ECE 3317
Fall 2023

Homework #1

Assigned: Thursday, Aug. 24
Due: Thursday, Aug. 31

1) Given $c_1 = 2 + j$ and $c_2 = -2 + j3$, calculate the following:

   (a) $c_1 + c_2$
   (b) $c_1 - c_2$
   (c) $c_1 c_2$
   (d) c_1 / c_2

   Give the answers in both rectangular and polar forms.

2) Give answers to the four parts below in rectangular form.

   (a) Given the equation $z^2 = 1 + j$. Find the two values of z that satisfy this equation.

   (b) Calculate (4j)1/2 (there should be two answers here).

   (c) Calculate $\sqrt{4j}$ (there should be a unique answer here).

   (d) Calculate $(1 + 2j)^3$.

3) Let a be a real number. Use a Taylor series for $\sqrt{1+z}$ where z is complex number with a small magnitude, to show that $\sqrt{(1+ja)}$ is approximately equal to $(1 + ja/2)$ for $|a| \ll 1$, and approximately equal to $(1+j)\sqrt{a/2}$ for $|a| \gg 1$. Note that the same Taylor series for a real variable x also works for a complex variable z. Also note that for $|a| \gg 1$ it might be helpful to first factor out a term ja from inside the square root.

4) Give the complex number (F, G, H, E) that corresponds to the phasor representation of the following time-harmonic (sinusoidally varying) functions. Give the complex number in rectangular form.

   Helpful identity: $\cos(x - \pi/2) = \sin x$

   (a) $\mathscr{F}(t) = 3\sin(\omega t - \pi/2)$
   (b) $\mathscr{G}(t) = -2\cos(\omega t)$

(c) $\mathscr{H}(t) = 4\cos(60\pi t - \pi/4)$
(d) $\mathscr{E}(t) = 4\sin\omega t + \cos\omega t$

5) Obtain the sinusoidally-varying function $\mathscr{K}(t)$ corresponding to the following phasors:

(a) $K = 1 - j$
(b) $K = e^{j0.4}$
(c) $K = 2e^{-j\pi/4} + 3e^{j0.3}$

Your answers should be in the standard IEEE format ($A\cos(\omega t + \phi)$). Put the phase angle ϕ in radians.

6) Let $\underline{A} = \underline{\hat{x}} - 2\underline{\hat{y}} + 3\underline{\hat{z}}$ and $\underline{B} = -\underline{\hat{x}} - 2\underline{\hat{y}} + 3\underline{\hat{z}}$

(a) Find $\underline{A} + \underline{B}$
(b) Find $\underline{A} - \underline{B}$
(c) Find $\underline{A} \cdot \underline{B}$
(d) Find $\underline{A} \times \underline{B}$
(e) Find the angle between $\underline{A}$ and $\underline{B}$.

(Note: For part (e), you can only do this because the vectors are real!)

7) Find a real vector that is perpendicular to $\underline{A} = \underline{\hat{x}} + 2\underline{\hat{y}} - 3\underline{\hat{z}}$, has no $\underline{\hat{z}}$ component, has a magnitude equal to 1, and has a positive $\underline{\hat{y}}$ component.

8) Find the real vector $\underline{F}$ that is parallel to $\underline{B} = \underline{\hat{x}} - 2\underline{\hat{y}} + 3\underline{\hat{z}}$ and has a magnitude equal to 1, and also has a positive $\underline{\hat{z}}$ component.

9) Find the complex phasor vector $\underline{A}$ for the following time-harmonic (sinusoidally varying) vectors:

(a) $\underline{\mathscr{A}}(t) = \cos(\omega t)\underline{\hat{x}} - 2\cos(\omega t - \pi/2)\underline{\hat{y}} - 2\sin(\omega t)\underline{\hat{z}}$
(b) $\underline{\mathscr{A}}(t) = (\sin\omega t - 3\sin\omega t)\underline{\hat{y}} + (\sin\omega t - \cos\omega t)\underline{\hat{z}}$
(c) $\underline{\mathscr{A}}(t) = 0.3\sin(ky - \omega t)\underline{\hat{x}}$

10) From the following complex phasor vectors, find the sinusoidal vector $\mathscr{K}(t)$ in terms of ωt:

(a) $\underline{K} = j\underline{\hat{y}} + \underline{\hat{z}}$

(b) $\underline{K} = j\left(\underline{\hat{y}} + j\underline{\hat{z}}\right)$

(c) $\underline{K} = e^{-jky}\,\underline{\hat{x}} - je^{-jky}\,\underline{\hat{z}}$

Each vector should have components that are in the standard IEEE format ($A\cos\left(\omega t + \phi\right)$). Put the phase angle ϕ in radians.

11) Sketch (in the xy plane) the trace of the tip of the time-harmonic vector $\underline{\mathscr{A}}(t)$, where the corresponding complex phasor vector is

(a) $\underline{A} = 3\underline{\hat{x}} + 3\underline{\hat{y}}$

(b) $\underline{A} = 3\underline{\hat{x}} + j3\underline{\hat{y}}$

(c) $\underline{A} = 2\underline{\hat{x}} + j3\underline{\hat{y}}$

Make sure you pick enough values of ωt so that you can be sure what the curve looks like in the xy plane. If you want to try to solve analytically for what the curve looks like that is also fine. (In other words, if you wish, you can eliminate the time variable and get an equation that relates $\mathscr{A}_x$ to $\mathscr{A}_y$).