ECE 3317
Fall 2023

Homework #4

Assigned: Tuesday, Sep. 19
Due: Thursday, Sep. 28

Assigned: Probs. 2, 4–11. (You may do the other problems for practice if you wish, but only turn in the assigned problems).

Note: In all plotting problems, you will be graded on both the *accuracy* and the *quality* of your plots. Your plots should be drawn neatly and to scale. Please use graph paper.

1) Use the formula for the load reflection coefficient to show that

$$-1 \le \Gamma_L \le 1 .$$

The load reflection coefficient formula is

$$\Gamma_L = \frac{R_L - Z_0}{R_L + Z_0} .$$

Hint: Consider both possible cases, $R_L \ge Z_0$ and $R_L \le Z_0$.

2) Assume a 300 [Ω] twin lead is connected directly to a 75 [Ω] coaxial cable without using a matching transformer. What percentage of the power gets reflected back on the twin lead transmission line from the junction? (Note that power is proportional to the square of the voltage. Therefore, the power reflection coefficient is the square of the magnitude of the reflection coefficient.)

3) Consider a wave v^+ that is traveling in the positive z direction, which is incident on a junction with another transmission line. The wave is on transmission line 1 in the region $z < 0$, with characteristic impedance Z_{01}, and to the right of the junction ($z > 0$) is transmission line 2 with characteristic impedance Z_{02}. Assume for simplicity that the incident voltage wave v^+ on line 1 is a step function $u(t-z/c_{d1})$ with a voltage of 1.0 [V]. Solve for the reflected and transmitted voltages and currents on both sides of the junction at $z = 0$, using the known reflection and transmission coefficients. From this, show that the total current is continuous at the junction. Also show that the total power flowing in the z direction at $z = 0$ is the same on either side of the junction. Recall that the voltage transmission coefficient is related to the voltage reflection coefficient by

$$T_J^+ = 1 + \Gamma_J^+ .$$

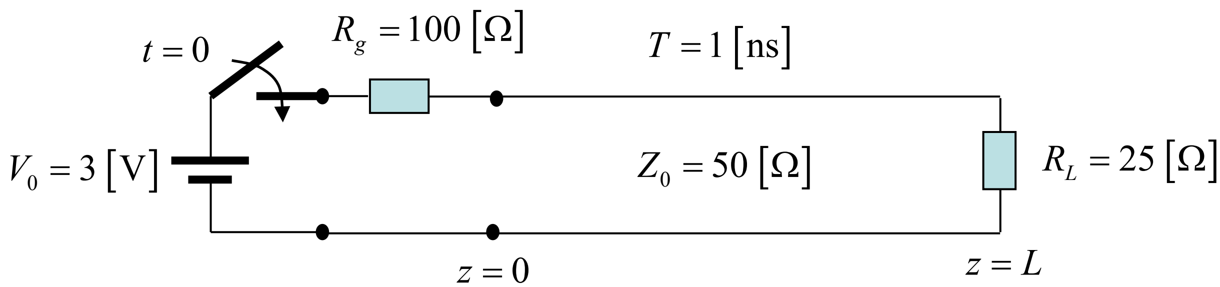
Note that the total voltage and current on the left line (line 1) are found from summing the voltage and current for the incident and reflected waves, while the total voltage and current on the right line (line 2) are those of the transmitted wave.

Also, note that power is given by

$$p(t) = v(t)\,i(t),$$

where v and i are the total voltage and current in the time domain.

4) Consider the problem shown below. Construct a bounce diagram for this problem. Include in your diagram at least three bounces from each end.



5) For the same problem as above, use the bounce diagram to make an accurate oscilloscope trace if the oscilloscope were attached to the input end at $z = 0$. Repeat for $z = L/2$. Plot out to 5 [ns].

6) For the same problem as above, use the bounce diagram to make a snapshot of the voltage on the line at the time $t = 2.25$ [ns]. Put an arrow on your diagram to indicate which way the wave front is traveling.

7) For the same problem as above, now assume that we have a digital pulse of amplitude 3.0 [V] and duration W = 0.5 [ns] applied at the input. Use the bounce diagram to construct an accurate oscilloscope trace for an oscilloscope connected at $z = L/2$. Plot out to 5 [ns].

8) For the same problem as in Prob. 7 (with the digital pulse), use the bounce diagram to make an accurate snapshot of the voltage on the line at $t = 0.75$ [ns]. Repeat for $t = 1.25$ [ns].

9) For the same problem as in Prob. 7 (with the same digital pulse), use the general formula discussed in Notes 7 to describe mathematically the oscilloscope trace for an oscilloscope connected at $z = L/2$. The general formula is

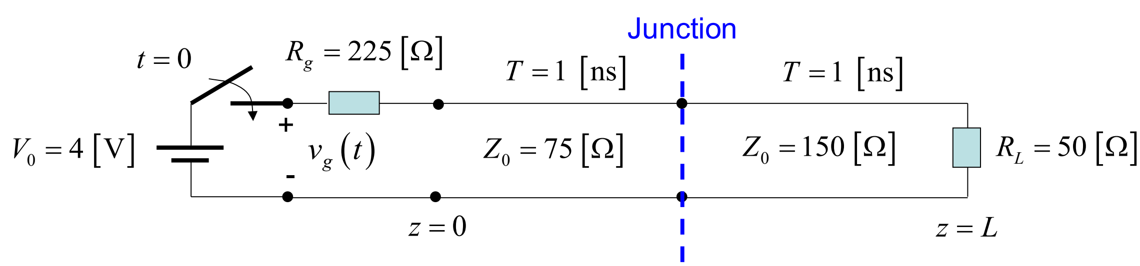
$$
\begin{aligned}
v(z,t) = {} & Av_g\left(t - z/c_d\right) \\
& + \Gamma_L A v_g\left(t - L/c_d - (L-z)/c_d\right) \\
& + \Gamma_g \Gamma_L A v_g\left(t - 2L/c_d - z/c_d\right) \\
& + \Gamma_g \Gamma_L^2\, A v_g\left(t - 3L/c_d - (L-z)/c_d\right) \\
& + \ldots
\end{aligned}
$$

where

$$A = \frac{Z_0}{R_g + Z_0}.$$

Keep four terms in your answer (as shown in the formula above). Make sure that you evaluate all of the constants appearing in your answer (both the constants in front of the v_g functions and the constants inside the arguments of the v_g function). Note that in the above general formula the length L and the velocity c_d appear, and these were not given. However, you do not need to know what these values are because you should be able to figure out what the ratios are, e.g., L/c_d. In this problem the function v_g(t) means the same digital pulse function as in Prob. 7. Give an equation for it in terms of the unit step function $u(t)$.

10) Consider the problem shown below (the same one that was shown in Notes 8), which has a junction of two transmission lines. Use a bounce diagram to make a snapshot of the voltage on the line at t = 1.5 [ns]. Repeat for t = 3.5 [ns]. Put arrows on your diagram to indicate which way the wave fronts are traveling on both lines. Note that your horizontal scale should go from zero to L. (It is not necessary to have a numerical value for L.)



11) Consider the same problem as in Prob. 4 above, but with the 25 [Ω] load resistor replaced by a capacitor of 10 [pF] and the 100 [Ω] source resistor R_g replaced by a 50 [Ω] resistor. Construct a plot of the voltage at z = 0 vs. time. Plot out to 5 [ns].

12) A clock signal generator puts out a periodic square-wave clock signal $v_g(t)$ starting at t = 0, as shown below. This signal travels on a microstrip line that is on a motherboard to reach a device that has an input resistance of 100 [Ω]. The microstrip line has a characteristic impedance of 25 [Ω]. Assume that the effective relative permittivity seen by the microstrip line is 2.25. Assume that the frequency of the clock signal is $f = 1/T_c$ = 2.0 [GHz], where T_c is the period of the clock signal. Also, assume that the length of the microstrip line is 12.5 [cm]. Furthermore, assume that the signal generator has a 50 [Ω] internal Thévenin resistance. Make a plot of what the voltage would look like at a point halfway between the signal

generator and the device, as a function of time. Plot from zero to 2 [nS]. Do this by using the general formula from Notes 7 (see Prob. 9 above).

Hint: First consider how many bounces (i.e., how many terms in Prob. 9) that you need to consider. Calculating the time T (the time it takes a signal to go across the line) will be helpful for this. Is reflection from the generator important here, when you plot out to 2 [nS]?

Compare your answer with what the voltage would look like at the same point halfway down the line if the load resistance was <u>matched</u> to the characteristic impedance of the microstrip line, i.e., the device now has an input resistance of 25 [Ω].