Forward Wave (cont.)

$$v^{+}(z,t) = |A| e^{-\alpha z} \cos(\omega t - \beta z + \phi)$$

Snapshot of Waveform:



The distance λ_g is the distance it takes for the waveform to "repeat" itself in meters.

λ_g = guided wavelength

Wavelength

The wave "repeats" (except for the amplitude decay) when:

$$\beta \lambda_g = 2\pi$$

Hence:

$$\beta = \frac{2\pi}{\lambda_g}$$

Note: This equation can be used to find λ_g if we already know β:

$$\lambda_g = \frac{2\pi}{\beta} = \frac{2\pi}{\mathrm{Im}(\gamma)} = \frac{2\pi}{\mathrm{Im}\left(\sqrt{(R + j\omega L)(G + j\omega C)}\right)}$$

Wavelength (cont.)

Lossless case:

$$\lambda_g = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{LC}} = \frac{2\pi}{2\pi f\sqrt{LC}} = \frac{1}{f\sqrt{LC}} = \frac{1}{f\sqrt{\mu\varepsilon}} = \frac{c_d}{f} = \frac{c}{\sqrt{\mu_r\varepsilon_r}}\frac{1}{f} = \frac{\lambda_0}{\sqrt{\mu_r\varepsilon_r}} = \lambda_d$$

Summary for lossless case:

$$\lambda_g = \lambda_d$$

$$\lambda_d = \frac{\lambda_0}{\sqrt{\mu_r\varepsilon_r}}$$

$$\lambda_0 = \frac{c}{f}$$

λ_d = wavelength in dielectric

λ_0 = wavelength in free - space (air)

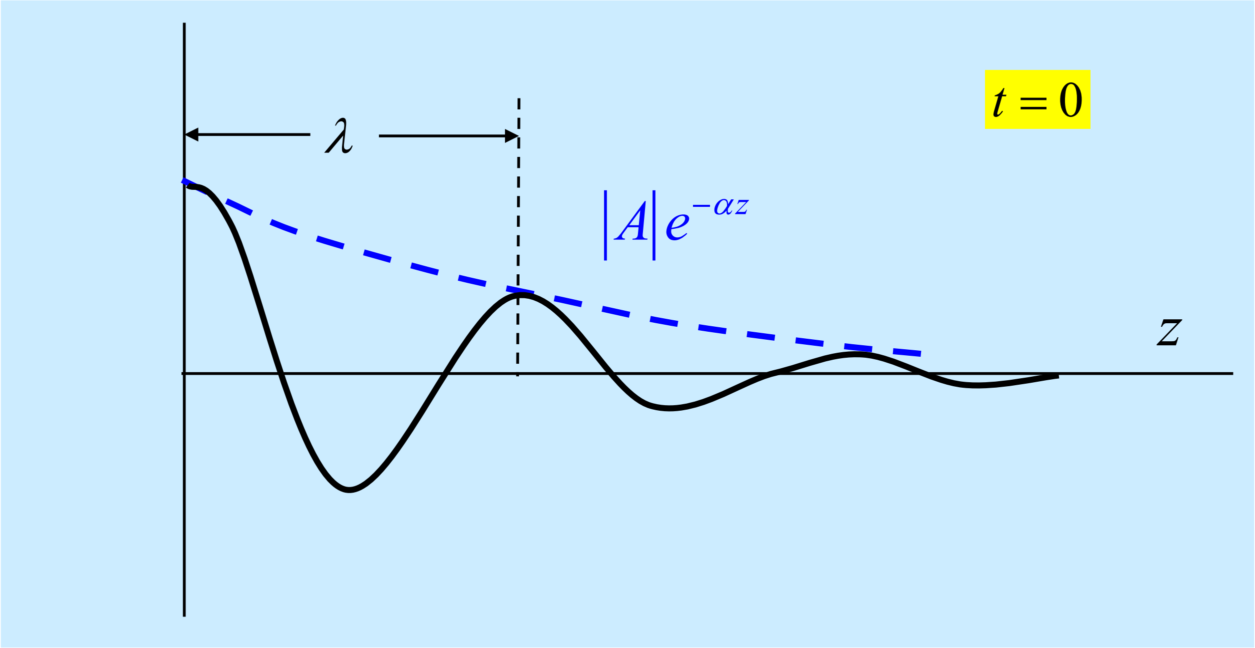
$$c \equiv 2.99792458 \times 10^8 \; [\text{m/s}]$$

Attenuation Constant

The attenuation constant controls how fast the wave decays.

$$v^{+}(z,t) = |A| e^{-\alpha z} \cos(\omega t - \beta z + \phi)$$

$$\text{envelope} = |A| e^{-\alpha z}$$



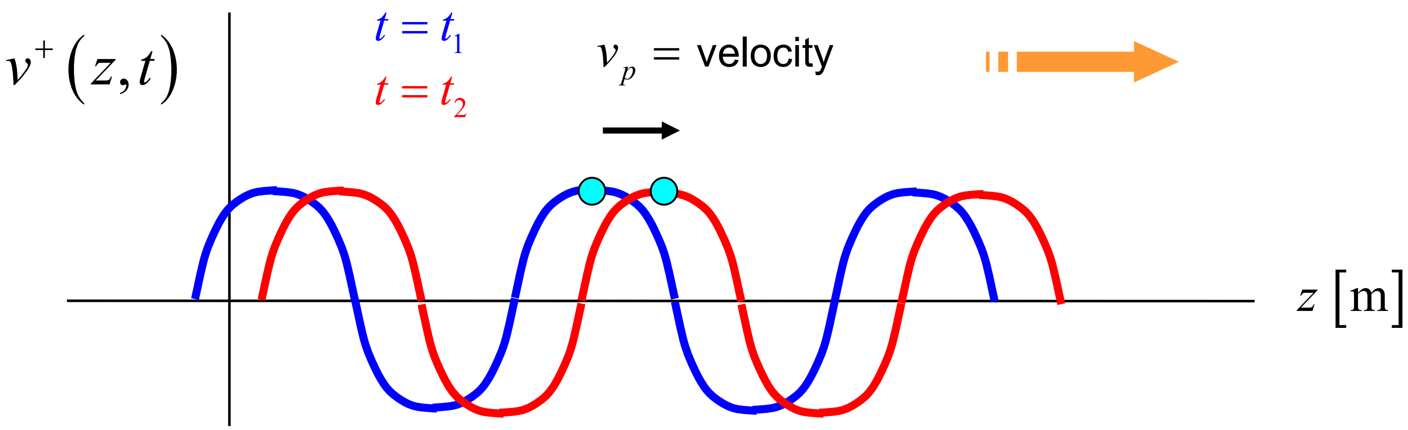
$$\alpha = \mathrm{Re}(\gamma) = \mathrm{Re}\left(\sqrt{(R + j\omega L)(G + j\omega C)}\right)$$

Phase Velocity

The forward-traveling wave is moving in the positive z direction.

Consider a sinusoidal wave moving on a transmission line
(shown in the figure below for a lossless line ($\alpha = 0$) for simplicity):

$$v^{+}(z,t) = |A| e^{-\alpha z} \cos(\omega t - \beta z + \phi)$$



Crest of wave: $\omega t - \beta z + \phi = 0$

Phase Velocity (cont.)

The phase velocity v_{phase} is the velocity of a point on the wave, such as the crest.

Set $\omega t - \beta z = -\phi = \text{constant}$

Take the derivative with respect to time: $\omega - \beta \dfrac{dz}{dt} = 0$

Hence $$\frac{dz}{dt} = \frac{\omega}{\beta}$$

We thus have $$v_{\text{phase}} = \frac{\omega}{\beta}$$

Note:
This result holds for a general lossy line.

Recall: $\beta = \operatorname{Im}\left(\sqrt{(R + j\omega L)(G + j\omega C)}\right)$

Phase Velocity (cont.)

Let's calculate the phase velocity for a lossless line:

$$v_{\text{phase}} = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{LC}} = \frac{1}{\sqrt{LC}}$$

Lossless line:

$$\alpha = 0$$
$$\beta = \omega\sqrt{LC}$$

Also, we know that $LC = \mu\varepsilon = \dfrac{1}{c_d^2}$

Hence $v_{\text{phase}} = c_d$ (lossless line)

Recall: $c_d = \dfrac{1}{\sqrt{\mu\varepsilon}} = \dfrac{1}{\sqrt{\mu_0\varepsilon_0}}\dfrac{1}{\sqrt{\mu_r\varepsilon_r}} = \dfrac{c}{\sqrt{\mu_r\varepsilon_r}}$

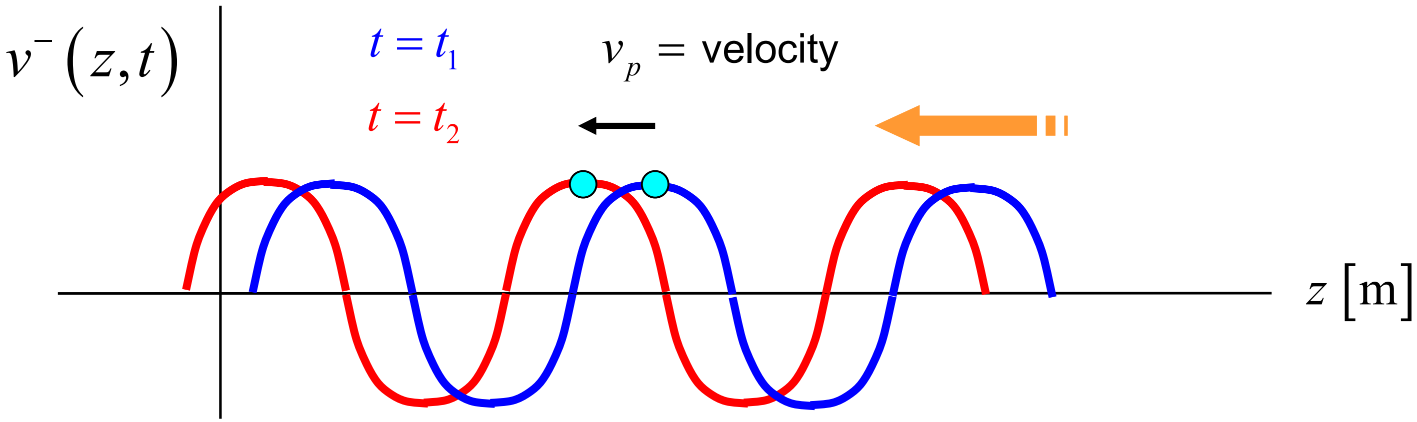
Backward Traveling Wave

Let's now consider the backward-traveling wave

(shown in the figure below for a lossless line ($\alpha = 0$) for simplicity):

$$V^-(z) = Be^{+\gamma z} = B\,e^{+\alpha z}\,e^{+j\beta z} = |B|\,e^{\phi}\,e^{+\alpha z}\,e^{+j\beta z}$$

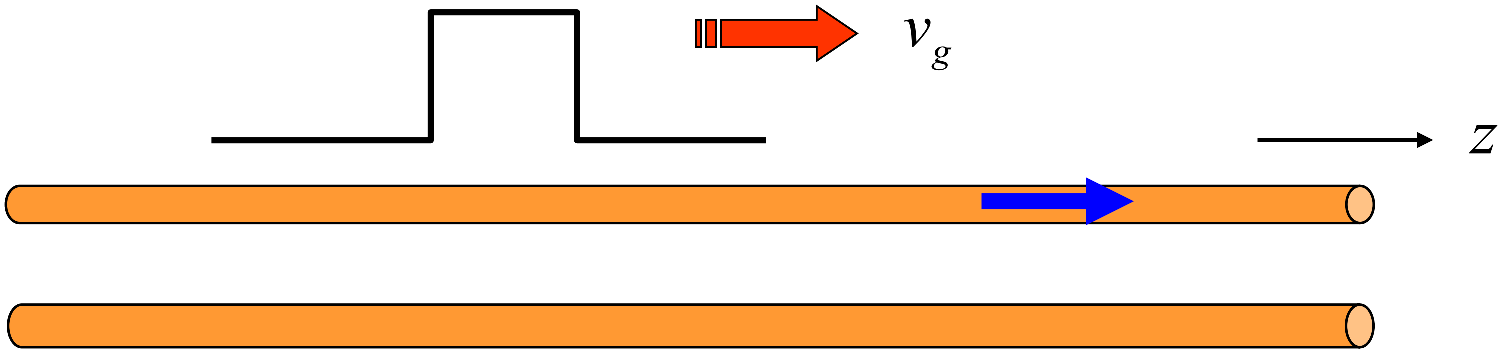
$$\Rightarrow \quad v^-(z,t) = |B|\,e^{+\alpha z}\cos(\omega t + \beta z + \phi)$$



This wave has the same phase velocity, but it travels backwards.

Group Velocity

The group velocity v_{group} is the velocity of a pulse.



We have (derivation omitted):

$$v_{\text{group}} = \frac{d\omega}{d\beta}$$

Note:
This result holds for a general lossy line.

Note: for a lossless line we have: $v_{\text{group}} = v_{\text{phase}} = 1/\sqrt{LC} = 1/\sqrt{\mu\varepsilon} = c_d$

$$\left(\text{Lossless line}: \beta = \omega\sqrt{LC}\right)$$

Attenuation in dB/m

$$V^{+}(z) = Ae^{-\gamma z} = Ae^{-jk_z z} = Ae^{-\alpha z}e^{-j\beta z}$$

Gain in dB: $\mathrm{dB} = 20\log_{10}\left|\dfrac{V^{+}(z)}{V^{+}(0)}\right| = 20\log_{10}\left(e^{-\alpha z}\right)$

Use the following logarithm identity: $\log_{10} x = \dfrac{\ln x}{\ln 10}$

Therefore, the "gain" is: $\mathrm{dB} = 20\dfrac{\ln\left(e^{-\alpha z}\right)}{\ln 10} = 20\dfrac{(-\alpha z)}{\ln 10} = -\left(\dfrac{20}{\ln 10}\alpha\right)z$

Hence, we have: $\mathrm{Attenuation} = \left(\dfrac{20}{\ln 10}\right)\alpha \quad [\mathrm{dB/m}]$

Attenuation in dB/m (cont.)

Final attenuation formulas:

$$\text{Attenuation} = \left(\frac{20}{\ln 10}\right)\alpha \qquad [\text{dB/m}]$$

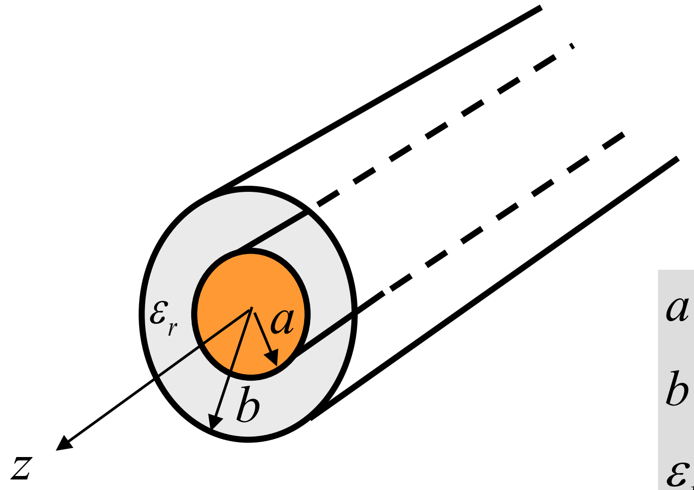
$$\text{Attenuation} \approx \left(8.686\right)\alpha \qquad [\text{dB/m}]$$

Example: Coaxial Cable

$$C = \frac{2\pi\varepsilon_0\varepsilon_r}{\ln\left(\frac{b}{a}\right)} \quad [\text{F/m}]$$

$$L = \frac{\mu_0}{2\pi}\ln\left(\frac{b}{a}\right) \quad [\text{H/m}]$$

$$G = \frac{2\pi\sigma_d}{\ln\left(\frac{b}{a}\right)} \quad [\text{S/m}]$$



TV coax

Copper conductors (nonmagnetic: $\mu_m = \mu_0$)

$a = 0.5\ [\text{mm}]$

$b = 3.2\ [\text{mm}]$

$\varepsilon_r = 2.2$

$\tan\delta_d = 0.001$

$\sigma_{ma} = \sigma_{mb} = 5.8\times10^7\ [\text{S/m}]$

$f = 500\ [\text{MHz}]$ (UHF)

$$R = \left(\frac{1}{2\pi a\,\sigma_{ma}\delta_{ma}} + \frac{1}{2\pi b\,\sigma_{mb}\delta_{mb}}\right) \quad [\Omega/\text{m}]$$

Note:
The "loss tangent" of the dielectric is called $\tan\delta_d$.

$$\delta_{ma} = \sqrt{\frac{2}{\omega\mu_{ma}\sigma_{ma}}} \qquad \delta_{mb} = \sqrt{\frac{2}{\omega\mu_{mb}\sigma_{mb}}}$$

(skin depth of the two conductors)

Example: Coaxial Cable (cont.)

Dielectric conductivity is often specified in terms of the loss tangent:

$$\tan\delta_d \equiv \frac{\sigma_d}{\omega\varepsilon} = \frac{\sigma_d}{\omega\varepsilon_0\varepsilon_r}$$

σ_d = effective conductivity of the dielectric material*

Note:
The loss tangent of practical insulating materials (e.g., Teflon) is approximately constant over a wide range of frequencies. (For example, $\tan\delta_d \approx 0.001$ for Teflon.)

*The effective conductivity accounts for the actual conductivity as well as molecular friction and other effects.

Example: Coaxial Cable (cont.)

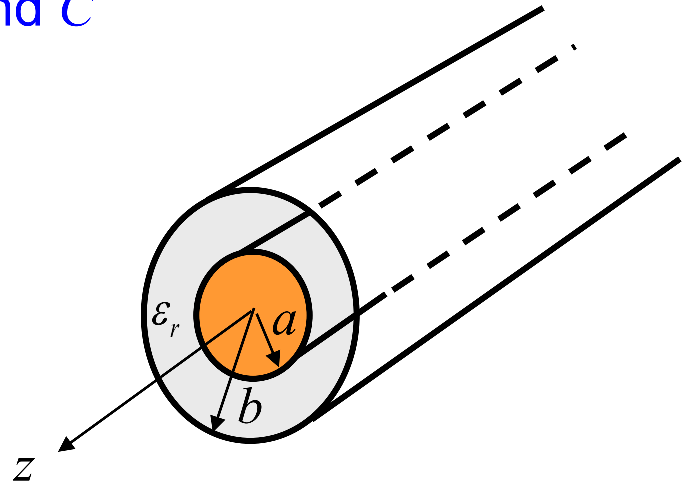
Relation between G and C

$$C = \frac{2\pi\varepsilon_0\varepsilon_r}{\ln\left(\frac{b}{a}\right)} \quad [\text{F/m}]$$

$$G = \frac{2\pi\sigma_d}{\ln\left(\frac{b}{a}\right)} \quad [\text{S/m}]$$

$$G = C\left(\frac{\sigma_d}{\varepsilon_0\varepsilon_r}\right)$$

$$\Rightarrow \frac{G}{\omega C} = \frac{\sigma_d}{\omega\varepsilon_0\varepsilon_r}$$



Recall

$$\tan\delta_d \equiv \frac{\sigma_d}{\omega\varepsilon} = \frac{\sigma_d}{\omega\varepsilon_0\varepsilon_r}$$

Hence

$$\frac{G}{\omega C} = \tan\delta_d$$

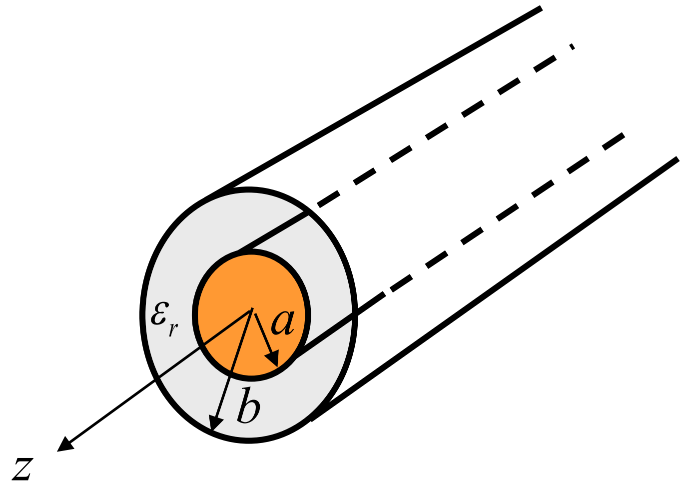
This relationship holds for <u>any</u> type of transmission line.

Example: Coaxial Cable (cont.)

Characteristic impedance (ignore R and G for this):

$$Z_0 = \sqrt{\frac{L}{C}} = \frac{\eta_0}{2\pi\sqrt{\varepsilon_r}} \ln\left(\frac{b}{a}\right)$$

$$Z_0 = 75\ [\Omega]$$



Skin depth of metal:

$$\delta_m = \sqrt{\frac{2}{\omega\mu\sigma_m}} \qquad (\mu = \mu_0)$$

$$\delta_m = 2.955\times10^{-6}\ [\text{m}]$$

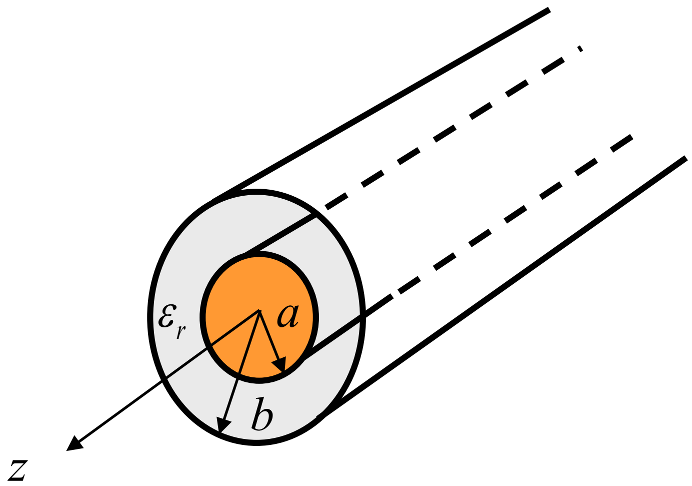
Effective conductivity of dielectric:

$$\sigma_d = (\omega\varepsilon_0\varepsilon_r)\tan\delta_d$$

$$\sigma_d = 6.12\times10^{-5}\ [\text{S/m}]$$

$$a = 0.5\ [\text{mm}]$$
$$b = 3.2\ [\text{mm}]$$
$$\varepsilon_r = 2.2$$
$$\tan\delta_d = 0.001$$
$$\sigma_{ma} = \sigma_{mb} = 5.8\times10^7\ [\text{S/m}]$$
$$f = 500\ [\text{MHz}]\ (\text{UHF})$$

Example: Coaxial Cable (cont.)



$$a = 0.5 \ [\text{mm}]$$
$$b = 3.2 \ [\text{mm}]$$
$$\varepsilon_r = 2.2$$
$$\tan\delta_d = 0.001$$
$$\sigma_{ma} = \sigma_{mb} = 5.8 \times 10^7 \ [\text{S/m}]$$
$$f = 500 \ [\text{MHz}] \ (\text{UHF})$$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)}$$

Results for (R, L, G, C):

$$R = 2.147 \ [\Omega/\text{m}]$$
$$L = 3.713 \times 10^{-7} \ [\text{H/m}]$$
$$G = 2.071 \times 10^{-4} \ [\text{S/m}]$$
$$C = 6.593 \times 10^{-11} \ [\text{F/m}]$$

$$\gamma = 0.022 + j(15.543) \ [1/\text{m}]$$

$$\alpha = 0.022 \ [\text{nepers/m}]$$
$$\beta = 15.544 \ [\text{rad/m}]$$

$$\text{Attenuation} = 0.191 \ [\text{dB/m}]$$
$$\lambda_g = 0.404 \ [\text{m}]$$

Current

Use the first Telegrapher equation:

$$\frac{\partial v}{\partial z} = -Ri - L\frac{\partial i}{\partial t}$$

$$\Downarrow \quad \frac{\partial}{\partial t} \rightarrow j\omega$$

$$\frac{\partial V}{\partial z} = -RI - j\omega LI$$

Next, use

$$V(z) = Ae^{-\gamma z} + B\,e^{+\gamma z}$$

so

$$\frac{\partial V(z)}{\partial z} = -\gamma\left[Ae^{-\gamma z} - B\,e^{+\gamma z}\right]$$

Current (cont.)

Hence, we have

$$-\gamma\left[Ae^{-\gamma z}-B\,e^{+\gamma z}\right]=-RI-j\omega LI$$

Solving for the phasor current I, we have

$$I=\left(\frac{\gamma}{R+j\omega L}\right)\left[Ae^{-\gamma z}-B\,e^{+\gamma z}\right]$$

$$=\left(\frac{\sqrt{(R+j\omega L)(G+j\omega C)}}{R+j\omega L}\right)\left[Ae^{-\gamma z}-B\,e^{+\gamma z}\right]$$

$$=\sqrt{\frac{G+j\omega C}{R+j\omega L}}\left[Ae^{-\gamma z}-B\,e^{+\gamma z}\right]$$

Characteristic Impedance

Define the (complex) characteristic impedance Z_0 in the frequency domain for a lossy line:

$$Z_0 \equiv \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

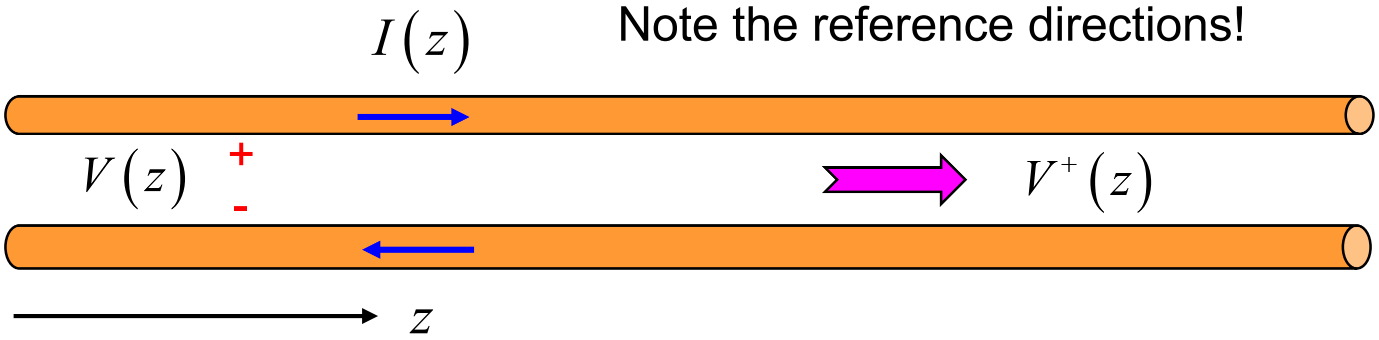
Then we have:

$$I(z) = \left(\frac{1}{Z_0}\right)\left[A e^{-\gamma z} - B\, e^{+\gamma z}\right]$$

Note: In the time domain, we only define Z_0 for a lossless line. In the frequency domain, we can define it for a lossy line.

Characteristic Impedance (cont.)

Note the reference directions!



$$\frac{V^{+}(z)}{I^{+}(z)} = Z_0$$

The characteristic impedance is the ratio of the voltage to the current, for a wave traveling in the <u>positive</u> z direction.

Practical note: Even though Z_0 is always complex for a practical line (due to loss), we usually neglect this and take it to be real.

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} \approx \sqrt{\frac{L}{C}}$$

Summary of Solution

Characteristic Impedance

Lossy

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

Lossless:

$$Z_0 = \sqrt{\frac{L}{C}}$$

Voltage and Current

$$V(z) = Ae^{-\gamma z} + B\, e^{+\gamma z}$$

$$I(z) = \left(\frac{1}{Z_0}\right)\left[Ae^{-\gamma z} - B\, e^{+\gamma z}\right]$$

Appendix: Summary of Formulas

General Lossy Case

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}}$$

$$\tan \delta_d \equiv \frac{\sigma_d}{\omega \varepsilon} = \frac{\sigma_d}{\omega \varepsilon_0 \varepsilon_r}$$

$$V(z) = A e^{-\gamma z} + B\, e^{+\gamma z}$$

$$\frac{G}{\omega C} = \tan \delta_d$$

$$I(z) = \left(\frac{1}{Z_0}\right)\left[A e^{-\gamma z} - B\, e^{+\gamma z}\right]$$

$$\beta = \frac{2\pi}{\lambda_g}$$

$$\gamma \equiv \sqrt{(R + j\omega L)(G + j\omega C)}$$

$$v_{\text{phase}} = \frac{\omega}{\beta}$$

$$\gamma = \alpha + j\beta$$

$$\text{Attenuation} = (8.686)\,\alpha \quad [\text{dB/m}]$$

Appendix: Summary of Formulas

Lossless Case

$$Z_0 = \sqrt{\frac{L}{C}}$$

$$V(z) = Ae^{-j\beta z} + B\,e^{+j\beta z}$$

$$I(z) = \left(\frac{1}{Z_0}\right)\left[Ae^{-j\beta z} - B\,e^{+j\beta z}\right]$$

$$\gamma = j\beta$$

$$\beta = \omega\sqrt{LC}$$

$$\lambda_g = \lambda_d$$

$$\beta = \frac{2\pi}{\lambda_d}$$

$$\lambda_d = \frac{\lambda_0}{\sqrt{\varepsilon_r \mu_r}}$$

$$\lambda_0 = \frac{c}{f}$$

$$v_{\text{phase}} = c_d$$

$$c_d = \frac{c}{\sqrt{\varepsilon_r \mu_r}}$$

$$c = 2.99792458 \times 10^8 \;\; [\text{m/s}]$$